

## Trip Distribution Model

### Introduction

The decision to travel for a given purpose is called trip generation. These generated trips from each zone is then distributed to all other zones based on the choice of destination. This is called trip distribution which forms the second stage of travel demand modeling. There are a number of methods to distribute trips among destinations; and two such methods are growth factor model and gravity model. Growth factor model is a method which respond only to relative growth rates at origins and destinations and this is suitable for short term trend extrapolation. In gravity model, we start from assumptions about trip making behavior and the way it is influenced by external factors. An important aspect of the use of gravity models is their calibration that is the task of fixing their parameters so that the base year travel pattern is well represented by the model.

After all available and relevant information on the number of trips departing or arriving in each zone has been collected, the next step in transport modeling is to distribute these trips over origin-destination (OD) cells. This can be done either at disaggregated or at aggregate level. In disaggregated trip distribution, destination choice proportions are simulated using information on individual characteristics. In aggregate trip distribution individual characteristics are not considered. In this course we will only discuss aggregate trip distribution models.

The point of departure to the aggregate trip distribution step are the margins of the origin-destination (OD) table computed in the trip generation step. This implies estimates are available for either:

- ✚ the number of trip departures,
- ✚ the number of trip arrivals,
- ✚ the number of trip departures and the number of trip arrivals,
- ✚ none.

### Trip matrix

The trip pattern in a study area can be represented by means of a trip matrix or origin-destination (O-D) matrix. This is a two dimensional array of cells where rows and columns represent each of the zones in the study area. The notation of the trip matrix is given in figure 1. The cells of each row  $i$  contain the trips originating in that zone which have as destinations the zones in the corresponding columns.  $T_{ij}$  is the number of trips between origin  $i$  and destination  $j$ .  $O_i$  is the total number of trips between originating in zone  $i$  and  $D_j$  is the total number of trips attracted to zone  $j$ . The sum of the trips in a row should be equal to the total number of trips emanating from that zone. The sum of the trips in a column is the number of trips attracted to that zone. These two constraints can be represented as:

$$\sum_j T_{ij} = O_i$$

$$\sum_i T_{ij} = D_j$$

If reliable information is available to estimate both  $O_i$  and  $D_j$ , the model is said to be doubly constrained. In some cases, there will be information about only one of these constraints, the model is called singly constrained.

| Zones                 | 1        | 2        | ... | $j$      | ... | $n$      | $O_i = \sum_j T_{ij}$  |
|-----------------------|----------|----------|-----|----------|-----|----------|------------------------|
| 1                     | $T_{11}$ | $T_{12}$ | ... | $T_{1j}$ | ... | $T_{1n}$ | $O_1$                  |
| 2                     | $T_{21}$ | $T_{22}$ | ... | $T_{2j}$ | ... | $T_{2n}$ | $O_2$                  |
| ⋮                     | ...      | ...      | ... | ...      | ... | ...      | ⋮                      |
| $i$                   | $T_{i1}$ | $T_{i2}$ | ... | $T_{ij}$ | ... | $T_{in}$ | $O_i$                  |
| ⋮                     | ...      | ...      | ... | ...      | ... | ...      | ⋮                      |
| $n$                   | $T_{n1}$ | $T_{n2}$ | ... | $T_{nj}$ | ... | $T_{nn}$ | $O_n$                  |
| $D_j = \sum_i T_{ij}$ | $D_1$    | $D_2$    | ... | $D_j$    | ... | $D_n$    | $T = \sum_{ij} T_{ij}$ |

**Figure 1:** Notation of a trip matrix.

Two basic categories of aggregate trip distribution methods predominate in urban transportation planning:

✚ The first basic category of aggregate trip distribution methods is based on the gravity model. For gravity models, typical inputs include one or more flow matrices, an impedance matrix reflecting the distance, time, or cost of travel between zones, and estimates of future levels of productions and attractions. The gravity model explicitly relates flows between zones to interzonal impedance to travel.

The gravity model was originally motivated by the observation that flows decrease as a function of the distance separating zones, just as the gravitational pull between two objects decreases as a function of the distance between the objects. As implemented for planning models, the Newtonian analogy has been replaced with the hypothesis that the trips between zones  $i$  and  $j$  are a function of trips originating in zone  $i$  and the relative attractiveness and/or accessibility of zone  $j$  with respect to all zones.

Modern derivations of the gravity model show that it can be understood as the most likely spatial arrangement of trips given limited information available on zonal origin totals, zonal destination totals, and various supporting assumptions or constraints about mean trip lengths (Ortúzar en Willumsen, 1994).

Many different measures of impedance can be used, such as travel distance, travel time, or travel cost. There are also several potential impedance functions used to describe the relative attractiveness of each zone. Popular choices are the exponential functions typically used in entropy models. As an alternative to impedance functions, one can use a friction factor lookup table (essentially a discrete impedance function) that relates the impedance between zones to the attractiveness between zones.

Prior to applying a gravity model, one has to calibrate the impedance function. Typically, calibration entails an iterative process that computes coefficients such that the gravity model replicates the trip length frequency distribution and matches base year productions, or attractions, or both.

✚ The second basic category of aggregate trip distribution are the growth factor method. These involve scaling an existing matrix (called base matrix) by applying

multiplicative factors (often derived from predicted productions and/or attractions) to matrix cells.

### Generalized cost

One of the factors that influences trip distribution is the relative travel cost between two zones. This cost element may be considered in terms of distance, time or money units. It is often convenient to use a measure combining all the main attributes related to the dis-utility of a journey and this is normally referred to as the generalized cost of travel. This can be represented as

$$c_{ij} = \alpha_1 t_{ij}^v + \alpha_2 t_{ij}^w + \alpha_3 t_{ij}^t + \alpha_4 t_{nij} + \alpha_5 F_{ij} + \alpha_6 \phi_j + \delta \quad 1$$

where  $t_{ij}^v$  is the in-vehicle travel time between i and j,  $t_{ij}^w$  is the walking time to and from stops,  $t_{ij}^t$  is the waiting time at stops,  $F_{ij}$  is the fare charged to travel between i and j,  $\phi_j$  is the parking cost at the destination, and  $\delta$  is a parameter representing comfort and convenience, and  $\alpha_1, \alpha_2, \dots$  are the weights attached to each element of cost function.

### Growth factor methods

As an alternative to trip distribution models, growth factor models may be used. In this approach a base year matrix is needed. Each cell of this matrix is multiplied by a growth factor. Growth factors may be computed in a number of ways, e.g. as the output of an economic model, a trend model, etc. The base year matrix contains an estimate of the trips being made in the base year. Theoretically it is possible to directly observe a base year matrix using a travel survey. However, if the study area is decomposed into many zones, and the travel survey represents only a part of all travel, directly observing a base year matrix would result in a matrix mainly consisting of zero cells.

## 1. Uniform growth factor

If the only information available is about a general growth rate for the whole of the study area, then we can only assume that it will apply to each cell in the matrix that is a uniform growth rate. The equation can be written as:

$$T_{ij} = f \times t_{ij} \quad 2$$

Where

$f$  is the uniform growth factor  $t_{ij}$  is the previous total number of trips,

$T_{ij}$  is the expanded total number of trips.

Advantages are that they are simple to understand, and they are useful for short-term planning. Limitation is that the same growth factor is assumed for all zones as well as attractions.

## 2. Doubly constrained growth factor model

When information is available on the growth in the number of trips originating and terminating in each zone, we know that there will be different growth rates for trips in and out of each zone and consequently having two sets of growth factors for each zone. This implies that there are two constraints for that model and such a model is called doubly constrained growth factor model. One of the methods of solving such a model is given by Furness who introduced balancing factors  $a_i$  and  $b_j$  as follows:

$$T_{ij} = t_{ij} \times a_i \times b_j \quad 3$$

In such cases, a set of intermediate correction coefficients are calculated which are then appropriately applied to cell entries in each row or column. After applying these corrections to say each row, totals for each column are calculated and compared with the target values. If the differences are significant, correction coefficients are calculated and applied as necessary. The procedure is given below:

1. Set  $b_j = 1$
2. With  $b_j$  solve for  $a_i$  to satisfy trip generation constraint.
3. With  $a_i$  solve for  $b_j$  to satisfy trip attraction constraint.
4. Update matrix and check for errors. 5. Repeat steps 2 and 3 till convergence.

Here the error is calculated as:  $E = \sum |O_i - O_i^1| + \sum |D_j - D_j^1|$  where  $O_i$  corresponds to the actual productions from zone  $i$  and  $O_i^1$  is the calculated productions from that zone. Similarly  $D_j$  are the actual attractions from the zone  $j$  and  $D_j^1$  are the calculated attractions from that zone.

### Advantages and limitations of growth factor model

The advantages of this method are:

1. Simple to understand.
2. Preserve observed trip pattern.
3. useful in short term-planning.

The limitations are:

1. Depends heavily on the observed trip pattern.
2. It cannot explain unobserved trips.
3. Do not consider changes in travel cost.
4. Not suitable for policy studies like introduction of a mode.

### Gravity model

This model originally generated from an analogy with Newton's gravitational law. Newton's gravitational law says,

$F = GM_1M_2/d_2$  Analogous to this,

$$T_{ij} = CO_i D_j / c_{ij}^n \quad 4$$

Introducing some balancing factors,

$$T_{ij} = A_i O_i B_j D_j f(c_{ij}) \quad 5$$

Where

$A_i$  and  $B_j$  are the balancing factors,

$f(c_{ij})$  is the generalized function of the travel cost.

This function is called deterrence function because it represents the disincentive to travel as distance (time) or cost increases. Some of the versions of this function are:

$$f(c_{ij}) = e^{-\beta c_{ij}} \quad 6$$

$$f(c_{ij}) = c_{ij}^{-n} \quad 7$$

$$f(c_{ij}) = c_{ij}^{-n} \times e^{-\beta c_{ij}} \quad 8$$

The first equation is called the exponential function, second one is called power function whereas the third one is a combination of exponential and power function.

As in the growth factor model, here also we have singly and doubly constrained models. The expression

$T_{ij} = A_i O_i B_j D_j f(c_{ij})$  is the classical version of the doubly constrained model. Singly constrained versions can be produced by making one set of balancing factors  $A_i$  or  $B_j$  equal to one. Therefore we can treat singly constrained model as a special case which can be derived from doubly constrained models. Hence we will limit our discussion to doubly constrained models.

As seen earlier, the model has the functional form,  $T_{ij} = A_i O_i B_j D_j f(c_{ij})$

$$\sum_i T_{ij} = \sum_i A_i O_i B_j D_j f(c_{ij}) \quad 9$$

But

$$\sum_i T_{ij} = D_j \quad 10$$

Therefore,

$$D_j = B_j D_j \sum_i A_i O_i f(c_{ij}) \quad 11$$

From this we can find the balancing factor  $B_j$  as:

$$B_j = 1 / \sum_i A_i O_i f(c_{ij}) \quad 12$$

$B_j$  depends on  $A_i$  which can be found out by the following equation:

$$A_i = 1 / \sum_j B_j D_j f(c_{ij}) \quad 13$$

We can see that both  $A_i$  and  $B_j$  are interdependent. Therefore, through some iteration procedure similar to that of Furness method, the problem can be solved. The procedure is discussed below:

1. Set  $B_j = 1$ , find  $A_i$  using equation 1.11
2. Find  $B_j$
3. Compute the error as  $E = \sum |O_i - O_i^1| + \sum |D_j - D_j^1|$  where  $O_i$  corresponds to the actual productions from zone  $i$  and  $O_i^1$  is the calculated productions from that zone. Similarly  $D_j$  are the actual attractions from the zone  $j$  and  $D_j^1$  are the calculated attractions from that zone.
4. Again set  $B_j = 1$  and find  $A_i$ , also find  $B_j$ .

Repeat these steps until the convergence is achieved.

**Example** The productions from zone 1, 2 and 3 are 98, 106, 122 and attractions to zone 1, 2 and 3 are 102, 118, 106. The function  $f(c_{ij})$  is defined as  $f(c_{ij}) = 1/c_{ij}^2$  The cost matrix is as shown below:

$$\begin{bmatrix} 1.0 & 1.2 & 1.8 \\ 1.2 & 1.0 & 1.5 \\ 1.8 & 1.5 & 1.0 \end{bmatrix}$$

Solution:

The second step is to find  $B_j$ . This can be found out as  $B_j = 1/\sum A_i O_i f(c_{ij})$ , where  $A_i$  is obtained from the previous step.

The function  $f(c_{ij})$  can be written in the matrix form as:

$$\begin{bmatrix} 1.0 & 0.69 & 0.31 \\ 0.69 & 1.0 & 0.44 \\ 0.31 & 0.44 & 1.0 \end{bmatrix}$$

Then  $T_{ij}$  can be computed using the formula:

$$T_{ij} = A_i O_i B_j D_j f(c_{ij})$$

For eg,  $T_{11} = 102 \times 1.0397 \times 0.00462 \times 98 \times 1 = 48.01$ .

$O_i$  is the actual productions from the zone and  $O_i^1$  is the computed ones. Similar is the case with attractions also.



| $j$ | $i$ | $A_i$   | $O_i$ | $f(c_{ij})$ | $A_i O_i f(c_{ij})$ | $\Sigma A_i O_i f(c_{ij})$ | $B_j = 1/\Sigma A_i O_i f(c_{ij})$ |
|-----|-----|---------|-------|-------------|---------------------|----------------------------|------------------------------------|
| 1   | 1   | 0.00462 | 98    | 1.0         | 0.4523              | 0.9618                     | 1.0397                             |
|     | 2   | 0.00425 | 106   | 0.694       | 0.3117              |                            |                                    |
|     | 3   | 0.00527 | 122   | 0.308       | 0.1978              |                            |                                    |
| 2   | 1   | 0.00462 | 98    | 0.69        | 0.3124              | 1.0458                     | 0.9562                             |
|     | 2   | 0.00425 | 106   | 1.0         | 0.4505              |                            |                                    |
|     | 3   | 0.00527 | 122   | 0.44        | 0.2829              |                            |                                    |
| 3   | 1   | 0.00462 | 98    | 0.31        | 0.1404              | 0.9815                     | 1.0188                             |
|     | 2   | 0.00425 | 106   | 0.44        | 0.1982              |                            |                                    |
|     | 3   | 0.00527 | 122   | 1.00        | 0.6429              |                            |                                    |

|         | 1      | 2       | 3       | $A_i$   | $O_i$ | $O_i^1$ |
|---------|--------|---------|---------|---------|-------|---------|
| 1       | 48.01  | 35.24   | 15.157  | 0.00462 | 98    | 98.407  |
| 2       | 32.96  | 50.83   | 21.40   | 0.00425 | 106   | 105.19  |
| 3       | 21.14  | 31.919  | 69.43   | 0.00527 | 122   | 122.489 |
| $B_j$   | 1.0397 | 0.9562  | 1.0188  |         |       |         |
| $D_j$   | 102    | 118     | 106     |         |       |         |
| $D_j^1$ | 102.11 | 117.989 | 105.987 |         |       |         |

$O_i$  is the actual productions from the zone and  $O_i^1$  is the computed ones. Similar is the case with attractions also

Therefore error can be computed as;

$$\text{Error} = \Sigma |O_i - O_i^1| + \Sigma |D_j - D_j^1|$$

$$\text{Error} = |98 - 98.407| + |106 - 105.19| + |122 - 122.489| + ||102 - 102.11| + |118 - 117.989| + |106 - 105.987| = 2.03$$