Q9/The I cross-section shown in Figure is a beam subjected to a maximum <u>shearing force</u> of 100 kN.Determine the shearing stress

(a) at the junction between web and flange.

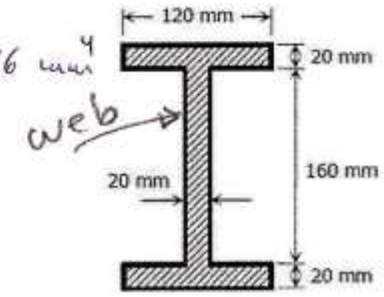
(b) at the neutral axis

Solution:

$$\tau = \frac{VQ}{Ib}$$

$$V = 100 \times 1000 = 100,000 \ N$$

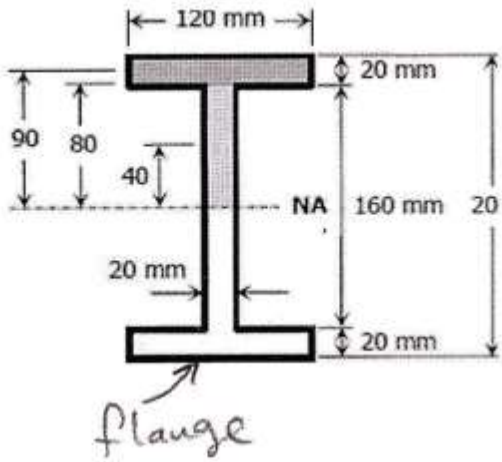
$$I = \frac{bh^3}{12} = \frac{120(200)^3}{12} - \frac{100(160)^3}{12} = 45,866,666 \ mm^4$$



(a) $Q_{\cancel{web}}^{flange} = 120 \times 20 \times 90 = 216\,000 \ mm^3$

$$\tau_{\cancel{web}\ flange} = \frac{100\,000 \times 216\,000}{45,866,666 \times 120} =$$

$$\tau_{web\ \cancel{flange}} = \frac{100,000 \times 216\,000}{45,866,666 \times 20} =$$



(b) $Q_{\cancel{flange}}^{web} = 120(20)(90) + 20(80)(40)$

$$= 280,000 \ mm^3$$

$$\tau_{N.A} = \frac{100,000 \times 280,000}{45,866,666 \times 20} =$$

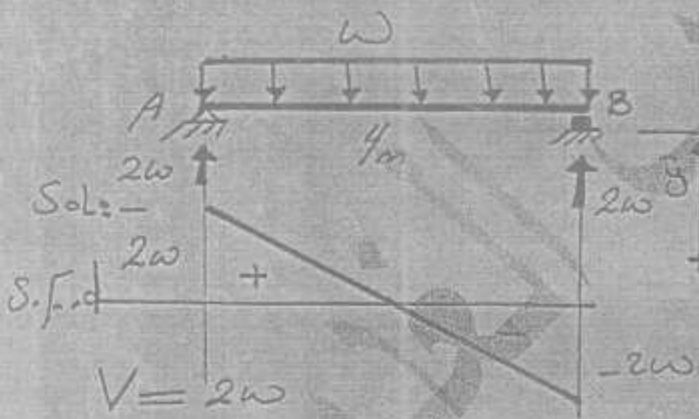
Shear. Stress. 4 — أجهادات القص

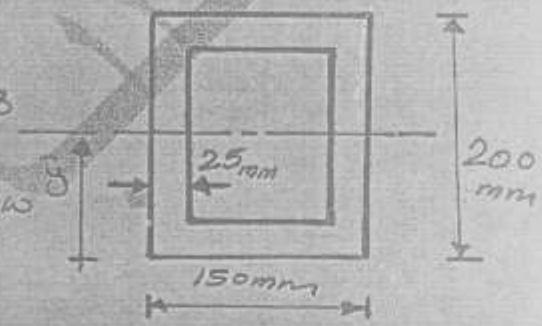
Strength of mater — مقاومة ثاني

Ex. 2

A Simply supported beam 4m Long has the cross section shown in fig. Determine the maximum uniform distributed Load which can be applied over the entir Length of the beam if the Shearing stress is Limited to $1.2 \frac{N}{mm^2}$?

Sol:-



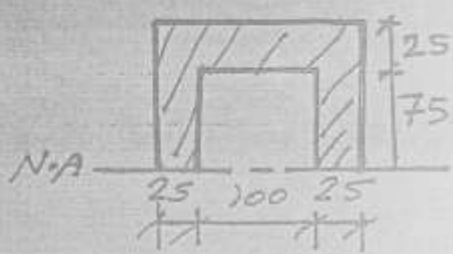


$V = 2\omega$

$\bar{y} = \frac{200}{2} = 100\,mm$

$I = A \cdot d^2 + I_c = \frac{150 * 200^3}{12} - \frac{100 * 150^3}{12}$

$I_{N.A} = 71875000\ mm^4$



$Q = 150 * 100 * 50 - 100 * 75 * \frac{75}{2}$

$Q = 468750\ mm^3$

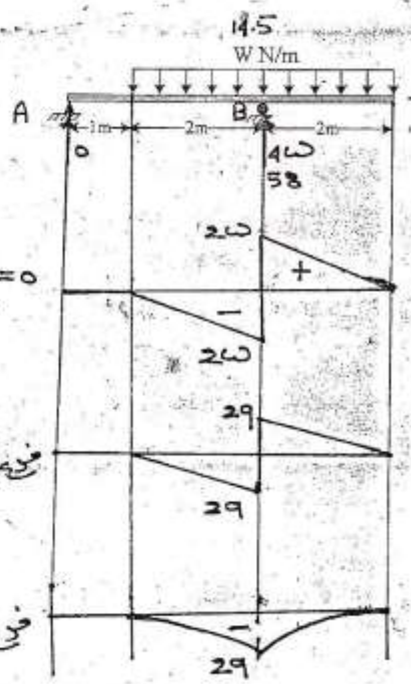
$\tau = \frac{V \cdot Q}{I \cdot b}$

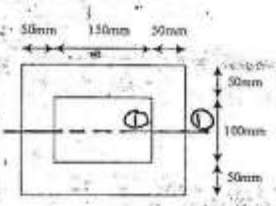
$1.2 = \frac{2\omega * 10^3 * 468750}{71875000 * 50} \Rightarrow \omega = 4.6\ KN/m$

Q6/The distributed load shown in figure, is supported by a box beam with the dimensions given.

a-Determine the maximum value of w that will not cause a shearing stress greater than 2 MPa in the beam

b-With the value of w calculated above, draw the shear and moment diagrams.





$\Sigma M_B = 0$

$A(3) + 2w - 2w = 0$

$A = 0$

$\Sigma F_y = 0 \uparrow$

$B = 4w$

$$\tau = \frac{VQ}{Ib}$$

$V = 2w$ kN

$Q = 2 \times 100 \times 50 \times 50 + 150 \times 50 \times 75$

$= 1062500$

$$I = \frac{250(200)^3}{12} - \frac{150(100)^3}{12}$$

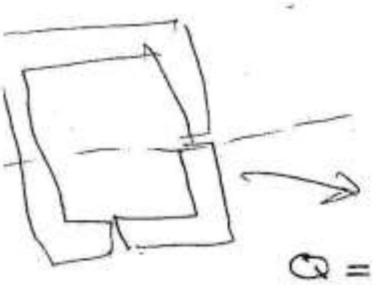
$= 1.54 \times 10^8\ mm^4$

$$2 = \frac{2w \times 10^3 \times 1062500}{1.54 \times 10^8 \times 100}$$

عرض المقطعين

$w = 14.5$ kN/m

أو بطريقة أخرى



$Q = 125 \times 50 \times 75 + 50 \times 50 \times 25 = 531250$

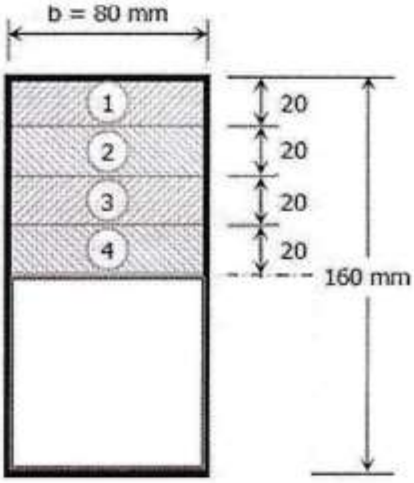
لإيجاد الإجهاد في مقطع ① ②

كونه هو الاكبر

$$2 = \frac{2w \times 10^3 \times 531250}{1.54 \times 10^8 \times 50}$$

$w = 14.5$ kN/m

Q10/ A timber beam 80 mm wide by 160 mm high as shown in the figure, is subjected to a vertical shear V = 40 kN. Determine the shearing stress developed at the top and bottom for each layer of the section.



Solution: $I = \frac{bh^3}{12} = \frac{80(160)^3}{12} = 27.31 \times 10^6 \text{ mm}^4$

$\tau = \frac{VQ}{Ib}$, $V = 40 \times 1000 = 40 \times 10^3$ N (given)

τ @ Top of layer ① $= \frac{40 \times 10^3 \times [\text{zero}]}{Ib} = \text{zero}$

τ @ bottom of layer ① $= \frac{40 \times 10^3 \times [80 \times 20 \times 70]}{27.31 \times 10^6 \times 80} =$

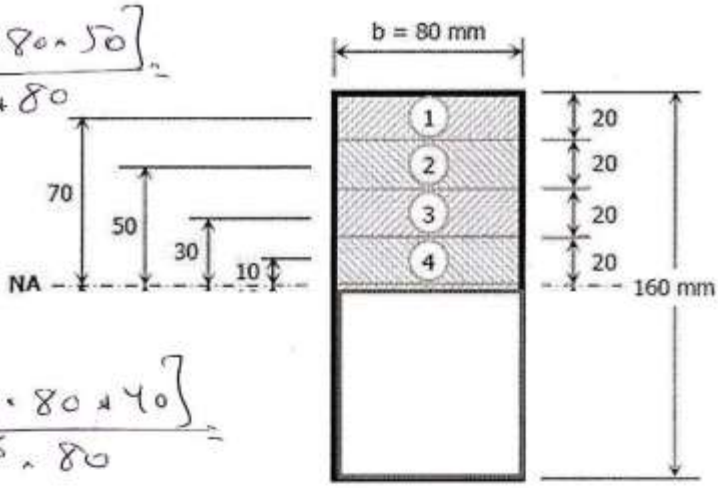
$= \tau$ @ Top of layer ②

τ @ bottom of layer ② $= \frac{40 \times 10^3 [40 \times 80 \times 60]}{27.31 \times 10^6 \times 80} =$

$= \tau$ @ Top of layer ③

τ @ bottom of layer ③ $= \frac{40 \times 10^3 [60 \times 80 \times 50]}{27.31 \times 10^6 \times 80} =$

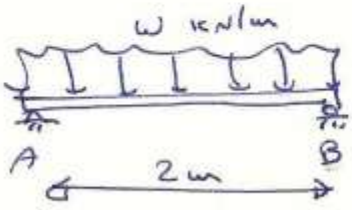
$= \tau$ @ Top of lay ④

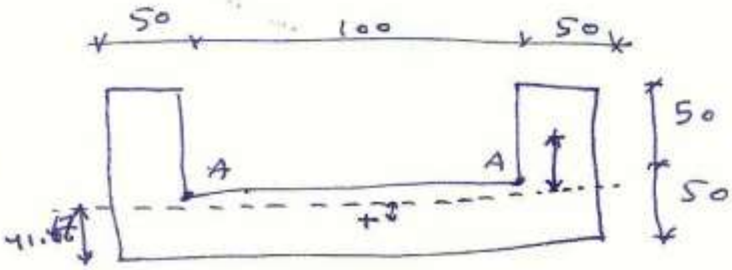


τ @ bottom of lay ④ $= \frac{40 \times 10^3 [80 \times 80 \times 40]}{27.31 \times 10^6 \times 80} =$

~~$= \tau$ @ bottom of lay~~

Example: Determine max. value of ω.
if the allowable shearing stress = 1.6 MPa.



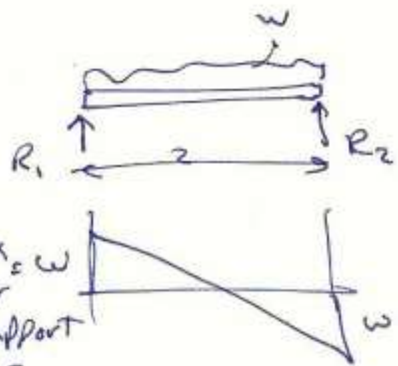


Solution: $\bar{y} = \dfrac{\Sigma Ay}{\Sigma A} = \dfrac{(200*50)*25 + 2*(50*50)*75}{(200*50) + 2*(50*50)} = 41.67\,mm$

$I = \dfrac{200*50^3}{12} + (200*50)*(41.67-25)^2 +$
$2\left(\dfrac{50*50^3}{12} + (50*50)*(100-41.67-25)^2\right) =$

$I = 2*10^6 + 2.77*10^6 + 2*578.58*10^6 = 11.46*10^6\,mm^4$

$R_1 = R_2 = \dfrac{\omega \ell}{2} = \dfrac{\omega*2}{2} = \omega\ kN$



max. w shear @ support = V

Note: Max shearing stress developed @ the N.A

$\tau = \dfrac{VQ}{Ib} = \dfrac{\omega*1000*(41.67*200)*\frac{41.67}{2}}{11.46*10^6*200}$

$\tau = 1.6\ N/mm^2$ (given)

$\therefore \omega = 21.12\ kN/m$

Now find the τ @ max section A-A

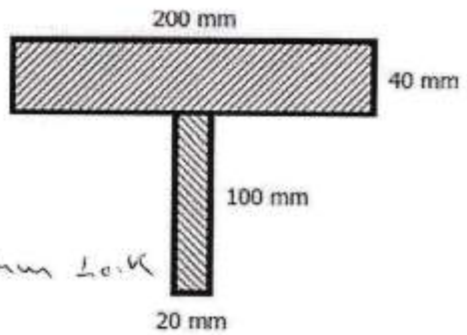
$\tau = \dfrac{\omega*10^3*(2*50*50)\left(\frac{50}{2} + (50-41.67)\right)}{11.46*10^6*2*50} = 1.6\ N/mm^2$

$\therefore \omega = 11\ kN/m$

Q8/ The T section shown in Figure is the cross-section of a beam formed by joining two rectangular pieces of wood together. The beam is subjected to a maximum shearing force of 60 kN.

1- Prove that the NA is 34 mm from the top.
2- Prove that the I = 10.57 × 106 mm4.

Using these values, determine the shearing stress
(a) at the neutral axis
(b) at the junction between the two pieces of wood.



Solution: $A_1 = 200*40 = 8000\ mm^2$

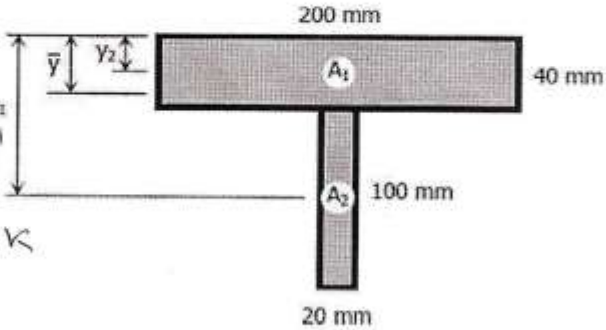
$y_1 = 20\ mm$

$A_2 = 20(100) = 2000\ mm^2$

$y_2 = 90\ mm$

$A = A_1 + A_2 = 10,000\ mm^2$

$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{8000*20 + 2000*90}{10\,000} = 34\ mm$ o.k



$I = \bar{I} + Ad^2 = \frac{bh^3}{12} + Ad^2$

$I_1 = \frac{200(40)^3}{12} + 8000(14)^2 = 2634666.6$

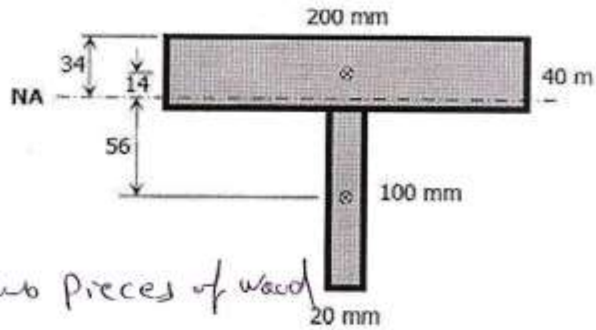
$I_2 = \frac{20(100)^3}{12} + 2000(56)^2 = 7938666.67\ mm^4$

$\therefore I_{N.A} = I_1 + I_2 = 10.57\times10^6\ mm^4$ ∴ o.k

(a) at the Nuteral axis :-

$Q = 200(34)(17) = 115600\ mm^3$

$V = 60(1000) = 60\,000\ N$ (given)



$\tau = \frac{VQ}{Ib} = \frac{60\,000(115\,600)}{10.57*10^6(200)}$

$\tau_{N.A} = 3.28\ MPa$

(b) at the Junction between the two pieces of wood

(1) Flange :- $b = 200\ m$

$\tau_{flange} = \frac{60\,000*(100*20*56)}{10.57*10^6*200} = 3.1788\ MPa$

(2) Web :- $b = 20\ mm$

$\tau_{web} = \frac{60000(100*20*56)}{10.57*10^6*20} = 31.7881\ MPa$