Fluid in motion

The behavior of fluid in motion is very complex due to the large number of variable involved. Since viscosity introduces a great complexity to fluid flow behavior, we assume the fluid is an ideal one, this mean no viscosity and no shear stress as it flow. We also assume incompressible flow which mean that the density of fluid does not change throughout the system.

Properties of flow:

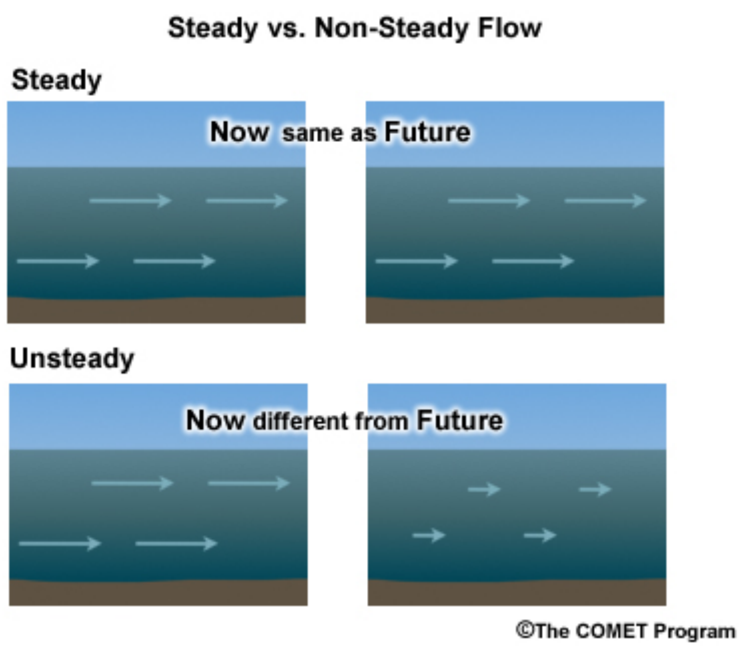
Steady flow: when there is no change in flow properties(Q,v,h) with time.

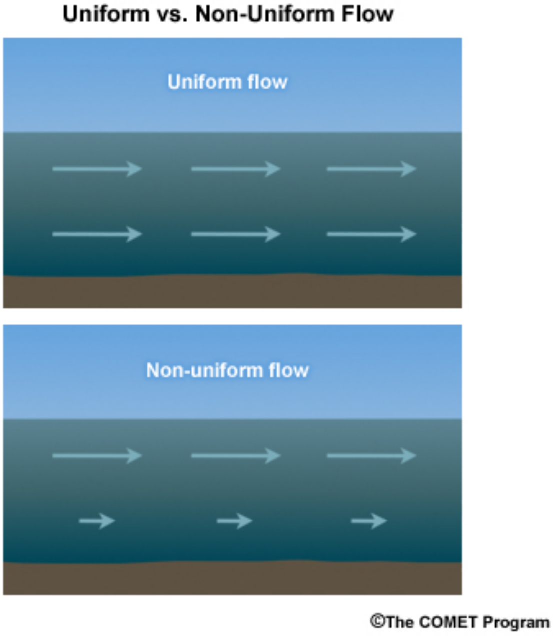
Unsteady flow: when there is change in flow properties with time.

Uniform flow: when there is no change in fluid properties with distance.

Non-uniform flow(2D): flow with two directions (x,y) and velocity distribution become parabola section with maximum value at the center and minimum at the wall of pipe.

Three-dimensions flow(3D): flow with 3 directions (x, y, z) as flow through orifice.





The continuity equation

This theorem state that the mass can neither be created nor destroyed. For steady flow the rate which mass enters a control volume equals the rate at which mass leaves this volume. in fig.1, fluid is flowing from left to right the pipe has two different size (A1 and A2) the volume between 1 and 2 is the control volume (CV), the rate at which the mass enter equals the rate at which the mass leaves CV.

Maas flow rate

From fig.1 let M represent the rate at which mass enter or leave the CV, we have M1=M2, thus for steady flow the mass flow rate at the inlet to the CV equals mass flow rate at the exit from the CV.

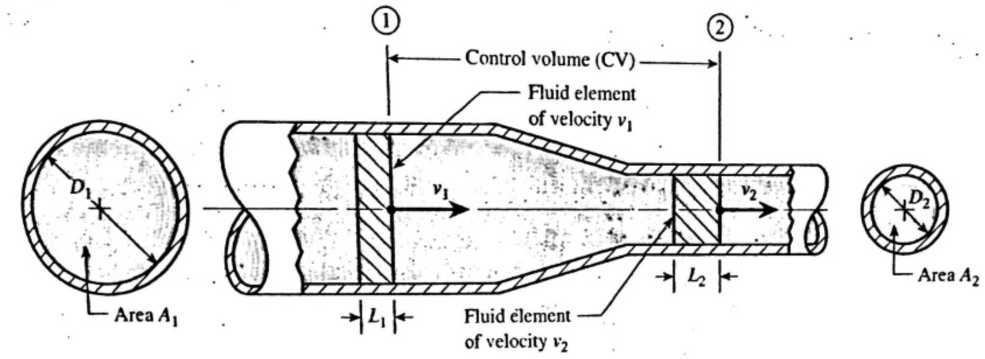


Fig.1

$M_1 = \frac{m_1}{t} = \frac{\rho_1 A_1 L_1}{t} = \rho_1 A_1 v_1$ and

$M_2 = \frac{m_2}{t} = \frac{\rho_2 A_2 L_2}{t} = \rho_2 A_2 v_2$

So $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

Weight flow rate

Since specific weight equals times the acceleration of gravity ($\gamma=\rho*g$)

The continuity equation shows that we also have equality of weight flow rates entering and leaving the CV, W1=W2

$\gamma_1 A_1 v_1 = \gamma_1 A_1 v_1$

volume flow rate

for steady incompressible flow the volume flow rate is also constant represented by Q(flow), because flow =volume per time.

Q1=Q2

$\frac{Vol.1}{t} = \frac{vol.2}{t}$

$\frac{vol_1}{vol_2} = \frac{A_2}{A_1} = (\frac{D_2}{D_1})^2$

D is the diameter for circular section.

Or $Q = v*A$ (v is velocity)

Examples:

Ex1:

For the fig.1 the following data are given: D_1=4 in. D_2=2in. v_1=4ft/s

Find the volume flow rate, fluid velocity at sec. 2, weight flow rate and mass flow rate?

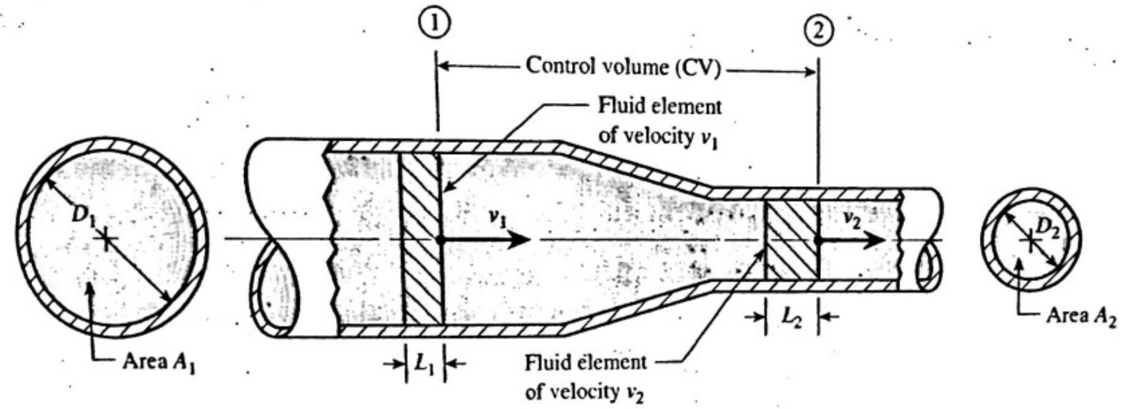


Fig.1

$Q=Q_1=A_1v_1$

$A_1=\frac{\pi}{4}D^2=\frac{\pi}{4}(4/12)^2=0.0873\ ft^2$

$Q = (0.0873)\ (4) = 0.349\ ft^3/s$

b- $v_2=v_1\ (D_1/D_2)^2$

$=4(4/2)^2=16ft/s$

c-$W=W_1=\gamma\ A_1\ v_1=\gamma\ Q_1=62.4*0.349=21.8$ lb/s

d-$M=M_1=\rho A_1v_1=\rho Q_1=1.94*0.349=0.677$ slug/s

Ex2: A pipe line of 300 mm diameter carrying water at an average velocity 4.5 m/s branches into two pipes of 150 mm, 200mm dia., if the average velocity in the 150mm is 5/8 of the velocity in the main pipe, find the average velocity of flow in 200 mm, and the total flow rate in the system by l/s?

Q=AV=Q1+Q2

AV=A1VA+A2V2

$\frac{1}{4}\pi(0.3)^2*4.5=\frac{1}{4}\pi(0.15)^2*\frac{5}{8}*4.5+\frac{1}{4}\pi(0.2)^2*V_2$

V_2=8.54 m/s

Total flow $Q=\frac{1}{4}\pi\ (0.3)^2*4.5$

=0.318 m^3/s

=318 l/s

Ex3:

The velocity of a liquid (s.g)=1.4, in 150mm pipeline is 0.8 m/s. Calculate the rate of flow in l/s , m^3/s, kg/s and KN/s?

$Q=VA=0.8(\frac{\pi}{4})(\frac{150}{1000})^2=0.01414\ m^3/s= 14.141\ l/s$

$M=\rho*Q= (1.4*1000) * 0.01414=19.79$ kg/s

$W=\gamma*Q= (1.4*9.8) * 0.01414=0.194$ KN/s

Example 4: (fig. 2) Two stream of water enter the mixing chamber if the inlet velocity 80 kg/sec. and 100 kg/sec. find the outlet mass flow? And outlet weight flow?

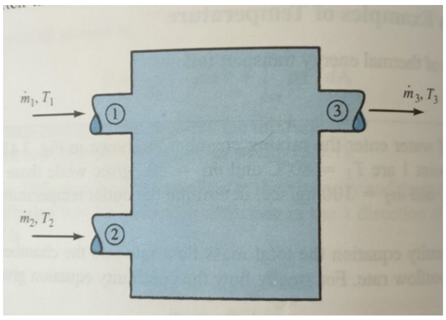


Fig. 2

Total mass flow rate= m_1+m_2

80 +100 =180 kg/sec.

W = mass * g

180 * g =180 * 9.8 = 1764 N/sec

Bernoullis equation

Bernoullis equation is based on the conservation of energy law, which states that energy can be neither created nor destroyed. The total energy possessed by a given mass of fluid can be considered to consist of three type: potential, kinetic and flow energy.

Fig.3 shows fluid flowing from left to right, the total energy by a given weight w of fluid entering CV at sta.1 and the same weight of fluid leaving CV at sta.2.

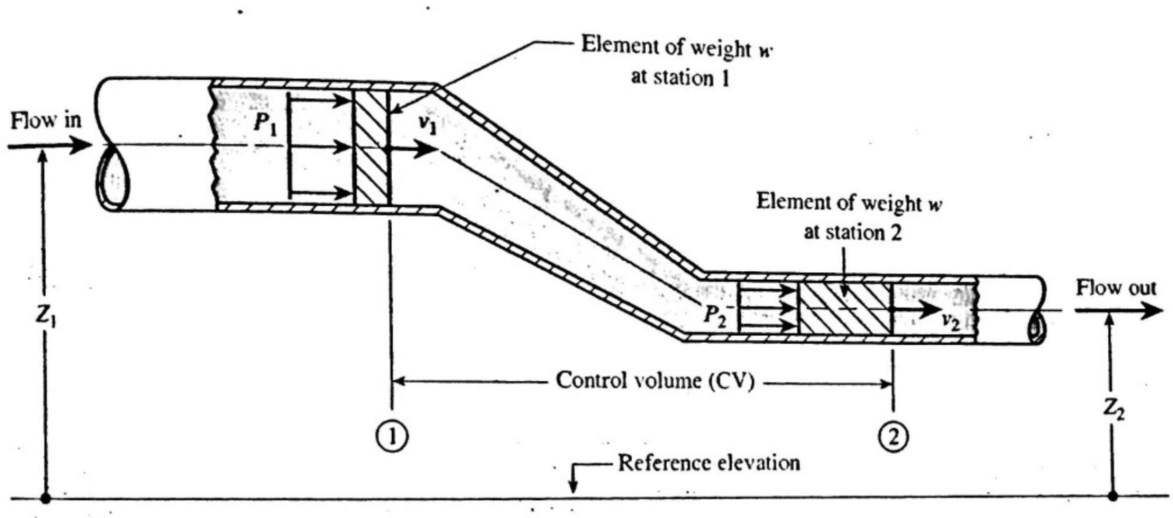


Fig.3

Potential energy: الطاقة الكامنة

The fluid element of weight w has a potential energy due to its elevation Z related to a reference plane.

PE= wz

Kinetic energy: الطاقة الحركية

The fluid element of weight w moving with a velocity

KE=$\frac{1}{2}$ mv^2

Flow energy; طاقه الجريان

It is the amount of work that pressure accomplishes by pushing the element of weight w at sta.1 into the CV or pushing the element of weight w at sta.2 out of CV.

FE= $\frac{Pw}{\gamma}$

Statement of Bernoullis equation:

Daniel Bernoulli an eighteen century Swiss scientist, formulated his equation by noting that the total energy possessed by the fluid in CV does not change with respect to time.

Total energy in element at 1 = total energy in element at 2

(PE+KE+FE)1=(PE+KE+FE)2

$wZ_1+\frac{1}{2}\frac{w}{g}v_1^2+\frac{P_1 w}{\gamma}= wZ_2+\frac{1}{2}\frac{w}{g}v_2^2+\frac{P_2}{\gamma}w$ divide by w

$$Z_1+\frac{v_1^2}{2g}+\frac{P_1}{\gamma}=Z_2+\frac{v_2^2}{2g}+\frac{P_2}{\gamma}$$

$$\frac{P_1}{\gamma}+\frac{v_1^2}{2g}+Z_1=\frac{P_2}{\gamma}\ \frac{v_2^2}{2g}+Z_2$$

Z: elevation head.

$\frac{P}{\gamma}$: pressure head.

$\frac{v_2^2}{2g}$: *velocity head*

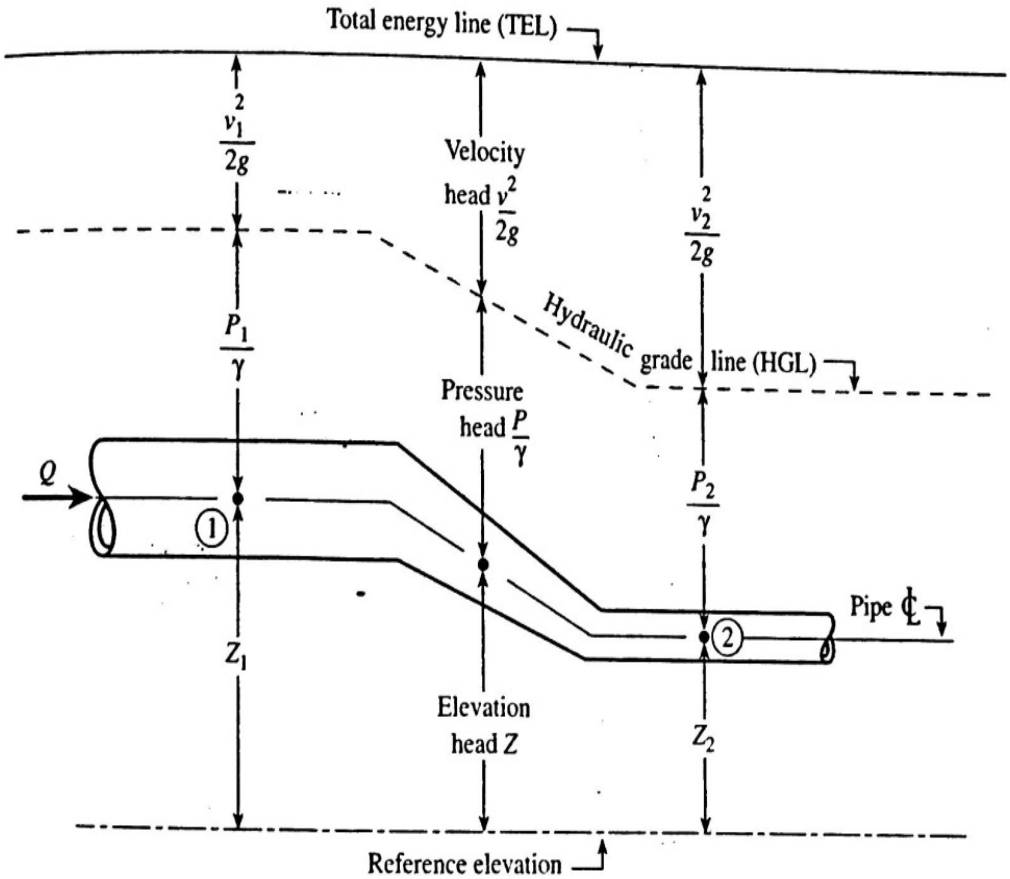


Fig.4

Ex5:For the pipe of fig.5 find P2 if the following data are given:p1= 20 psi, D2 =2 in and D2=1.5 in , Q=200 gpm of water?

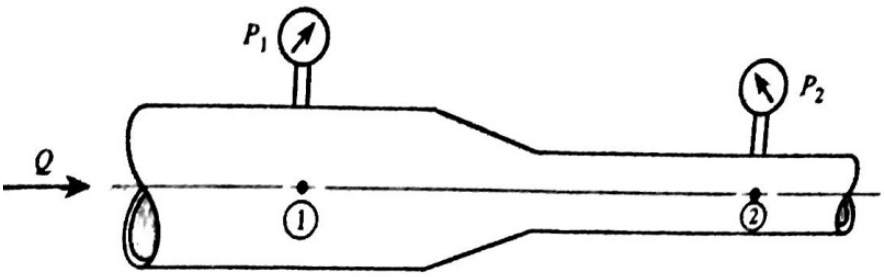


Fig.5

$Z_1 + \frac{v_1^{\,2}}{2g} + \frac{P_1}{\gamma} = Z_2 + \frac{v_2^{\,2}}{2g} + \frac{P_2}{\gamma}$

$V1 = \frac{Q}{A1} = \frac{200*1*231*1}{\frac{\pi}{4}\left(\frac{2}{12}\right)*60*1*1728}$

=20.4 ft/s

$v2 = v1(D1/D2)^2$

$\frac{20*144}{62.4} + \frac{20.4}{2(32.2)} = \frac{P2}{62.4} + \frac{36.2^{\,2}}{2(32.2)}$ +46.2+6.46 = $\frac{P2}{62.4}$ +20.3

Solving for P at 2 yields:

P2= 2020 lb/ft^2 gage=14 psig.

Torricillis theorem:

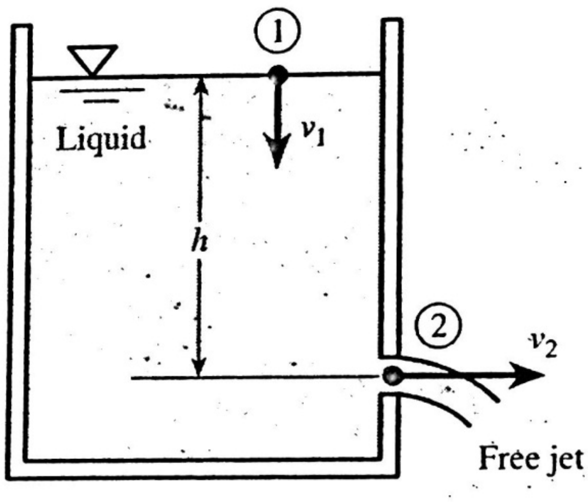


Fig.6

The velocity of free jet of fluid is equal to:

$V = \sqrt{2gh}$

Because:

1-P1=P2 atmosphere pressure

2-v1 negligible cross sec. A1 very large compared with A2 so

v1=$\frac{A2}{A1}$ v2 (very small number)

3-Z2 can be taken as a zero number.

EX6:for the system in fig.6, 198 h=36 ft and diameter of side opening is 2 in find the jet velocity and the volume flow rate in gpm?

V2=$\sqrt{2gh}$

=$\sqrt{2 * 32.2 * 36}$ =48.3 ft/s

Q=A2v2=$\frac{\pi}{4}$ $(2)^2$*48.3*12*$\frac{1}{231\ in^3}$*60

=473gpm

Ex.7

In fig. 7 whoe do the magnitude of the velocity of the three jets compare with each other?

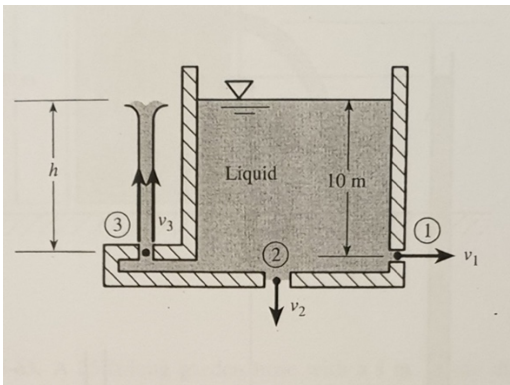


Fig. 7

From the torricillies theorem

$V=\sqrt{2\,g\,h}$

$V1=v2=v3=\sqrt{2*9.8*10} = 14$ m/s

Ex. 8

In fig. 8, a free jet of water going out of tank, find the distance x the jet make contact with ground?

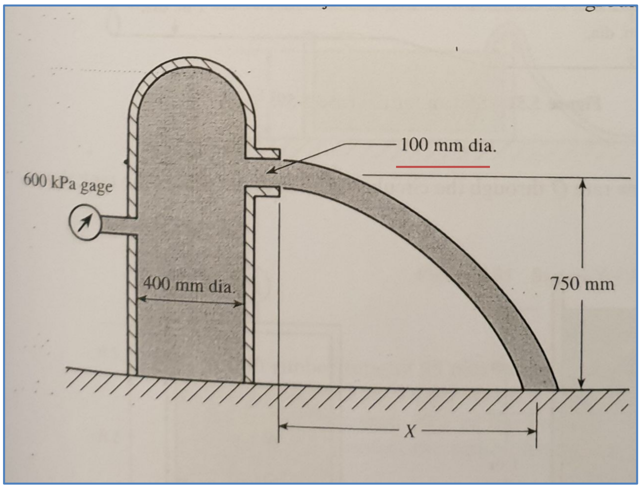


Fig. 8

$$\frac{P_1}{\gamma}+\frac{v_1^{\,2}}{2g}+Z_1=\frac{P_2}{\gamma}\ \frac{v_2^{\,2}}{2g}+Z_2$$

V1 A1 =V2 A2

V1= V2 A2/A1

$$\frac{P_1}{\gamma}+\frac{(V2\ A2/A1)^2}{2g}+Z_1=\frac{P_2}{\gamma}\ \frac{v_2^{\,2}}{2g}+Z_2$$

$$\frac{600*1000}{9800}+\frac{(V2*16)2}{2*9.8}+0=0+\frac{V2^2}{2*9.8}+0$$

$61.22 + 13.06\ V2^2 = 19.6*\ V2^2$

V2= 3.06 m/s

X=V/t t= X/V

$y=\frac{g}{2}t^{\,2}$ $t=\sqrt{(2\,h\ /g)}$

t = t

so $\frac{X}{V} = \sqrt{(2\,h\ /g)}$

$X = V * \sqrt{(2\,h\ /g)}$

$= 3.06\ \sqrt{(2\ 0.75\ /9.8)} = 1.197.\ m$

The siphon:

It is a device that is used to cause a liquid to flow from one container in an upward direction downward in to a second. as shown in fig.6,

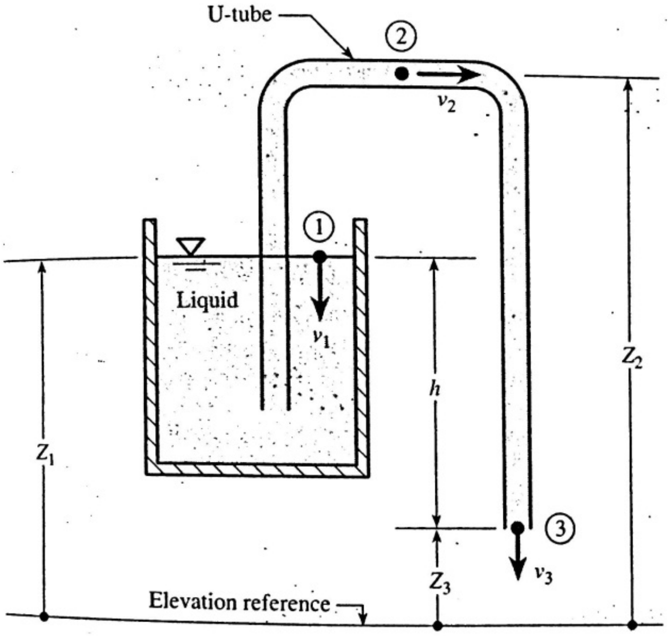


Fig.9

Point 1 lies in the free surface in the container.

Point 2 lies in the U-tube at its highest elevation.

Point 3 lies in the U-tube at the lowest elevation

The output at 3 is a free jet.

If we apply Bernoullis eq. for pointe 1 &3

P1=P3, $v1\approx 0$, Z1-Z3=h, so

V3=$\sqrt{2gh}$

So Q3 =A3v3

P2=γ(Z1-Z2)+γ($-v^2_2$/2g)

EX9:

Water is siphoned from a large storage tank through 50 mm diameter hose(fig.10). Find the maximum height H of a building over which the water can be siphoned?

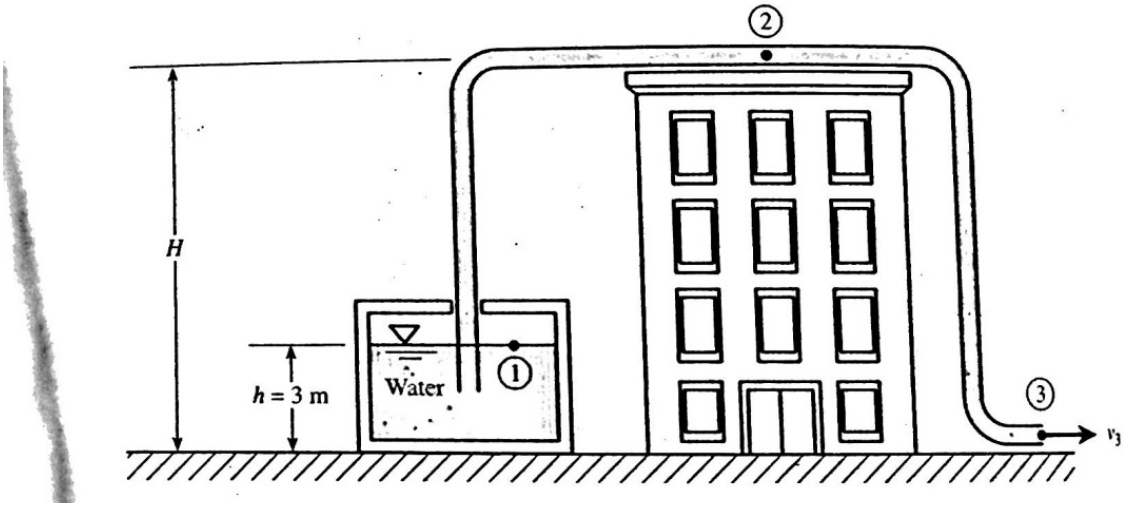


Fig.10

V3=$\sqrt{2gh}$

V3=$\sqrt{2*9.8*3}$ =7.67 m/s

V2=v3

P2=γ (Z1-Z2)-γ (v^2_2/2g)

2.34-101) KPa =9800 N/m^3(3-H)-9800*$\frac{7.67}{2*9.81}$

-98700=29400 – 9800 H -29380

H=10.1 m

Ex.10

The siphon in fig.11 is filled with water and discharged at 150 L/s, find the velocity in pipe, and the pressure at point 2?

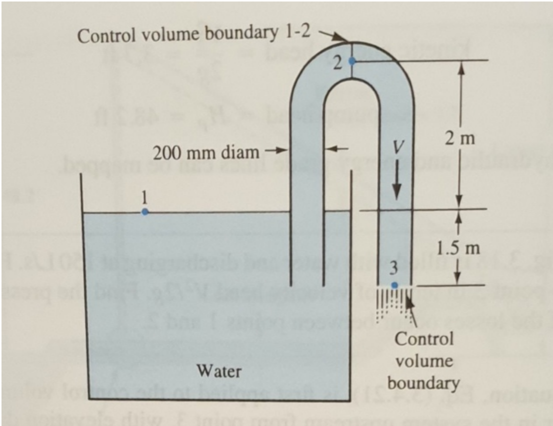


Fig.11

Q=V*A

V= .15/ $0.2^2 * \frac{\pi}{4}$ = 4.777 m/s

$$\frac{P_1}{\gamma} + \frac{v_1{}^2}{2g} + Z_1 = \frac{P_2}{\gamma} \ \frac{v_2{}^2}{2g} + Z_2$$

$$0+0+0 = \frac{P_2}{\gamma} + \frac{4.77^2}{2*9.8} + 2$$

P= 31010.03 Pa.

Ex11: Water is flowing upward through the pipeline (fig.12) ,a manometer measures the pressure difference p1-p1. Find the volume of flow rate?

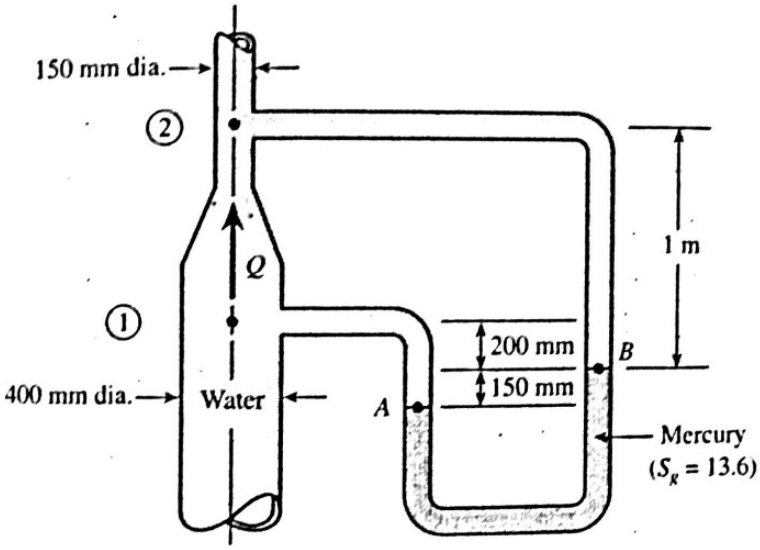


Fig.12

$P_B=P_2+9800*1=P_2+9800$

Pa=Pb+9800*13.6*0.15

Pa=Pb +20000P=P2+29800

P1=Pa-3430=P2+ 26370

P1-P2=26370 pa

$$Z_1+\frac{v_1^{\ 2}}{2g}+\frac{P_1}{\gamma}=Z_2+\frac{v_2^{\ 2}}{2g}+\frac{P_2}{\gamma}$$

A1v1=A2v2

V2=A1v1/A2

$=(\frac{400}{150})^{2}*v1=7.11\ v1$

$\frac{v_2^{\ 2}-v_1^{\ 2}}{2g}$=(Z1-Z2)+(P1-P2)/γ

$\frac{(7.11v_1)^{2}-v_1^{\ 2}}{2*9.81}=-(1-0.2)+\frac{26370}{9800}$

$\frac{49.6v_1^{\ 2}}{19.6}=-0.8+2.69=1.89$ m

V1=0.89 m/s

Q1=A1v1=$\frac{\pi}{4}(0.4)^2$*0.86=0.108 m^3/s

The pitot-static tube:

Fig.9, shows a pitot tube installed in a pipeline along with piezometer, both are pressure measuring device. Unlike, piezometer, pitot continues inward toward the centerline of the pipe. As a result the fluid that inters the inside of the pitot tube comes to a stop (stagnates), in contrast the piezometer tube allows the fluid to travel past without any change of velocity.

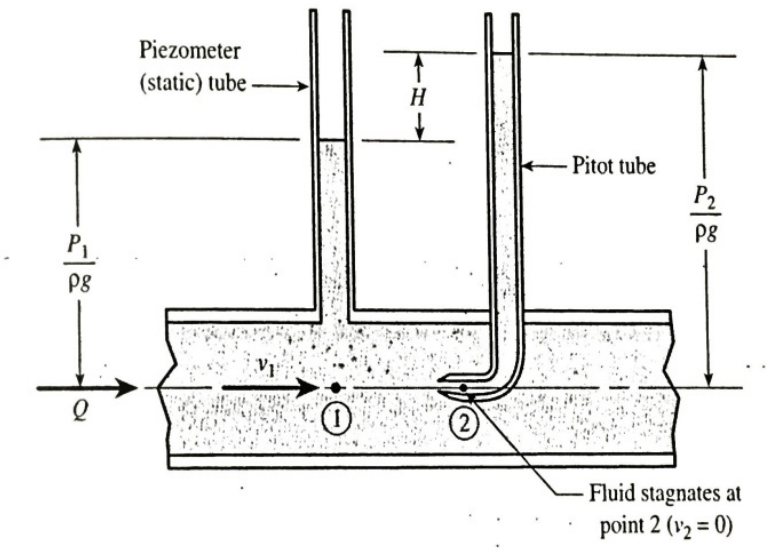


Fig.13

$$Z_1+\frac{v_1^{\ 2}}{2g}+\frac{P_1}{\gamma}=Z_2+\frac{v_2^{\ 2}}{2g}+\frac{P_2}{\gamma}$$

Z1=Z2 and v2=0 (stagnation point)

$$H=\frac{v_1^{\ 2}}{2g}$$

Ex 12:

Determine the flow rate through the pipe?

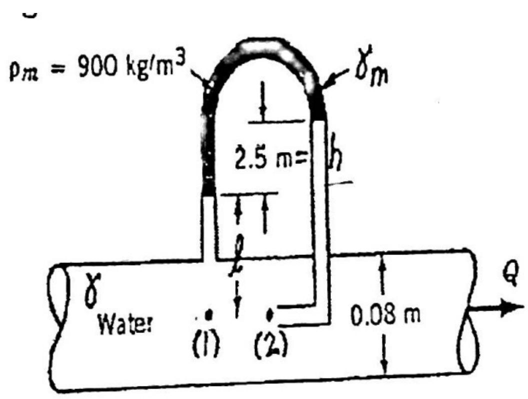


Fig. 14

Z1=Z2 v2=0

$$\frac{v_1{}^2}{2g} + \frac{P_1}{\gamma} = \frac{P_2}{\gamma}$$

$$V1=\sqrt{\frac{2g\ (P2-P1)}{\gamma}}$$

P1-γ L-γ $_m$h+γ(L+h)=P2

$$\text{So v1}=\sqrt{\frac{2g\left(1-\frac{\gamma m}{\gamma}\right)h}{}}$$

$=(2(9.81)(1\text{-}900/999)(2.5))^{0.5}$

=2.2m/s

Q=A1V1=$\frac{\pi}{4}(0.08)^2$ (2.2)=0.0111 m^3/s.

Example 13:

In the pipe in figure 15, a water manometer is connected to ends of a piezometer tube and pitot tube, find the water velocity and the flow ?

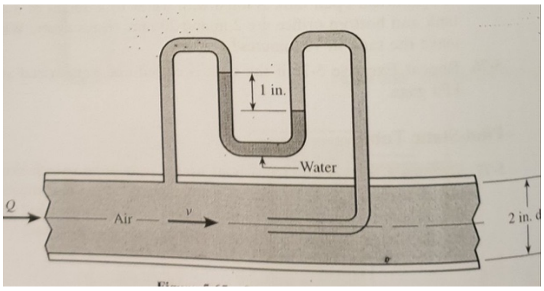


Fig.15

$$\frac{P_1}{\gamma} + \frac{v_1{}^2}{2g} + Z_1 = \frac{P_2}{\gamma} \frac{v_2{}^2}{2g} + Z_2$$

From manometer

P1 – γw h + γhg* h - γw γ* h + γw h = P 2

P1 - γw h +13.6 *62.4 *(1/12) – 62.4 (1/12) + γw *h =P2

P2-P 1= 65.52 ib/ ft^2

From birnoulis eq.

$$\frac{65.52}{62.3} = \frac{v1^2}{2*32.2}$$

V = 8.22 ft/s

Q= V* A

$$8.22 * \frac{2^2}{12^2}\frac{\pi}{4} = 1.0716\ ft^3/s$$

Verturi, nozzle and orifice flow meters:

Three additional used flow meters measuring that operate on the principles of Bernoullis eq., venture, nozzle and orifice flow meters. They based on the principle that as the velocity of flow increases, a drop in pressure occurred. The measurement of the pressure drop can be used to indicate the flow rate.

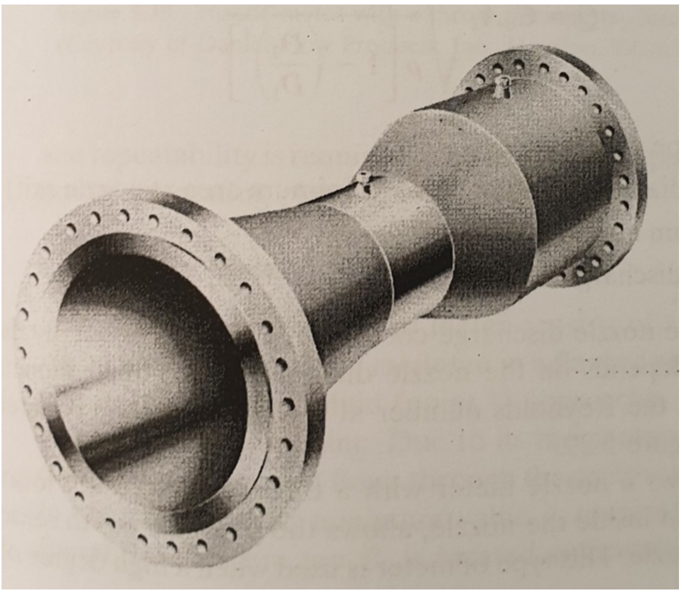


Fig. 16

Figs16, 7, show an actual venturi meter and a venturi meter is installed to the pipeline whose flow rate is measured, venture meter consists of a converging section followed by a constant diameter section (throat) followed by a diverging section.

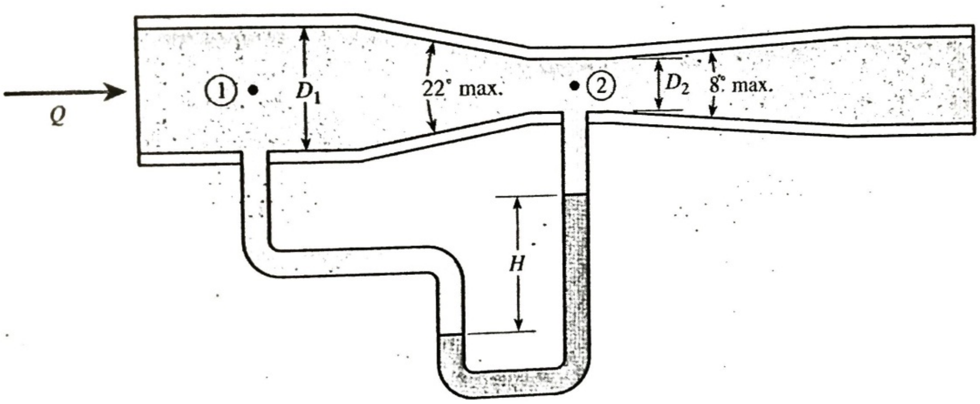


Fig. 17

$$Z_1+\frac{v_1^{\,2}}{2g}+\frac{P_1}{\gamma}=Z_2+\frac{v_2^{\,2}}{2g}+\frac{P_2}{\gamma}$$

Z1=Z2

A1v1=A2v2

So $Cv\ ideal = \sqrt{\frac{2(P1-P2)}{\rho(\ 1-\left(\frac{A2}{A1}\right)^{2}\ )}}$

To calculate the flow coefficient Cv=$\frac{V2\ actual}{V2\ ideal}$

Q=v2 actual A2= Cv ideal A2

$$Q=Cv\ A2\sqrt{\frac{2(P1-P2)}{\rho(\ 1-\left(\frac{A2}{A1}\right)^{2}\ )}}= Cv\ A2\sqrt{\frac{2(P1-P2)}{\rho(\ 1-\left(\frac{D2}{D1}\right)^{2}\ )}}$$

Fig.18, shows the nozzle meter contain a nozzle has a flange on its upstream face.

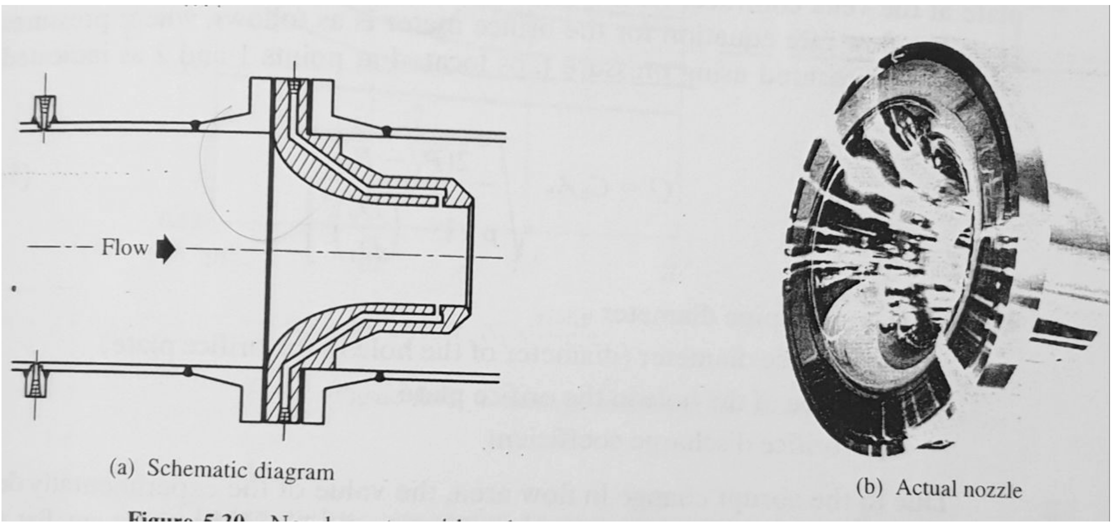
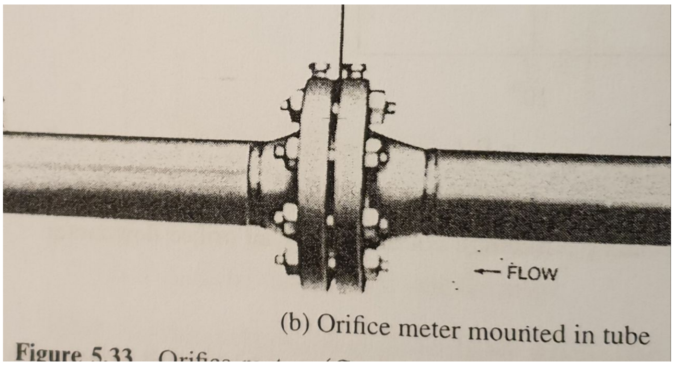


Fig.18

$$Q= Cn\ A2\sqrt{\frac{2(P1-P2)}{\rho(\ 1-\left(\frac{D2}{D1}\right)^{2}\ )}}$$ Cn discharge coefficient.

Fig.19, shows the orifice meter it is a simple flow meter consist of a circular plate containing a sharp edge, the upstream fluid approach the orifice it must turn inward to inter the orifice. The fluid jet cannot immediately change directions as its flow, thus the jet area continues to contract until its minimum value is obtained downstream:



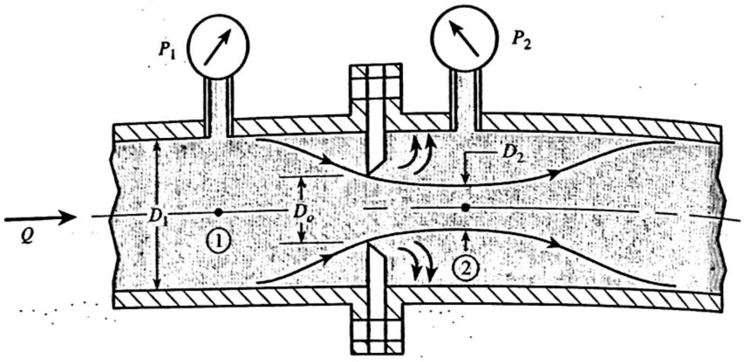


Fig.19

Q= $C0\ A0\sqrt{\frac{2(P1-P2)}{\rho\left(1-\left(\frac{D0}{D1}\right)^{2}\right)}}$ A0: area of the hole in the orifice plate.

Ex 14:

Venture meter of fig.17, has the following data: D1 3in, D20.75 in, Cv 0.98, fluid in manometer is mercury, P2-P1=10 psi, H=20 in Hg, find the flow rate in gallons per minute ?

$$Q=Cv\ A2\sqrt{\frac{2(P1-P2)}{\rho(\ 1-\left(\frac{A2}{A1}\right)^{2})}}$$

$A2=\frac{\pi}{4}(\frac{0.75}{12})^{2}=0.00307\ ft^2$

ρwater =1.94 slugs/ft^3

P1-P2=10*144=1440 lb/ft^2

$\left(\frac{A2}{A1}\right)^{2}=(\frac{0.75}{3})^{4}=0.00391$

$Q=0.98*0.00307*\sqrt{\frac{2*1440}{1.94(1-0.00391)}}=0.116\ ft^3/s$

$Q=0.116*1728*\frac{1}{231}*60=52.1$ gpm

P2-P1 =H (γ Hg – γ water) = 20*1/12*(847-62.4)=1308 ib/ft^2

$Q \propto \sqrt{P1-P2}$ $\qquad$ $Q=52.1*\sqrt{\frac{1308}{1440}}=49.7$ gpm .

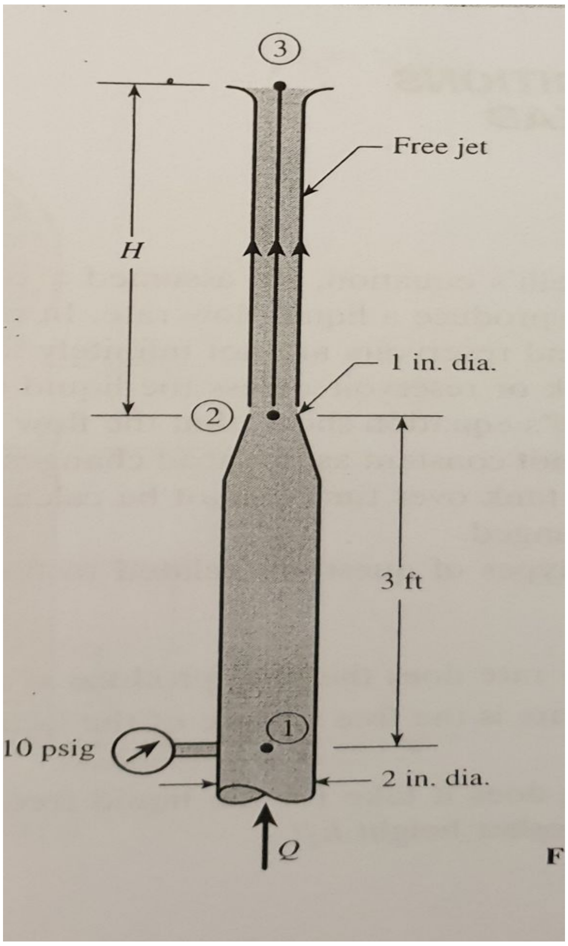
Ex. 15 An orifice flow meter consists of a 100 mm dia. pipe with 50 mm dia. sharp edge orifice. When water flow through orifice a U- tube manometer is connected indicates a differential head of 350 mm mercury, if the discharge coefficient is 0.65, find the flow?

$$Q=C0\ A0\sqrt{\frac{2(P1-P2)}{\rho(\ 1-\left(\frac{D0}{D1}\right)^{2})}}$$

$$Q=0.62*0.0025\ \frac{\pi}{4}\ \frac{\sqrt{2*(0.53*13.6*9800)}}{\sqrt{1000*(1-\frac{0.0025}{0.01})}}$$

Q= 0.0135 m3/s

Extra examplas:



1- For the figure above find Q?

2- Water flow through a venturi meter an inlet dia. = 100 mm and throat dia. =50 mm , P at inlet = 70 kpa. And at the throat 75 mm mercury, if the discharge coefficient = 0.95 , find the flow?