(1) The center of gravity
The center of Mass
The Centroid of the volume
} The Same Point

Centroid is always located on the axis of Symmetry.

The line Could be straight or curved

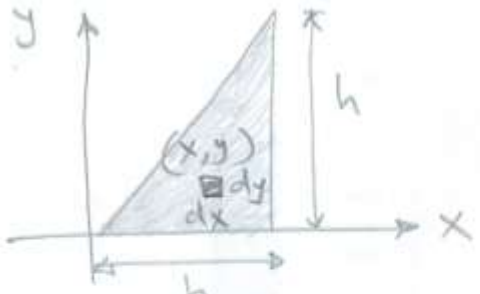
اذا كان الشكل اي متناظر يكون من السهل ايجاد ال Centroid ويكون في مركز هذه الاشكال مثلاً مركز المكعب او مركز الدائرة او مركز المستطيل او مركز الخط المستقيم.

لكن اذا كان الشكل غير متناظر كيف يمكن ايجاد مركزه؟

(2)

$$\bar{x} = \frac{\int x \cdot dA}{\int dA}$$

$$\bar{y} = \frac{\int y \cdot dA}{\int dA}$$



- سوف ناخذ element صغيرة مربعة □ داخل هذا الشكل وساحة هذا الـ element هي dx * dy .
- معادلة الخط المائل والذي يمثل الوتر لهذا المثلث هي ميل الخط المستقيم المائل وهي $y = \frac{h}{b}x$
- الان باستخدام التكامل لنجد قيمة المساحة لهذا الشكل وهي $\int dA$ وتساوي $\int dy \cdot dx$ ويكون هناك تكاملين احدهم باتجاه الـ x والاخر باتجاه الـ y .

لذلك فان الـ limit باتجاه الـ x يكون من صفر الى b والـ limit باتجاه الـ y يكون من صفر الى $\frac{h}{b}x$

(3)

$$\int dA = \int_0^b \int_0^{\frac{h}{b}x} dy \cdot dx$$

$$= \int_0^b |x|_0^{\frac{h}{b}x} \cdot dx$$

$$= \int_0^b \left[\frac{h}{b}x - 0\right] \cdot dx$$

$$= \int_0^b \frac{h}{b}x \cdot dx$$

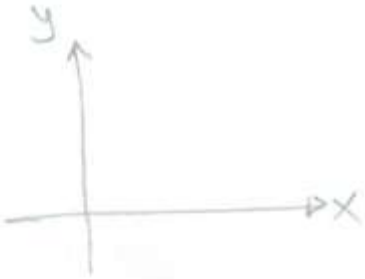
$$= \frac{h}{b} \int_0^b x \cdot dx$$

$$= \frac{h}{b}\left[\frac{x^2}{2}\right]_0^b$$

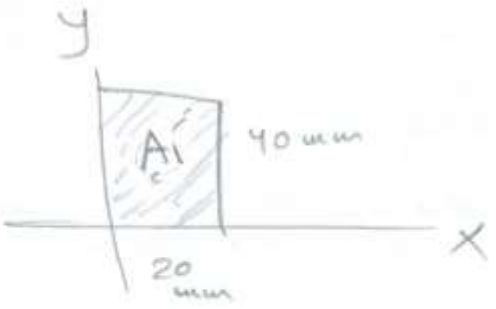
$$= \frac{h}{b}\left[\frac{b^2 - 0^2}{2}\right]$$

$$= \frac{h}{2b} \cdot b^2$$

$$= \frac{1}{2} h \cdot b$$



(4) Centroid of Composite or Complex shape



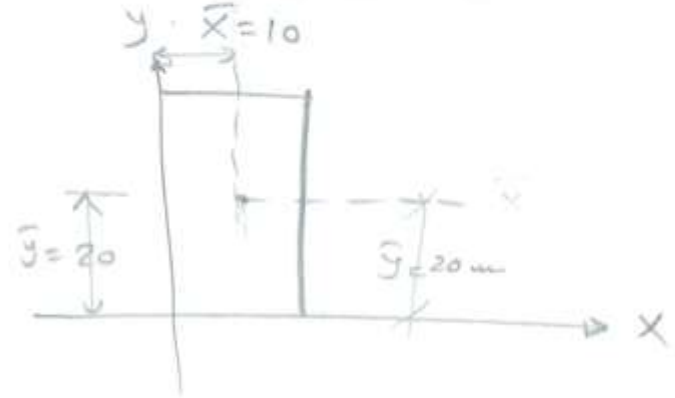
Example

Locate the Centroid of the shape

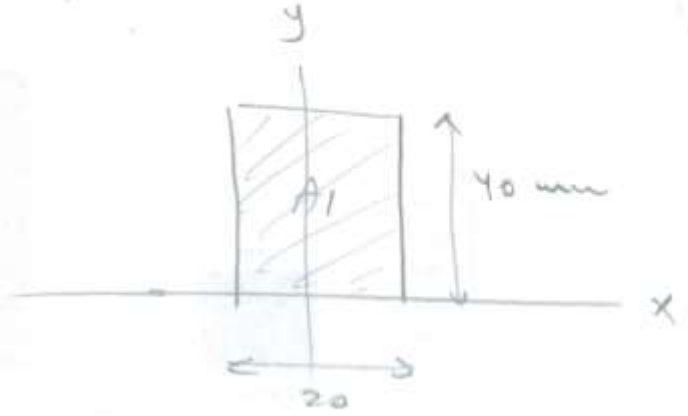
A type	Area (mm^2)	x (mm)	My = Area x x	y	Mx
(1)	40 * 20 = 800	10	800 * 10 = 8000 mm^3	20	800 * 20 = 16000

$$\bar{x} = \frac{\Sigma My}{\Sigma A} = \frac{8000}{800} = 10 \text{ mm}$$

$$\bar{y} = \frac{\Sigma Mx}{\Sigma A} = \frac{16000}{800} = 20 \text{ mm}$$



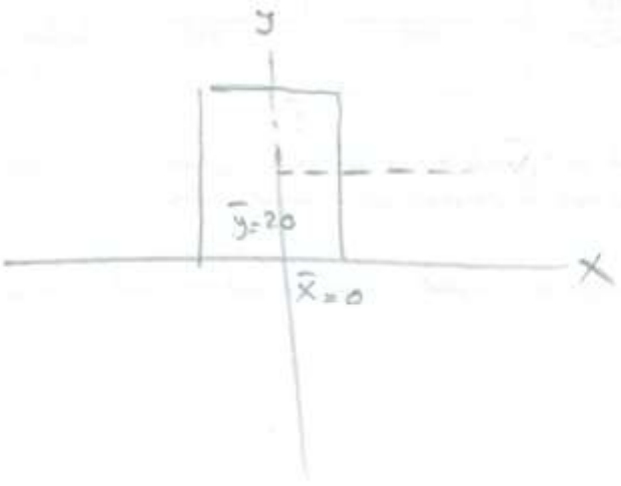
(5) locate the centroid of the shape



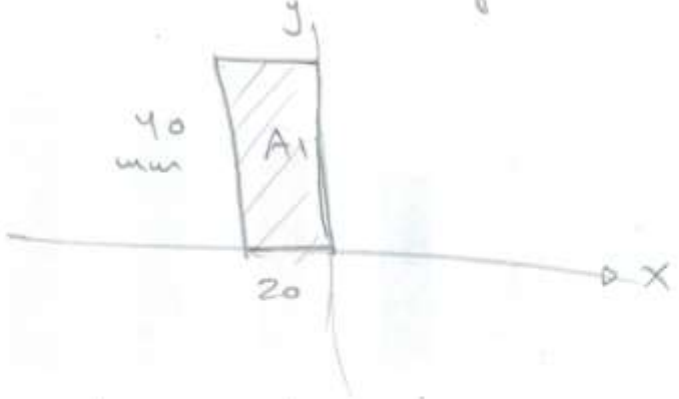
Area #	Area mm^2	(x) mm	My Area * X	(y) mm	MX = Area * Y
A1	40 * 20 = 800	0	800 * 0 = 0	20	800 * 20 = 16000

$$\bar{x} = \frac{\Sigma My}{\Sigma A} = \frac{0}{800} = 0$$

$$\bar{y} = \frac{\Sigma Mx}{\Sigma A} = \frac{16000}{800} = 20 \text{ mm}$$



(6) Locate the centroid of the shape



Area #	Area mm²	(X) mm	My Area × X	Y mm	Mx Area × y
	20×40 = 800	−10	800 × −10 = −8000	20	800×20 = 16000

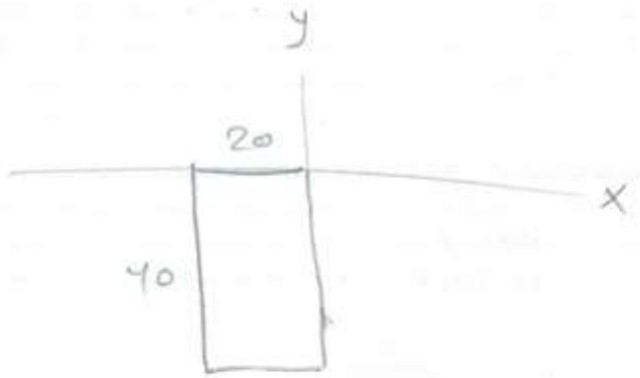
$$\bar{x} = \frac{\Sigma My}{\Sigma A} = \frac{-8000}{800} = -10 \text{ mm}$$

$$\bar{y} = \frac{\Sigma Mx}{\Sigma A} = \frac{16000}{800} = 20 \text{ mm}$$

MCQ :- The Area of shape above is

(1) 8000 m^2
(2) -8000 mm^2
(3) 60 mm^2
(4) All of these result above
(5) Not of These result above

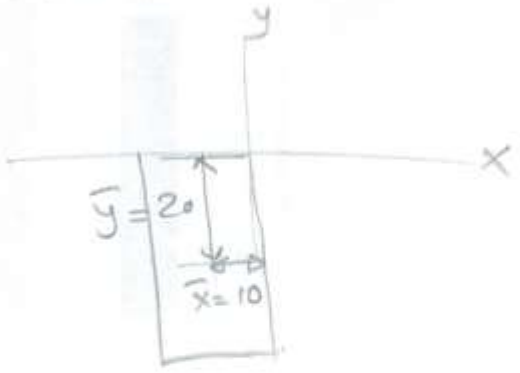
(7) locate the centroid of the shape?



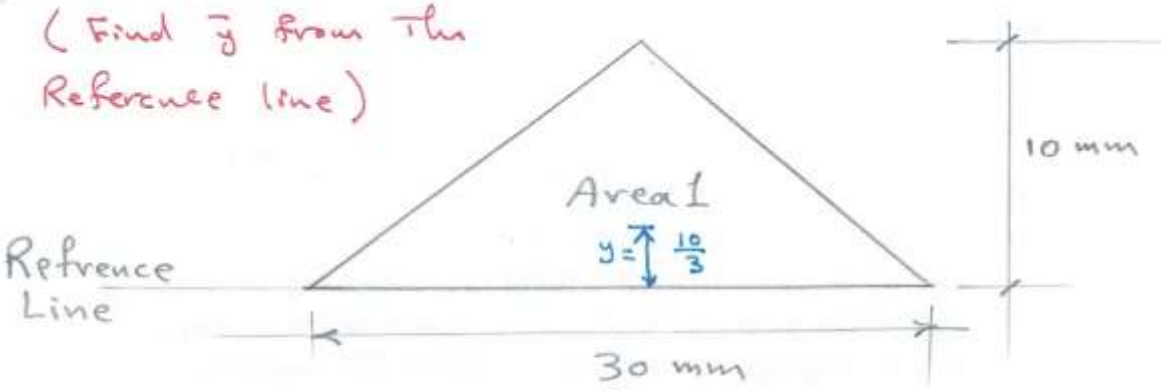
# Area	Area	X (mm)	My Area * X	y (mm)	Mx Area * Y
	20 * 40 = 800	-10	800 * -10 = -8000	-20	800 * -20 = -16000

$$\bar{x} = \frac{\Sigma My}{\Sigma A} = \frac{-8000}{800} = -10 \text{ mm}$$

$$\bar{y} = \frac{\Sigma Mx}{\Sigma A} = \frac{-16000}{800} = -20 \text{ mm}$$



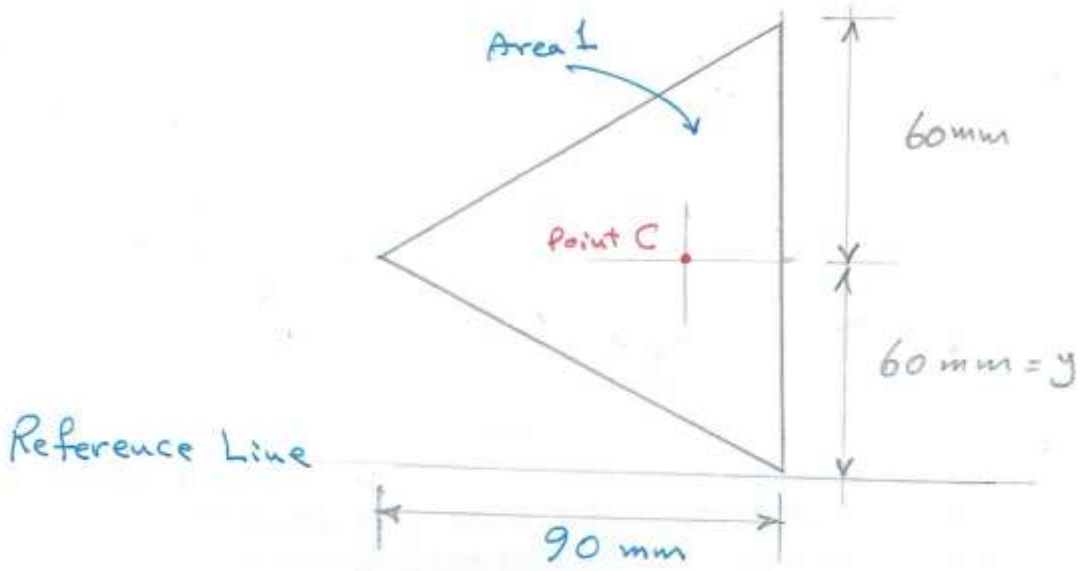
Find the center of gravity of this triangle
(Find $\bar{y}$ from the
Refernce line)



Area No.	Area value (mm^2)	Distance y (mm)	A * y
Area 1	$\frac{1}{2} * 30 * 10$ $= 150\ mm^2$	$\frac{10}{3}$	$150 * \frac{10}{3}$

$$\bar{y} = \frac{\sum A * y}{\sum A} =$$

$$= \frac{150 * \frac{10}{3}}{150} = \frac{10}{3}\ mm$$



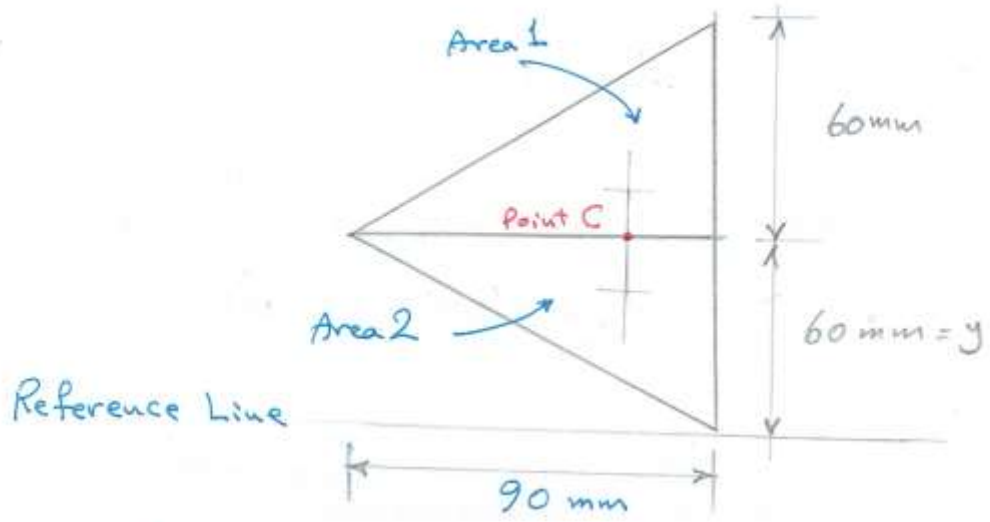
Find The center of gravity of this Traingle
(Find $\bar{y}$ from the Reference Line)

Area No.	Area value (mm^2)	Distance y (mm)	A * y (mm^3)
Area 1	$\frac{1}{2} * 90 * (60+60)$ $= 5400\ mm^2$	60	324000

$$\bar{y} = \frac{\sum A*y}{\sum A} = \frac{324000}{5400} = 60\ mm$$

$\bar{y}$ هي نفس قيمة y

وهي تمثل ال (Center of gravity) لهذا الشكل



Find the center of gravity of this Triangle
(Find $\bar{y}$ from the Reference Line)

Area No.	Area value (mm^2)	Distance y (mm)	A * y (mm^3)
Area 1	$\frac{1}{2} * 90 * (60+60)$ $= 5400\ mm^2$	60	324000

$$\bar{y} = \frac{\sum A*y}{\sum A} = \frac{324000}{5400} = 60\ mm$$

$\bar{y}$ هي نفس قيمة y

وهي تمثل ال (Center of gravity) لهذا الشكل

Area No.	Area value (mm^2)	y	A * y
1	$\frac{1}{2} * 90 * 60 = 2700$	$20 + 60 = 80$	216000
2	$\frac{1}{2} * 90 * 60 = 2700$	40	108000
	5400		324000

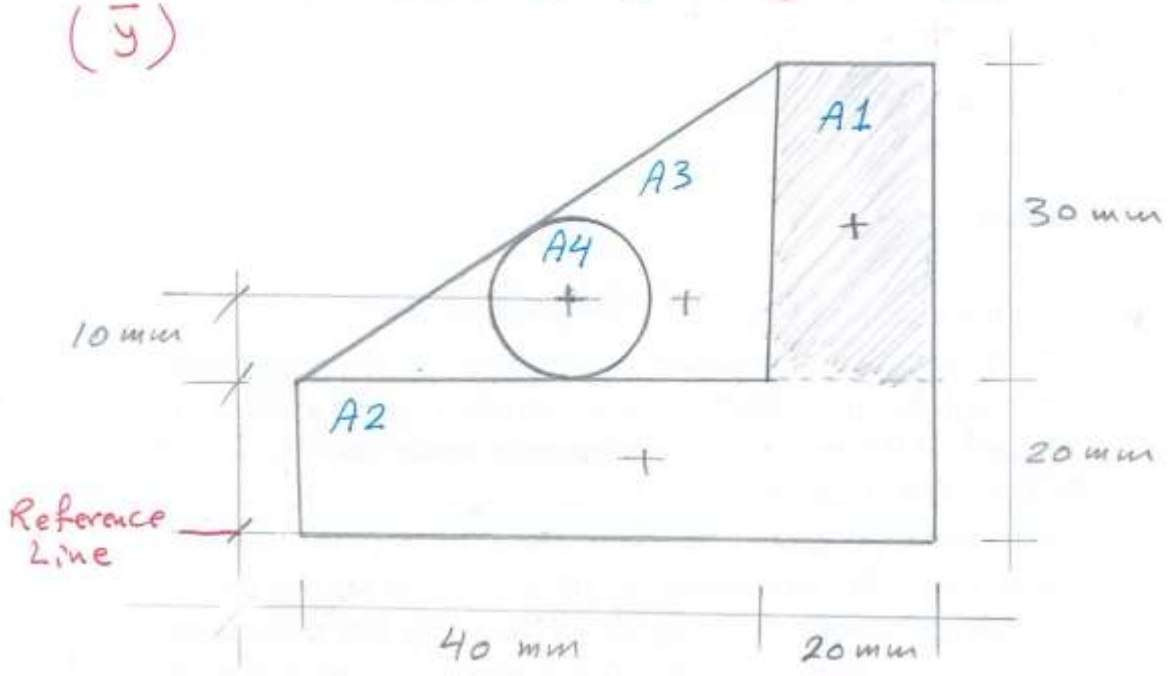
$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{324000}{5400} = 60\ mm$$

نفس الرقم السابق

المطلوب ايجاد مركز الثقل لهذا الشكل المركب

Find the center of gravity (c.g)

$(\bar{y})$



Area No.	Area value (mm^2)	المسافة من مركز ثقل الشكل الى خط المرجع $\bar{y}$	$A*\bar{y}$
A1	$30*20 = 600$	35	21000
A2	$60*20 = 1200$	10	12000
A3	$\frac{1}{2}*40*30 = 600$	30	18000
A4	$-\frac{\pi}{4}(20)^2 = -100\pi$	30	-3000π
	$\sum A = 2085.84$		$\sum A*\bar{y} = 41575.2$

$$\bar{y} = \frac{\sum A*\bar{y}}{\sum A} = \frac{41575.22}{2085.84} = 19.932 \text{ mm}$$

$$\simeq 20 \text{ mm}$$

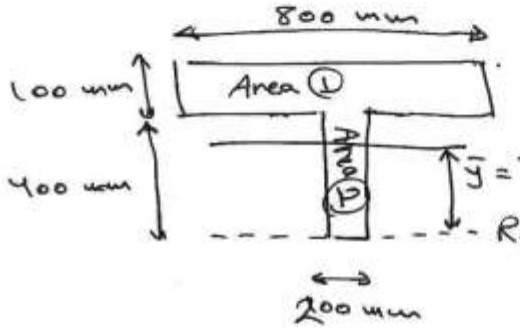
(6) Center of gravity كيفية ايجاد مركز الثقل

اذا كان الجسم غير متناظر حول محور X فنقف ايجاد شكل المقطع مؤثر محور X نختلف عن شكل القطع تحت محور X كان مركز الثقل للجسم لا يكون بالمنتصف لذلك يجب ايجاده عن طريق ايجاد قيمة $\bar{y}$ باستخدام القانون

$$\bar{y} = \frac{\Sigma A * y}{\Sigma A}$$ قانون

مثال :-

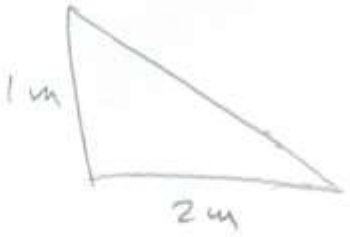
جد قيمة $\bar{y}$ هي مركز ثقل الجسم عن خط المرجع (Reference line)



Area Shape and No.	Area (mm^2)	y العدد بين مركز الجسم وخط المرجع (mm)	A*y	
Area ①	$100*800 = 80*10^3$	$\frac{100}{2}+400 = 450$	$80*10^3 * 450 = 36*10^6$	
Area ②	$400*200 = 80*10^3$	$\frac{400}{2} = 200$	$200*80*10^3 = 16*10^6$	
	$80*10^3 + 80*10^3 = 160*10^3$		$36*10^6 + 16*10^6 = 52*10^6$	Total

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{52*10^6}{160*10^3} = 325 \text{ mm up of the Reference line}$$

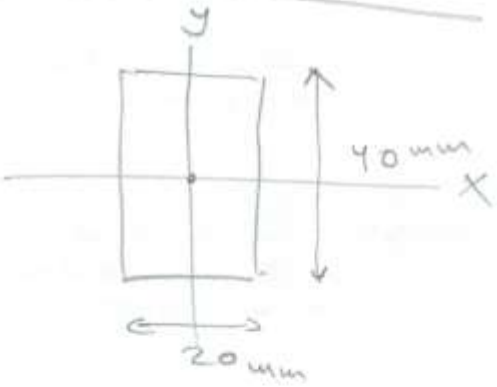
(8) MCQ/ The Area of the Rectangle is more than the area of Triangle



1- True
2- False
3- Not Applicaple
4- Not of Above Result

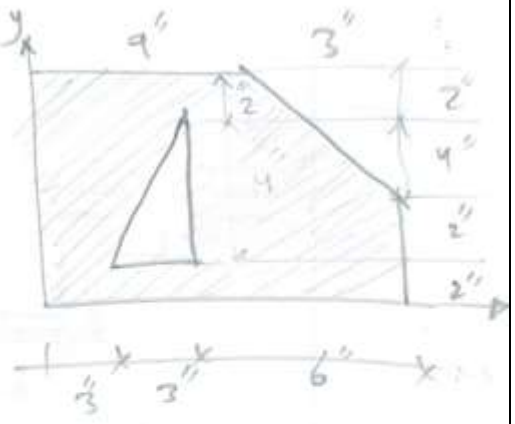
5-

MCQ The $\bar{y}$ for the figure shown is :-



1. 20 mm
2. 20 m
3. 10 mm
4. 10 m
5. 0
6. Not of the above Results

(9) locate the Centroid of the shaded Area

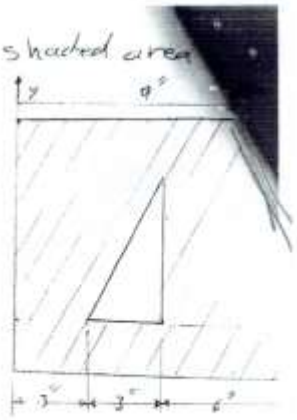


Area #	Area Sq in	x in	My Area·x	y in	Mx Area·y
□	12 * 10 = 120	6	120·6 = 720	5	120·5 = 600
◹	½·3·6 = 9	11	−99	8	−72
◺	½·3·6 = 9	5	−45	4	−36
	102		576		492

$$\bar{x} = \frac{\Sigma M_y}{\Sigma A} = \frac{576}{102} = 5.65 \text{ in}$$

$$\bar{y} = \frac{\Sigma M_x}{\Sigma A} = \frac{492}{102} = 4.82 \text{ in}$$

3-30 Locate the centroid of the shaded area fig.



solution

$A_{total} = (12 \times 10) - \frac{3\times6}{2} - \frac{3\times6}{2}$

$A_{Total} = 102 \ (in^2)$

$M_y = (12\times10)(6) - \frac{3\times6}{2}(5) - \frac{3\times6}{2}(11)$

$M_y = 576 \ (in^3)$

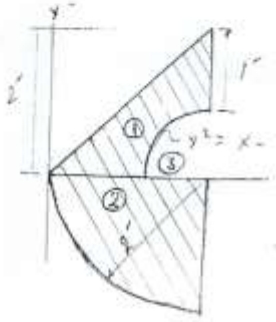
$\bar{x} = \frac{M_y}{A_{Total}} = \frac{576}{102} = 5.65 \ in$

$M_x = (10\times12)(5) - \frac{3\times6}{2}(4) - \frac{3\times6}{2}(8)$

$M_x = 492 \ in^3$

$\bar{y} = \frac{M_x}{A_{Total}} = 4.82 \ in$ (5.65, 4.82)

3.38 Determine the y coordinate of the centroid of the shaded area shown in fig. In the eq. $y^2 = x - 1$, x and are in feet.



$dA_3 = (2 - x)\,dy$

$A_3 = \int_0^1 (2 - (y^2+1))\,dy \Rightarrow A_3 = \int_0^1 (1 - y^2)\,dy$

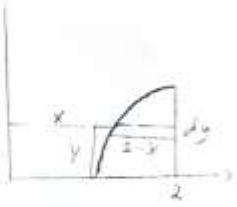
$A_3 = \left[y - \frac{y^3}{3}\right]_0^1 \Rightarrow A_3 = \left[1 - \frac{1}{3}\right] = \frac{2}{3} \ ft^2$

$dM_{x_3} = y\,dA$

$M_{x_3} = \int_0^1 y\,(1 - y^2)\,dy \Rightarrow M_{x_3} = \int_0^1 (y - y^3)\,dy$

$M_{x_3} = \left|\frac{y^2}{2} - \frac{y^4}{4}\right|_0^1 \Rightarrow M_{x_3} = \left(\frac{1}{2} - \frac{1}{4}\right) = \frac{1}{4} \ ft$

$\bar{y}_3 = \frac{M_{x_3}}{A_3} = \frac{0.25}{0.667} = 0.375 \ ft$



$A_{total} = \frac{2\times2}{2} + \frac{(2)^2\pi}{4} - \frac{2}{3} = 4.475 \ ft^2$

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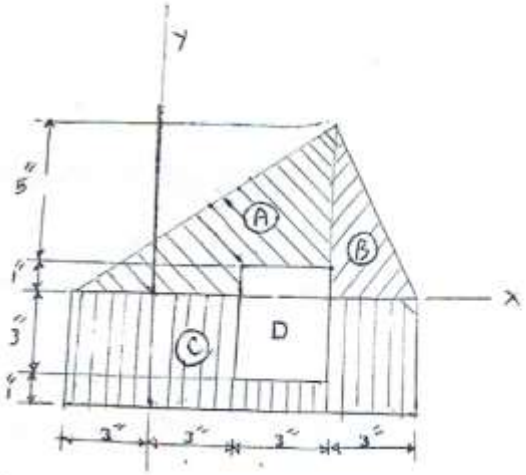
Ex Determine the Coordinates of the Centroid of the shaded area shown

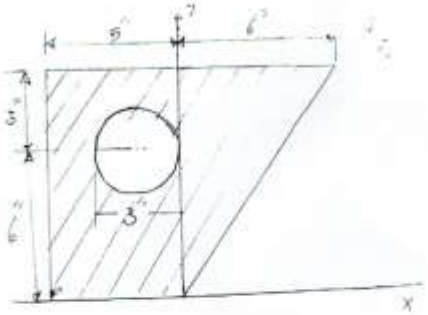
	Area m^2	X cm	M_y cm^3	Y cm	M_x cm^3
A	27	3	81	2	54
B	9	7	63	2	18
C	48	3	144	-2	-96
D	12	4.5	-54	-1	12
Total	72		234		-12



$$\bar{x} = \frac{\Sigma M_y}{\Sigma A} = \frac{234}{72} = 3.25 \text{ cm}$$

$$\bar{y} = \frac{\Sigma M_x}{\Sigma A} = \frac{-12}{72} = -0.1667 \text{ cm}$$

5.27 Locate the centroid of the shaded area shown.



$$A = \frac{6\times 9}{2} + 9\times 5 - (1.5)^2\pi$$

$$A_{total} = 65 \text{ (in}^2\text{)}$$

$$M_y = \frac{6\times 9}{2}(2) + 5\times 9(-2.5) + \left[-(1.5)^2\pi(-1.5)\right] = -47.8 \text{ (in}^3\text{)}$$

$$\bar{x} = \frac{M_y}{A_{total}} = \frac{-47.8}{65} = 0.73 \text{ in}$$

$$M_x = \frac{6\times 9}{2}(6) + 5\times 9(4.5) + \left[-(1.5)^2\pi(6)\right] = 322.08 \text{ in}^3$$

$$\bar{y} = \frac{M_x}{A_{total}} = \frac{322.08}{65} = 4.95 \text{ (in)}$$

The centroid of the fig is (0.73, 4.95)

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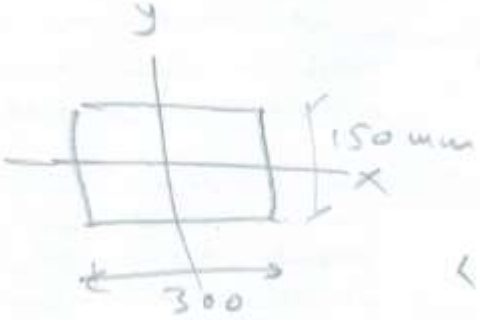
(10) MCQ the moment Area of the Fig. about X-AXIS is

(1) 45000 mm^2

(2) 75 mm

(3) 150 mm

(4) No of the above answers



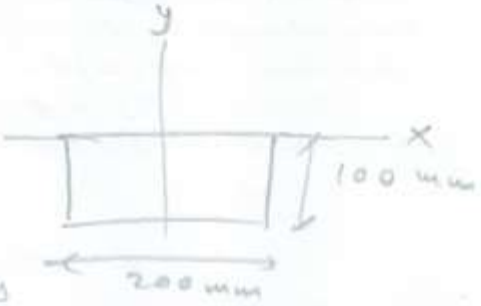
MCQ the $\bar{y}$ of the Fig shown is

1- 50 m

2- 25 mm

3- 100 mm

4- No of the above Answers



MCQ : The $\bar{x}$ of the above Fig is :-

1- 100 mm

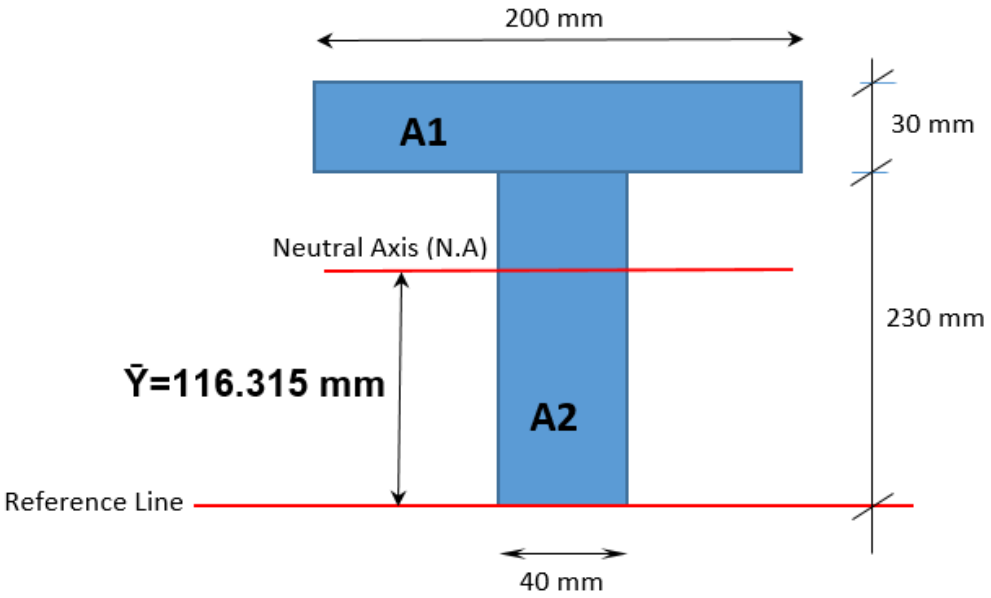
2- 50 mm

3- 200 mm

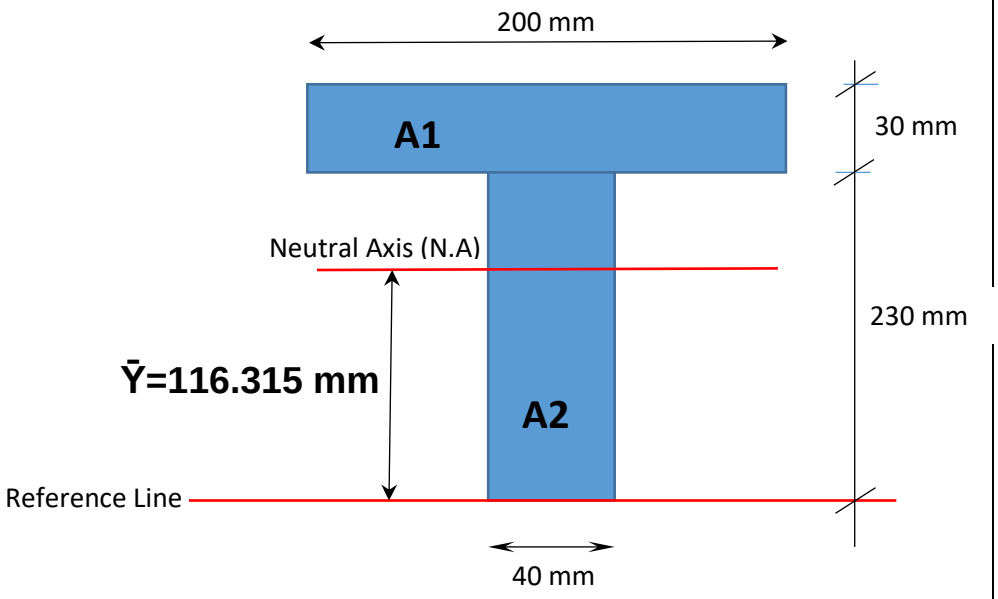
4- - 50 mm

5- - 100 mm

6- Not of the above Answers



Area No.	Area value (mm^2)	Distance from the center of area to the reference line (mm) y	A * y (mm^3)
Rectangle A1	6000	245	1,470,000
Rectangle A2	9200	115	1,058,000
	∑ A=15,200		∑Ay=2,528,000



Area No.	Area value (mm^2)	Distance from the center of area to the reference line (mm) y	A * y (mm^3)
Rectangle A1	6000	245	1,470,000
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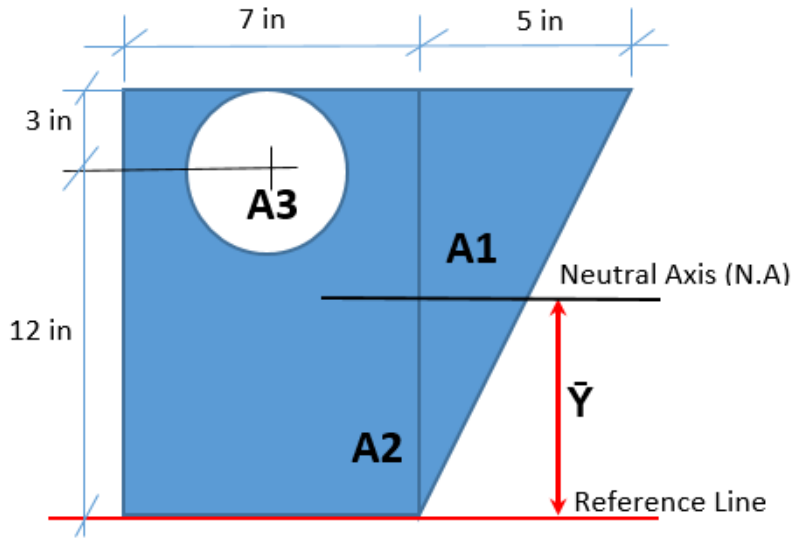
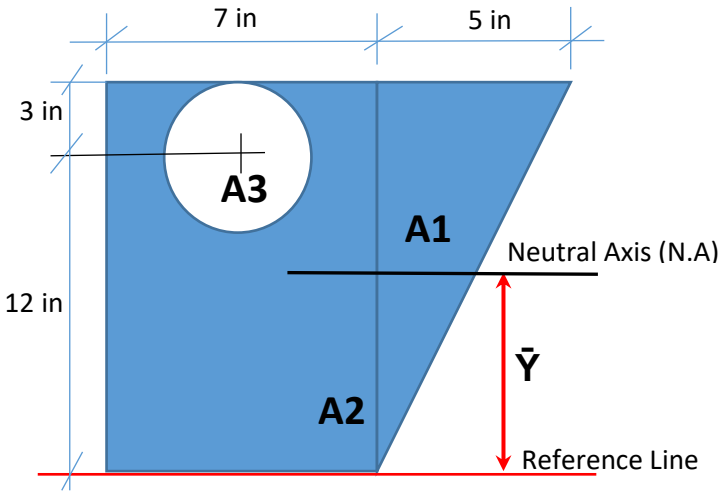
	∑ A=15,200		∑Ay=2,528,000



Area No.	Area value (in^2)	Distance from the center of area to the reference line (in) y	A * y (in^3)
A1	A1=	y1=	Ay1=
A2	A2=	y2=	Ay2=
A3	A3=	y3=	Ay3=
	∑ A=		∑Ay=



Area No.	Area value (in^2)	Distance from the center of area to the reference line (in) y	A * y (in^3)
A1	A1=	y1=	Ay1=
A2	A2=	y2=	Ay2=
A3	A3=	y3=	Ay3=
	∑ A=		∑Ay=

Area No.	Area value (in^2)	Distance from the center of area to the reference line (in) y	A * y (in^3)
 A1	A1=37.5	y1=10	Ay1=375
 A2	A2=105	y2=7.5	Ay2=787.5
 A3	A3=-28.27	y3=12	Ay3=-339.24
	∑ A=114.23		∑Ay=823.26

$\bar{Y}$=7.2 in