



**Strength of Materials**  
**2<sup>nd</sup> Stage (2021-2022)**  
**Civil Engineering Department**  
**Dr. Ali Hassan Ali**

# Syllabus

<b>Chapter</b>	<b>Topic</b>
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<b>2</b>	<b>Strain</b>
<b>3</b>	<b>Mechanical Properties of Materials</b>
<b>4</b>	<b>Axial Deformation</b>
<b>5</b>	<b>Shear Forces and Bending Moments diagrams</b>
<b>6</b>	<b>Torsion</b>
<b>7</b>	<b>Bending Stresses in Beams</b>
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<b>11</b>	<b>Strain Transformation</b>
<b>12</b>	<b>Beam Deflections</b>

# References

- **Philpot, T.A. (2017).** *Mechanics of Materials: An Integrated Learning System.* 4<sup>th</sup> Ed. John Wiley & Sons, Inc.
- **Hibbeler, R.C. (2011).** *Mechanics of Materials.* 8<sup>th</sup> Ed. Pearson Prentice Hall.



# Outcomes of courses

- **To understand the concept of stress and strain.**
- **To be familiar with the numerical equations of stress and strain and the background theory.**
- **To know how to draw a free body diagram for the structural members.**
- **To analyse different types and directions of loadings.**
- **To assess the mechanical properties of materials.**
- **To be familiar with the behaviour of beams in bending and torsion.**
- **To analyse the behaviour of beams in shear and deflection.**
- **To understand the classifications of beams, e.g., determinate, indeterminate...etc.**
- **To be familiar with the stress transformation by using Mohr's circle.**
- **To analyse axially compressed members, i.e., columns.**

# Chapter 9 Combined Loads

## 9.1 General Combined Loadings

In numerous industrial situations, axial, torsional, and flexural loads act simultaneously on machine components and the combined effects of these loads must be analyzed to determine the critical stresses developed in the component. Although an experienced designer can usually predict one or more points where high stress is likely, the most severely stressed point on any particular cross section may not be obvious. As a result, it is almost always necessary to analyze the stresses at more than one point before the critical stresses in the component can be known.

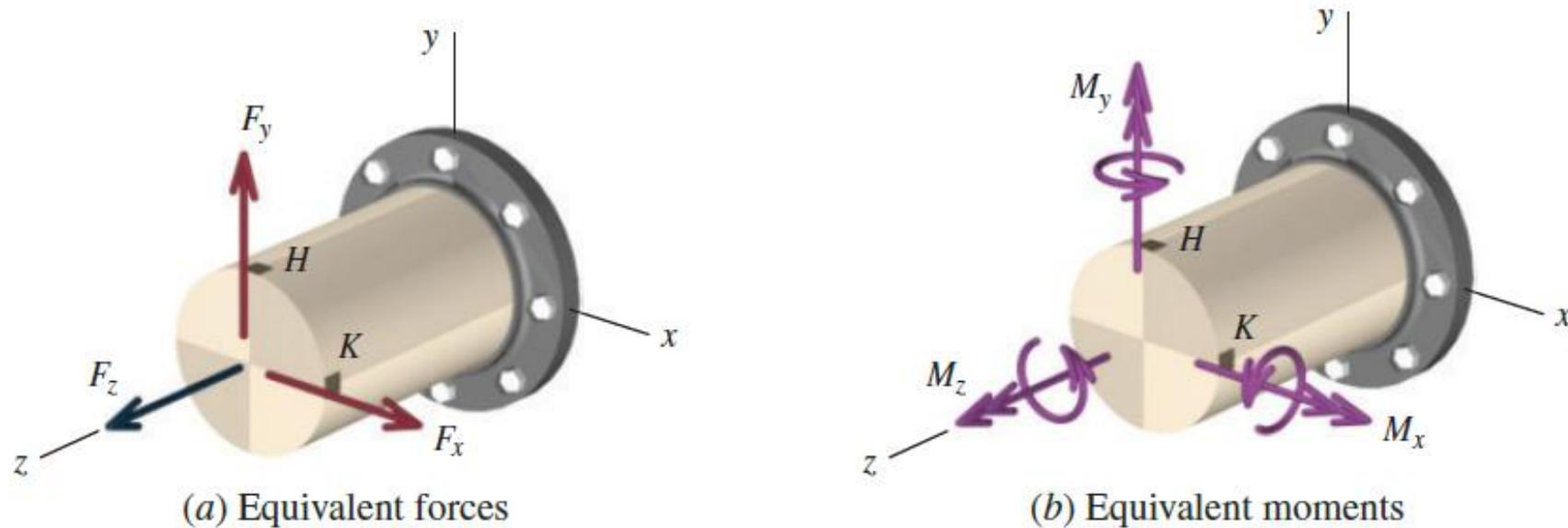
### Calculation Procedures

To determine the principal stresses and the maximum shear stress at a particular point in a component subjected to axial, torsion, bending, and pressure loads, the following procedures are useful:

1. Determine the statically equivalent forces and moments acting at the section of interest. In this step, a complicated three-dimensional component or structure subjected to multiple loads is reduced to a simple, prismatic member with no more than three forces and three moments acting at the section of interest.
  - a. In finding the statically equivalent forces and moments, it is often convenient to consider the portion of the structure or component that extends from the section of interest to the free end of the structure. The statically equivalent forces at the section of interest are found by summing the loads that act on this portion of the structure (i.e.,  $\Sigma F_x$ ,  $\Sigma F_y$ , and  $\Sigma F_z$ ). Note that these summations do not include the reaction forces.

# Chapter 9 Combined Loads

- b. The statically equivalent moments can be more difficult to determine correctly than the statically equivalent forces, since both a load magnitude and a distance term make up each moment component. One approach is to consider each load on the structure, in turn. The magnitude of the moment, the axis about which the moment acts, and the sign of the moment must be assessed for each load. In addition, a single load on the structure may create unique moments about two axes. After all moment

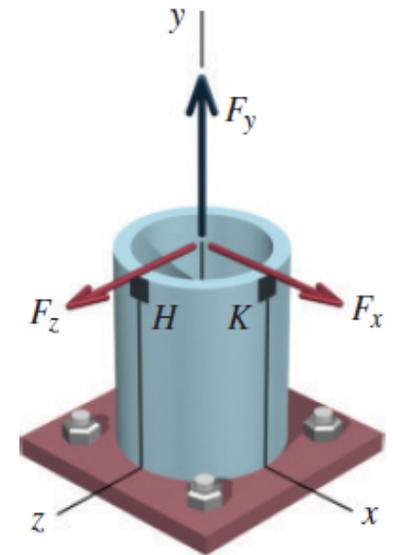


**FIGURE 9.1** Statically equivalent forces and moments at section of interest.

components have been determined, the statically equivalent moments at the section of interest are found by summing the moment components in each direction (i.e.,  $\Sigma M_x$ ,  $\Sigma M_y$ , and  $\Sigma M_z$ ).

# Chapter 9 Combined Loads

- c. As the geometry of the structure and of the loads becomes more complicated, it is often easier to use position vectors and force vectors to calculate equivalent moments. A position vector  $\mathbf{r}$  from the section of interest to the specific point of application of the load is determined, along with a vector  $\mathbf{F}$  describing the forces acting at that point. The moment vector  $\mathbf{M}$  is computed from the cross product of the position and force vectors; that is,  $\mathbf{M} = \mathbf{r} \times \mathbf{F}$ . If loads are applied at more than one location on the structure, then multiple cross products must be computed.
2. After the statically equivalent forces and moments at the section of interest have been determined, *prepare two sketches* showing the magnitude and direction of all forces and moments acting at the section of interest. Typical sketches are shown in Figures 9.1 and 9.2. These sketches help organize and clarify the results before the stresses are computed.
  3. Determine the stresses produced by each of the equivalent forces.
    - a. An axial force (force  $F_z$  in Figure 9.1a and force  $F_y$  in Figure 9.2a) produces either tensile or compressive normal stress given by  $\sigma = F/A$ .
    - b. Shear stresses computed with the equation  $\tau = VQ/It$  are associated with shear forces (forces  $F_x$  and  $F_y$  in Figure 9.1a and forces  $F_x$  and  $F_z$  in Figure 9.2a). Use the direction of the shear force arrow on the section of interest to establish the direction of  $\tau$  on the corresponding face of the stress element. Recall that  $\tau$  associated with shear forces is parabolically distributed on a cross section (e.g., see Figure 8.5). For circular cross sections,  $Q$  is calculated from Equation (8.4) or Equation (8.5) for solid cross sections and Equation (8.7) for hollow cross sections.



(a) Equivalent forces

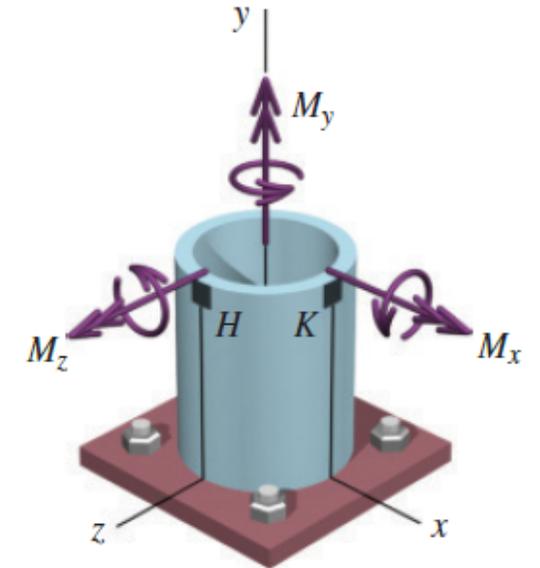
# Chapter 9 Combined Loads

4. Determine the stresses produced by the equivalent moments.
- a. Moments about the longitudinal axis of the component at the section of interest are termed *torques*. In Figure 9.1b,  $M_z$  is a torque; in Figure 9.2b,  $M_y$  is a torque. Torques produce shear stresses that are calculated from  $\tau = Tc/J$ , where  $J$  is the *polar moment of inertia*. Recall that the polar moment of inertia for a circular cross section is computed as follows:

$$J = \frac{\pi}{32}d^4 \quad (\text{for solid circular sections})$$

$$J = \frac{\pi}{32}[D^4 - d^4] \quad (\text{for hollow circular section})$$

Use the direction of the torque to determine the direction of  $\tau$  on the transverse face of the stress element at the point of interest.



(b) Equivalent moments

**FIGURE 9.2** Statically equivalent forces and moments at section of interest.

# Chapter 9 Combined Loads

- b. Bending moments produce normal stresses that are linearly distributed with respect to the axis of bending. In Figure 9.1b ,  $M_x$  and  $M_y$  are bending moments; in Figure 9.2b ,  $M_x$  and  $M_z$  are bending moments. Calculate the bending stress magnitude from  $\sigma = My/I$ , where  $I$  is the *area moment of inertia*. Recall that the area moment of inertia for a circular cross section is computed as follows:

$$I = \frac{\pi}{64} d^4 \quad (\text{for solid circular sections})$$

$$I = \frac{\pi}{64} [D^4 - d^4] \quad (\text{for hollow circular sections})$$

The sense of the stress (either tension or compression) can be determined by inspection. Recall that bending stresses act parallel to the longitudinal axis of the flexural member. Therefore, bending stresses in Figure 9.1b act in the  $z$  direction, while bending stresses in Figure 9.2b act in the  $y$  direction.

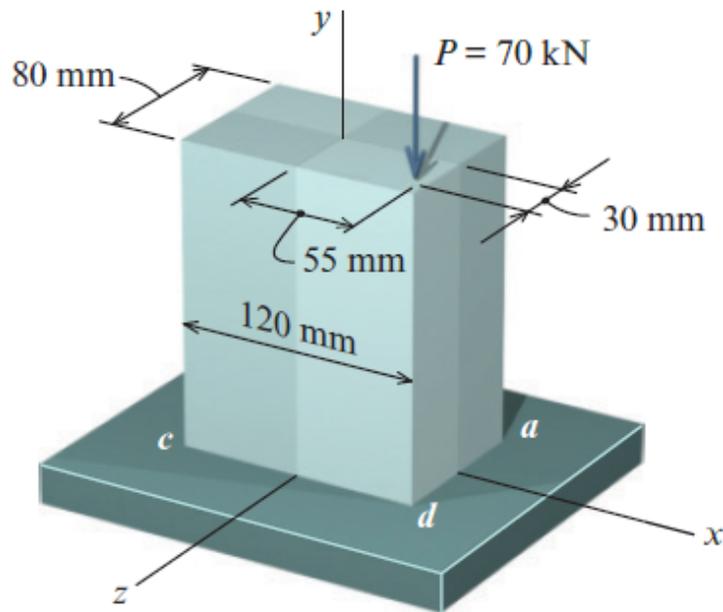
5. If the component is a hollow circular section that is subjected to internal pressure, then longitudinal and circumferential normal stresses are created. The longitudinal stress is calculated from  $\sigma_{\text{long}} = pd/4t$ , and the circumferential stress is given by  $\sigma_{\text{hoop}} = pd/2t$ , where  $d$  is the inside diameter of the component. Note that the term  $t$  in these two equations refers to the wall thickness of the pipe or tube. The term  $t$  appearing in the context of the shear stress equation  $\tau = VQ/It$  has a different meaning. For a pipe, the term  $t$  in the equation  $\tau = VQ/It$  is actually equal to the *wall thickness times 2!*

# Chapter 9 Combined Loads

6. Using the principle of superposition, summarize the results on a stress element, taking care to identify the proper direction of each stress component. As stated previously, it is generally more reliable to use *inspection* to establish the direction of normal and shear stresses acting on the stress element.
7. Once the stresses on orthogonal planes through the point are known and summarized on a stress element, the methods of Chapter 10 can be used to calculate the principal stresses and the maximum shear stresses at the point.

# Chapter 9 Combined Loads

## EXAMPLE 9.1



A short post supports a load  $P = 70$  kN as shown. Determine the normal stresses at corners  $a$ ,  $b$ ,  $c$ , and  $d$  of the post.

### Plan the Solution

The load  $P = 70$  kN will create normal stresses at the corners of the post in three ways. The axial load  $P$  will create compressive normal stress that is distributed uniformly over the cross section. Since  $P$  is applied 30 mm away from the  $x$  centroidal axis and 55 mm away from the  $z$  centroidal axis,  $P$  will also create bending moments about these two axes. The moment about the  $x$  axis will create tensile and compressive normal stresses that will be linearly distributed across the 80 mm width of the post. The moment about the  $z$  axis will create tensile and compressive normal stresses that will be linearly distributed across the 120 mm depth of the cross section. The normal stresses created by the axial force and the bending moments will be determined at each of the four corners, and the results will be superimposed to give the normal stresses at  $a$ ,  $b$ ,  $c$ , and  $d$ .

# Chapter 9 Combined Loads

## SOLUTION

### Section Properties

The cross-sectional area of the post is

$$A = (80 \text{ mm})(120 \text{ mm}) = 9,600 \text{ mm}^2$$

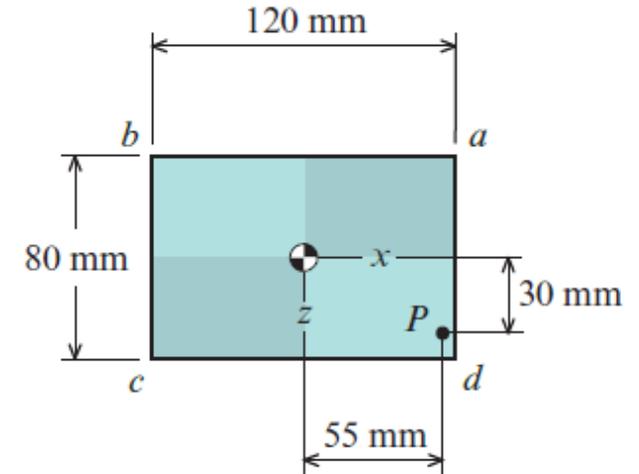
The moment of inertia of the cross-sectional area about the  $x$  centroidal axis is

$$I_x = \frac{(120 \text{ mm})(80 \text{ mm})^3}{12} = 5.120 \times 10^6 \text{ mm}^4$$

and the moment of inertia about the  $z$  centroidal axis is

$$I_z = \frac{(80 \text{ mm})(120 \text{ mm})^3}{12} = 11.52 \times 10^6 \text{ mm}^4$$

Since  $I_z > I_x$  for the coordinate axes shown, the  $x$  axis is termed the *weak axis* and the  $z$  axis is termed the *strong axis*.



**Cross-sectional dimensions and location of application of load.**

# Chapter 9 Combined Loads

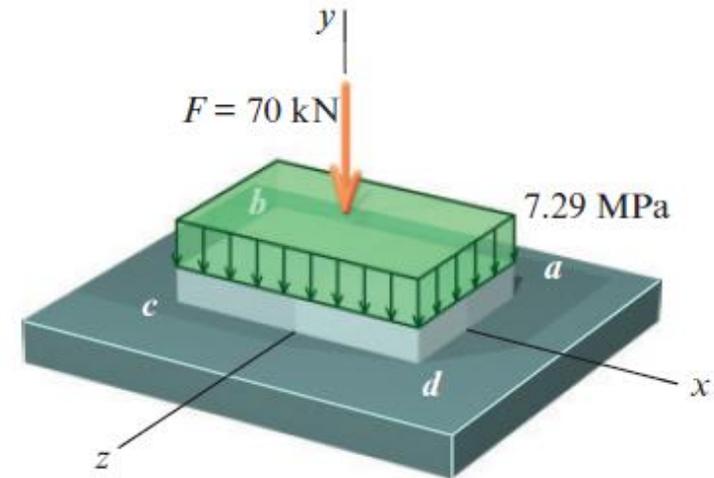
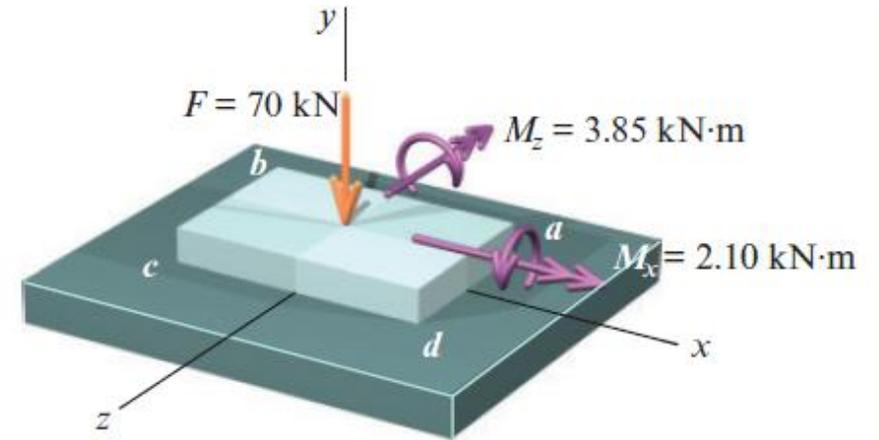
## Equivalent Forces in the Post

The vertical load  $P = 70$  kN applied 30 mm from the  $x$  axis and 55 mm from the  $z$  axis is statically equivalent to an internal axial force  $F = 70$  kN, an internal bending moment  $M_x = 2.10$  kN·m, and an internal bending moment  $M_z = 3.85$  kN·m. The stresses created by each of these will be considered in turn.

## Axial Stress Due to $F$

The internal axial force  $F = 70$  kN creates compressive normal stress that is uniformly distributed over the entire cross section. The magnitude of the stress is computed as

$$\sigma_{\text{axial}} = \frac{F}{A} = \frac{(70 \text{ kN})(1,000 \text{ N/kN})}{9,600 \text{ mm}^2} = 7.29 \text{ MPa (C)}$$



# Chapter 9 Combined Loads

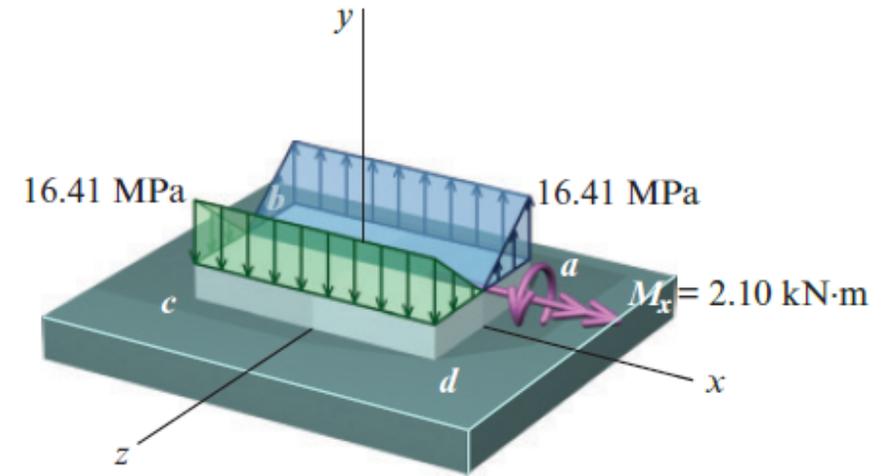
## Bending Stress Due to $M_x$

The bending moment acting as shown about the  $x$  axis creates compressive normal stress on side  $cd$ , and tensile normal stress on side  $ab$ , of the post. The maximum bending stress occurs at a distance  $z = \pm 40$  mm from the neutral axis (which is the  $x$  centroidal axis for  $M_x$ ). The maximum bending stress magnitude is

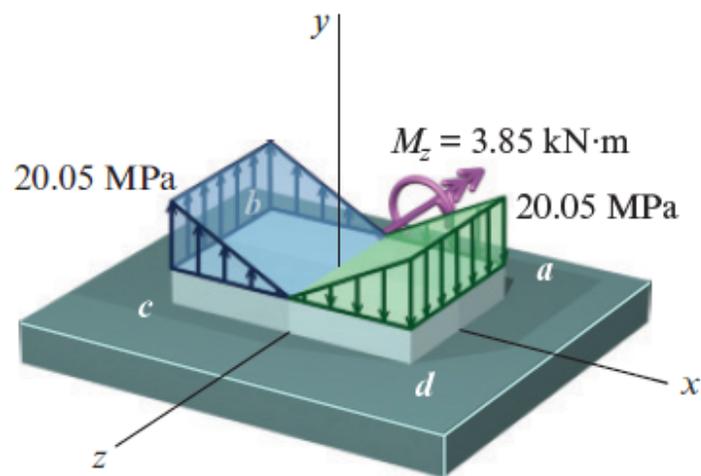
$$\begin{aligned}\sigma_{\text{bend}} &= \frac{M_x z}{I_x} = \frac{(2.10 \text{ kN} \cdot \text{m})(\pm 40 \text{ mm})(1,000 \text{ N/kN})(1,000 \text{ mm/m})}{5.120 \times 10^6 \text{ mm}^4} \\ &= \pm 16.41 \text{ MPa}\end{aligned}$$

## Bending Stress Due to $M_z$

The bending moment acting as shown about the  $z$  centroidal axis creates compressive normal stress on side  $ad$ , and tensile normal stress on side  $bc$ , of the post. The maximum bending stress occurs at a



# Chapter 9 Combined Loads



distance  $x = \pm 60$  mm from the neutral axis (which is the  $z$  centroidal axis for  $M_z$ ). The maximum bending stress magnitude is

$$\begin{aligned}\sigma_{\text{bend}} &= \frac{M_z x}{I_z} = \frac{(3.85 \text{ kN} \cdot \text{m})(\pm 60 \text{ mm})(1,000 \text{ N/kN})(1,000 \text{ mm/m})}{11.52 \times 10^6 \text{ mm}^4} \\ &= \pm 20.05 \text{ MPa}\end{aligned}$$

## Normal Stresses at Corners $a$ , $b$ , $c$ , and $d$

The normal stresses acting at each of the four corners of the post can be determined by superimposing the preceding results. In all instances, the normal stresses act in the vertical direction—that is, the  $y$  direction. The sense of the stress, either tension or compression, can be determined by inspection.

*Corner a:*

$$\begin{aligned}\sigma_a &= 7.29 \text{ MPa (C)} + 16.41 \text{ MPa (T)} + 20.05 \text{ MPa (C)} \\ &= -7.29 \text{ MPa} + 16.41 \text{ MPa} - 20.05 \text{ MPa} \\ &= -10.93 \text{ MPa} = 10.93 \text{ MPa (C)}\end{aligned}$$

**Ans.**

# Chapter 9 Combined Loads

*Corner b:*

$$\begin{aligned}\sigma_b &= 7.29 \text{ MPa (C)} + 16.41 \text{ MPa (T)} + 20.05 \text{ MPa (T)} \\ &= -7.29 \text{ MPa} + 16.41 \text{ MPa} + 20.05 \text{ MPa} \\ &= 29.17 \text{ MPa} = 29.17 \text{ MPa (T)}\end{aligned}$$

**Ans.**

*Corner c:*

$$\begin{aligned}\sigma_c &= 7.29 \text{ MPa (C)} + 16.41 \text{ MPa (C)} + 20.05 \text{ MPa (T)} \\ &= -7.29 \text{ MPa} - 16.41 \text{ MPa} + 20.05 \text{ MPa} \\ &= -3.65 \text{ MPa} = 3.65 \text{ MPa (C)}\end{aligned}$$

**Ans.**

*Corner d:*

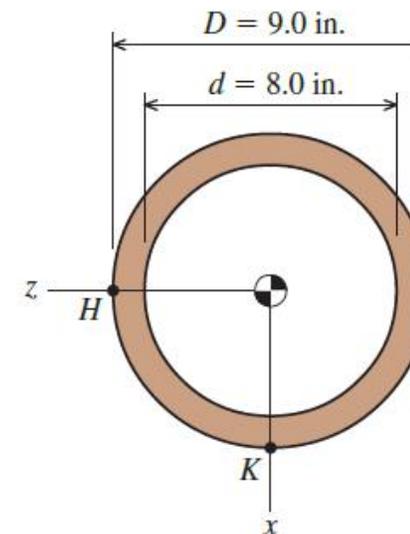
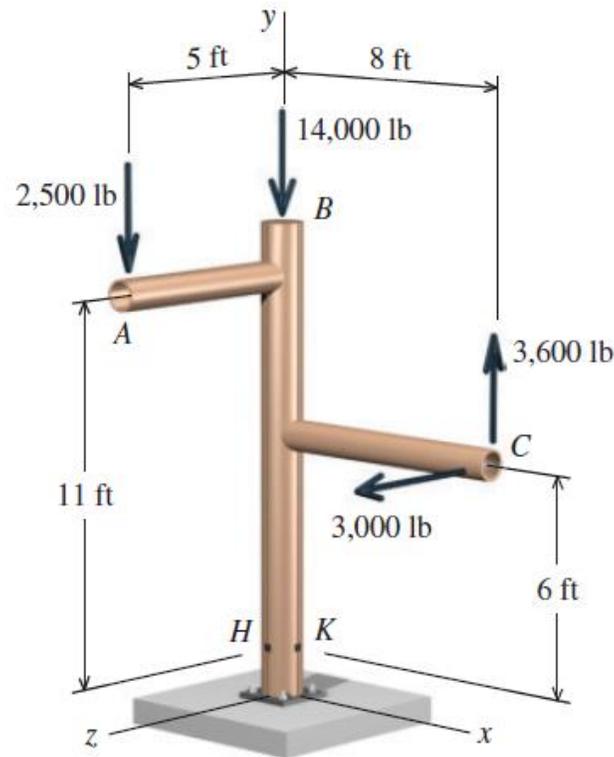
$$\begin{aligned}\sigma_d &= 7.29 \text{ MPa (C)} + 16.41 \text{ MPa (C)} + 20.05 \text{ MPa (C)} \\ &= -7.29 \text{ MPa} - 16.41 \text{ MPa} - 20.05 \text{ MPa} \\ &= -43.75 \text{ MPa} = 43.75 \text{ MPa (C)}\end{aligned}$$

**Ans.**

# Chapter 9 Combined Loads

## EXAMPLE 9.2

A vertical pipe column with an outside diameter  $D = 9.0$  in. and an inside diameter  $d = 8.0$  in. supports the loads shown. Determine the principal stresses and the maximum shear stress at points  $H$  and  $K$ .



Column cross-sectional dimensions.

# Chapter 9 Combined Loads

## Plan the Solution

Several loads act on the structure, making the analysis seem complicated. However, it can be simplified considerably by first reducing the system of four loads to a statically determinate system of forces and moments acting at the section of interest. The normal and shear stresses created by this equivalent force system will be computed and shown in their proper directions on stress elements for points  $H$  and  $K$ . Stress transformation calculations will be used to determine the principal stresses and maximum shear stress for each stress element.

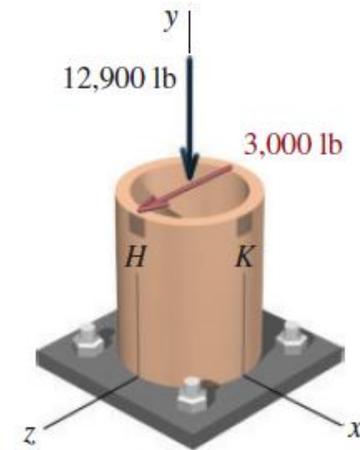
## SOLUTION

### Equivalent Force System

A system of forces and moments that is statically equivalent to the four loads applied at points  $A$ ,  $B$ , and  $C$  can be readily determined for the section of interest.

The equivalent forces are simply equal to the applied loads. There is no force acting in the  $x$  direction. The sum of the forces in the  $y$  direction is

$$\Sigma F_y = -2,500 \text{ lb} - 14,000 \text{ lb} + 3,600 \text{ lb} = -12,900 \text{ lb}$$



Equivalent forces at the section that contains points  $H$  and  $K$ .

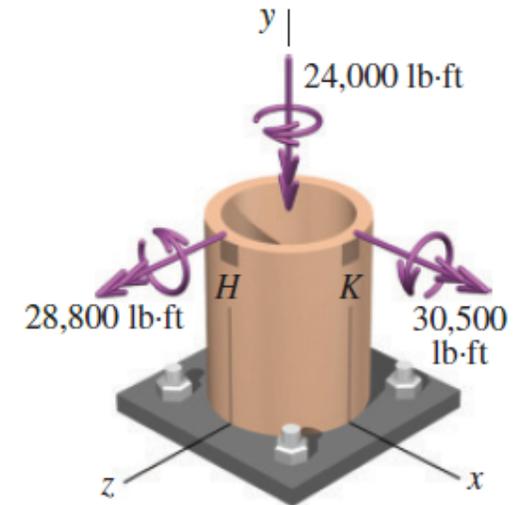
# Chapter 9 Combined Loads

In the  $z$  direction, the only force is the 3,000 lb load applied to point  $C$ . The equivalent forces acting at the section are shown in the accompanying figure.

The equivalent moments acting at the section of interest can be determined by considering each load in turn:

- The 2,500 lb load acting at  $A$  creates a moment of  $(2,500 \text{ lb})(5 \text{ ft}) = 12,500 \text{ lb} \cdot \text{ft}$ , which acts about the  $+x$  axis.
- The line of action of the 14,000 lb load passes through the section of interest; therefore, it creates no moments at  $H$  and  $K$ .
- The 3,600 lb load acting vertically at  $C$  creates a moment of  $(3,600 \text{ lb})(8 \text{ ft}) = 28,800 \text{ lb} \cdot \text{ft}$  about the  $+z$  axis.
- The 3,000 lb load acting horizontally at  $C$  creates two moment components.
  - One moment component has a magnitude of  $(3,000 \text{ lb})(8 \text{ ft}) = 24,000 \text{ lb} \cdot \text{ft}$  and acts about the  $-y$  axis.
  - A second moment component has a magnitude of  $(3,000 \text{ lb})(6 \text{ ft}) = 18,000 \text{ lb} \cdot \text{ft}$  and acts about the  $+x$  axis.
- The moments acting about the  $x$  axis can be summed to determine the equivalent moment:

$$M_x = 12,500 \text{ lb} \cdot \text{ft} + 18,000 \text{ lb} \cdot \text{ft} = 30,500 \text{ lb} \cdot \text{ft}$$



**Equivalent moments at the section that contains points  $H$  and  $K$ .**

# Chapter 9 Combined Loads

For the coordinate system used here, the axis of the pipe column extends in the  $y$  direction. Therefore, the moment component acting about the  $y$  axis is recognized as a torque; the components about the  $x$  and  $z$  axes are simply bending moments.

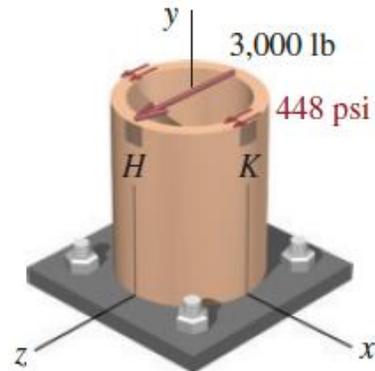
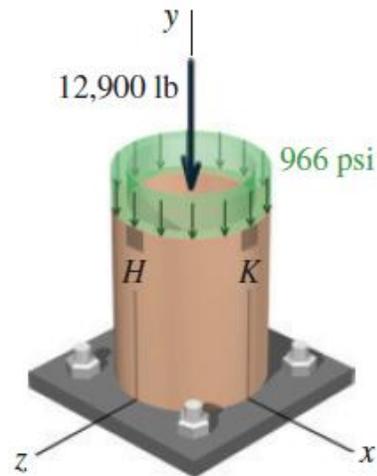
## Section Properties

The outside diameter of the pipe column is  $D = 9.0$  in., and the inside diameter is  $d = 8.0$  in. The area, the moment of inertia, and the polar moment of inertia for the cross section are, respectively, as follows:

$$A = \frac{\pi}{4}[D^2 - d^2] = \frac{\pi}{4}[(9.0 \text{ in.})^2 - (8.0 \text{ in.})^2] = 13.352 \text{ in.}^2$$

$$I = \frac{\pi}{64}[D^4 - d^4] = \frac{\pi}{64}[(9.0 \text{ in.})^4 - (8.0 \text{ in.})^4] = 121.00 \text{ in.}^4$$

$$J = \frac{\pi}{32}[D^4 - d^4] = \frac{\pi}{32}[(9.0 \text{ in.})^4 - (8.0 \text{ in.})^4] = 242.00 \text{ in.}^4$$



## Stresses at $H$

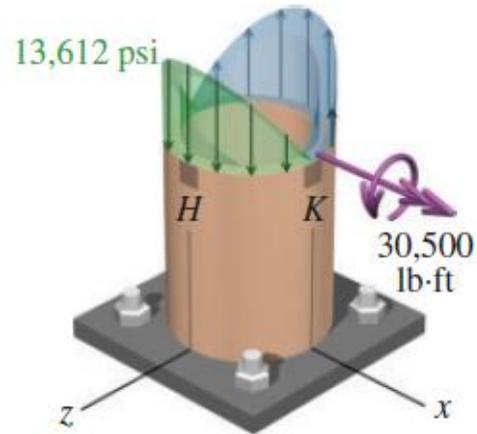
The equivalent forces and moments acting at the section of interest will be evaluated sequentially to determine the type, magnitude, and direction of any stresses created at  $H$ .

The 12,900 lb axial force creates compressive normal stress, which acts in the  $y$  direction:

$$\sigma_y = \frac{F_y}{A} = \frac{12,900 \text{ lb}}{13.352 \text{ in.}^2} = 966 \text{ psi (C)}$$

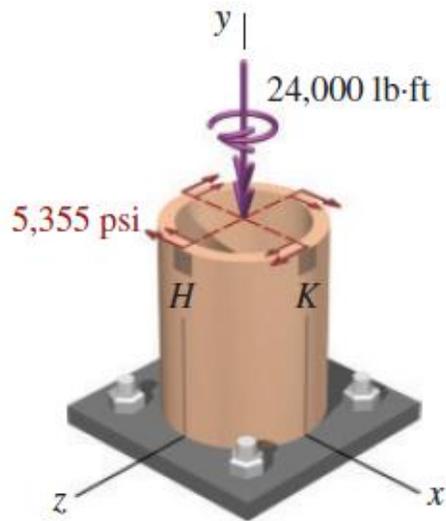
Although shear stresses are associated with the 3,000 lb shear force, the shear stress at point  $H$  is zero.

# Chapter 9 Combined Loads



The 30,500 lb·ft bending moment about the  $x$  axis creates compressive normal stress at  $H$ :

$$\sigma_y = \frac{M_x c}{I_x} = \frac{(30,500 \text{ lb}\cdot\text{ft})(4.5 \text{ in.})(12 \text{ in./ft})}{121.0 \text{ in.}^4} = 13,612 \text{ psi (C)}$$

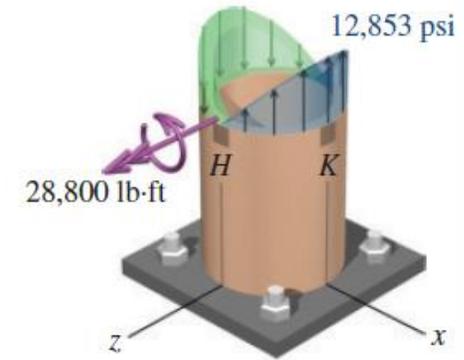


The 24,000 lb·ft torque acting about the  $y$  axis creates shear stress at  $H$ . The magnitude of this shear stress can be calculated from the elastic torsion formula:

$$\tau = \frac{Tc}{J} = \frac{(24,000 \text{ lb}\cdot\text{ft})(4.5 \text{ in.})(12 \text{ in./ft})}{242.0 \text{ in.}^4} = 5,355 \text{ psi}$$

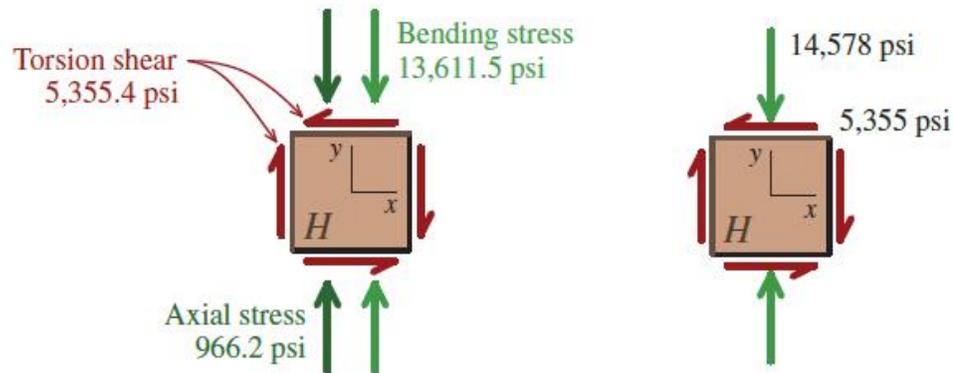
# Chapter 9 Combined Loads

The 28,800 lb·ft bending moment about the  $z$  axis creates bending stresses at the section of interest. Point  $H$ , however, is located on the neutral axis for this bending moment, and thus, the bending stress at  $H$  is zero.



## Combined Stresses at $H$

The normal and shear stresses acting at point  $H$  can be summarized on a stress element. Notice that the torsion shear stress acts in the  $-x$  direction on the  $+y$  face of the element. After the proper shear stress direction has been established on one face, the shear stress directions on the other three faces are known.



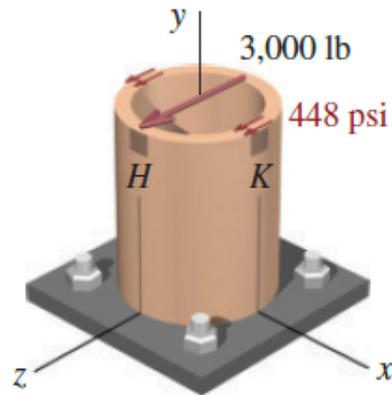
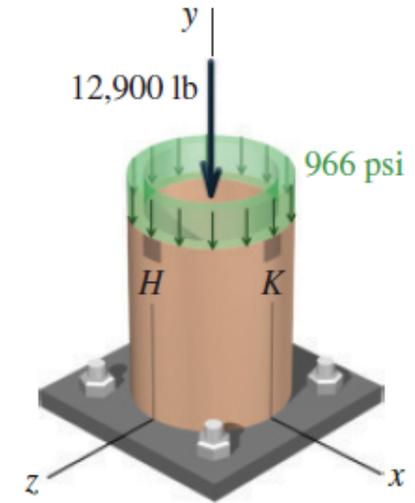
# Chapter 9 Combined Loads

## Stresses at *K*

The equivalent forces and moments acting at the section of interest will again be evaluated, this time to determine the type, magnitude, and direction of any stresses created at *K*.

The 12,900 lb axial force creates compressive normal stress, which acts in the *y* direction:

$$\sigma_y = \frac{F_y}{A} = \frac{12,900 \text{ lb}}{13.352 \text{ in.}^2} = 966 \text{ psi (C)}$$



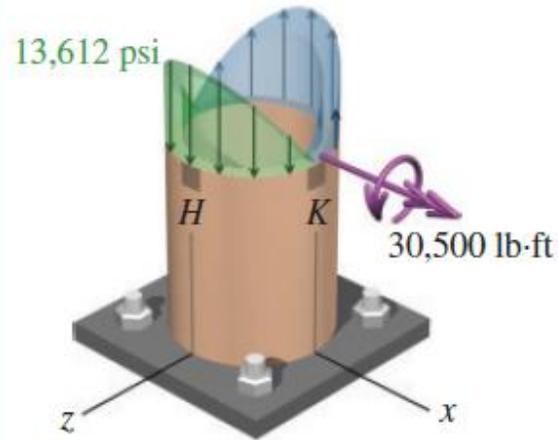
The 3,000 lb shear force acting horizontally at the section of interest is also associated with shear stress at point *K*. From Equation (9.10), the first moment of area for the hollow circular cross section is

$$Q = \frac{1}{12}[D^3 - d^3] = \frac{1}{12}[(9.0 \text{ in.})^3 - (8.0 \text{ in.})^3] = 18.083 \text{ in.}^3$$

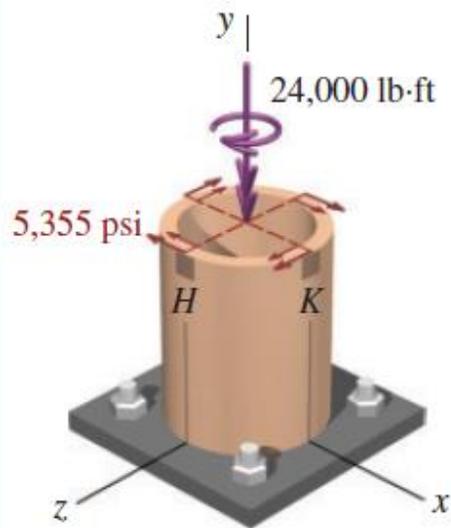
The shear stress formula [Equation (9.2)] is used to calculate the shear stress:

$$\tau = \frac{VQ}{I_x t} = \frac{(3,000 \text{ lb})(18.083 \text{ in.}^3)}{(121.0 \text{ in.}^4)(9 \text{ in.} - 8 \text{ in.})} = 448 \text{ psi}$$

# Chapter 9 Combined Loads



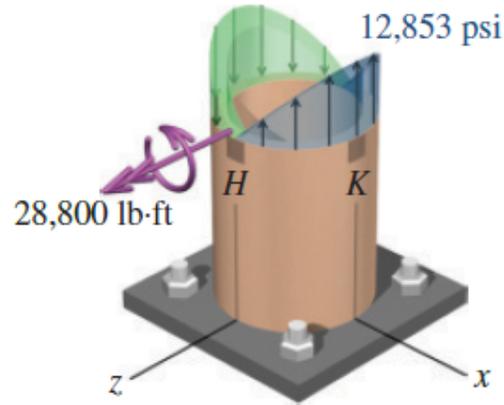
The 30,500 lb·ft bending moment about the  $x$  axis creates bending stresses at the section of interest. Point  $K$ , however, is located on the neutral axis for this bending moment, and consequently, the bending stress at  $K$  is zero.



The 24,000 lb·ft torque acting about the  $y$  axis creates shear stress at  $K$ . The magnitude of this shear stress can be calculated from the elastic torsion formula:

$$\tau = \frac{Tc}{J} = \frac{(24,000 \text{ lb}\cdot\text{ft})(4.5 \text{ in.})(12 \text{ in./ft})}{242.0 \text{ in.}^4} = 5,355 \text{ psi}$$

# Chapter 9 Combined Loads

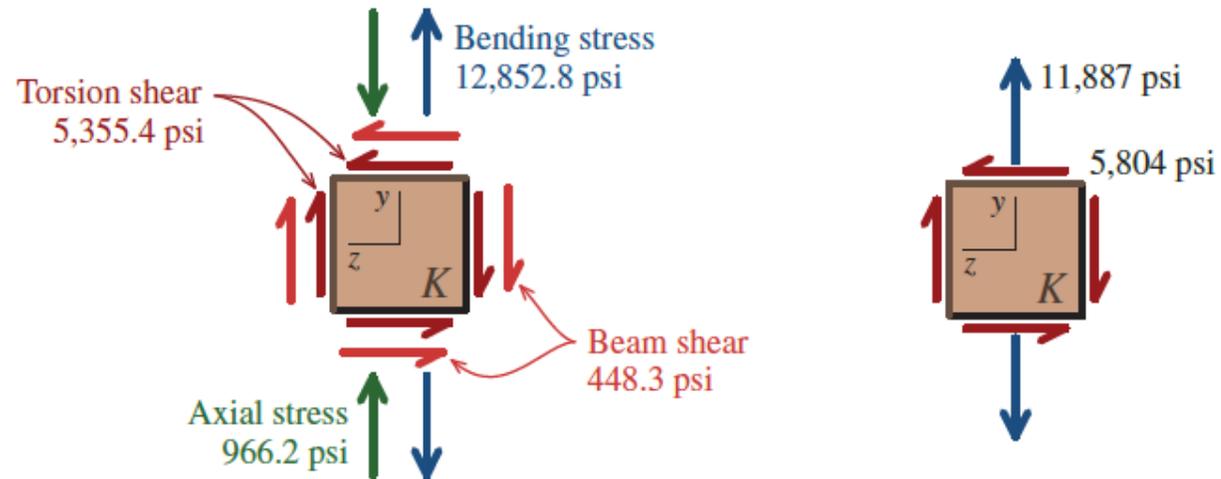


The 28,800 lb·ft bending moment about the z axis creates tensile normal stress at K:

$$\sigma_y = \frac{M_z c}{I_z} = \frac{(28,800 \text{ lb}\cdot\text{ft})(4.5 \text{ in.})(12 \text{ in./ft})}{121.0 \text{ in.}^4} = 12,853 \text{ psi (T)}$$

## Combined Stresses at K

The normal and shear stresses acting at point K can be summarized on a stress element.



# Chapter 9 Combined Loads

**Example 9.3:** A short 100 mm square steel bar with a 50 mm diameter axial hole is built at the base and is loaded at the top as shown in the figure. Determine the value of the force  $P$  so that the maximum normal stress at the fixed-end would not exceed 140 MPa.

Solution:

$$A = (100)^2 - \frac{\pi(50)^2}{4} = 8037\text{mm}^2$$

$$I = \frac{100(100)^3}{12} - \frac{\pi(25)^4}{4} = 8.03 \cdot 10^6\text{mm}^4$$

$$\sigma_A = -\frac{P\cos 30}{A} - \frac{(P\sin 30 \cdot 400) \cdot C}{I} + \frac{(P\cos 30 \cdot 50) \cdot C}{I} = 140\text{MPa}$$

$$140 = -\frac{P\cos 30}{8037} - \frac{(P\sin 30 \cdot 400) \cdot 50}{8.03 \cdot 10^6} + \frac{(P\cos 30 \cdot 50) \cdot 50}{8.03 \cdot 10^6}$$

$$140 = -1.078 \cdot 10^{-4}P - 1.25 \cdot 10^{-3}P + 2.7 \cdot 10^{-4}P = -1.088 \cdot 10^{-3}P$$

$$140 = \frac{P}{954.4} \Rightarrow P = 129\text{kN}$$

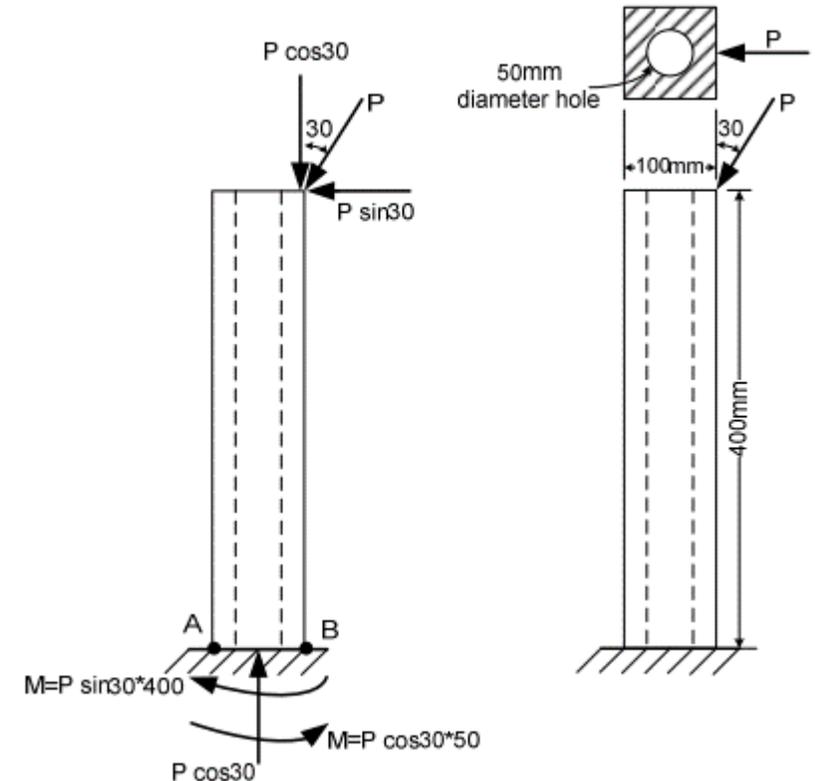
$$\sigma_B = -\frac{P\cos 30}{A} + \frac{(P\sin 30 \cdot 400) \cdot C}{I} - \frac{(P\cos 30 \cdot 50) \cdot C}{I} = 140\text{MPa}$$

$$140 = -\frac{P\cos 30}{8037} + \frac{(P\sin 30 \cdot 400) \cdot 50}{8.03 \cdot 10^6} - \frac{(P\cos 30 \cdot 50) \cdot 50}{8.03 \cdot 10^6}$$

$$140 = -1.078 \cdot 10^{-4}P + 1.25 \cdot 10^{-3}P - 2.7 \cdot 10^{-4}P = +8.72 \cdot 10^{-4}P$$

$$140 = \frac{P}{1147} \Rightarrow P = 161\text{kN}.$$

**The safe force  $P=129\text{kN}$**

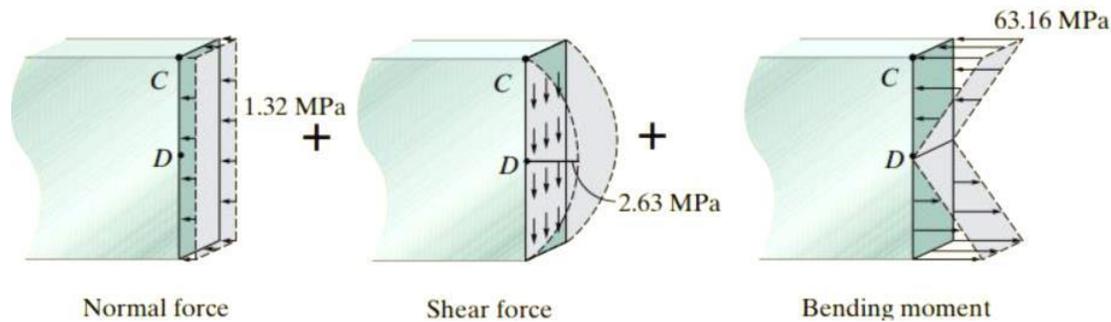


# Chapter 9 Combined Loads

**Example 9.4:** The member shown in the figure has a rectangular cross section. Determine the state of stress that the loading produces at point C and point D.

Solution:

$$N = 16.45 \text{ kN} \quad V = 21.93 \text{ kN} \quad M = 32.89 \text{ kN} \cdot \text{m}$$



**Stress Components at point C:**

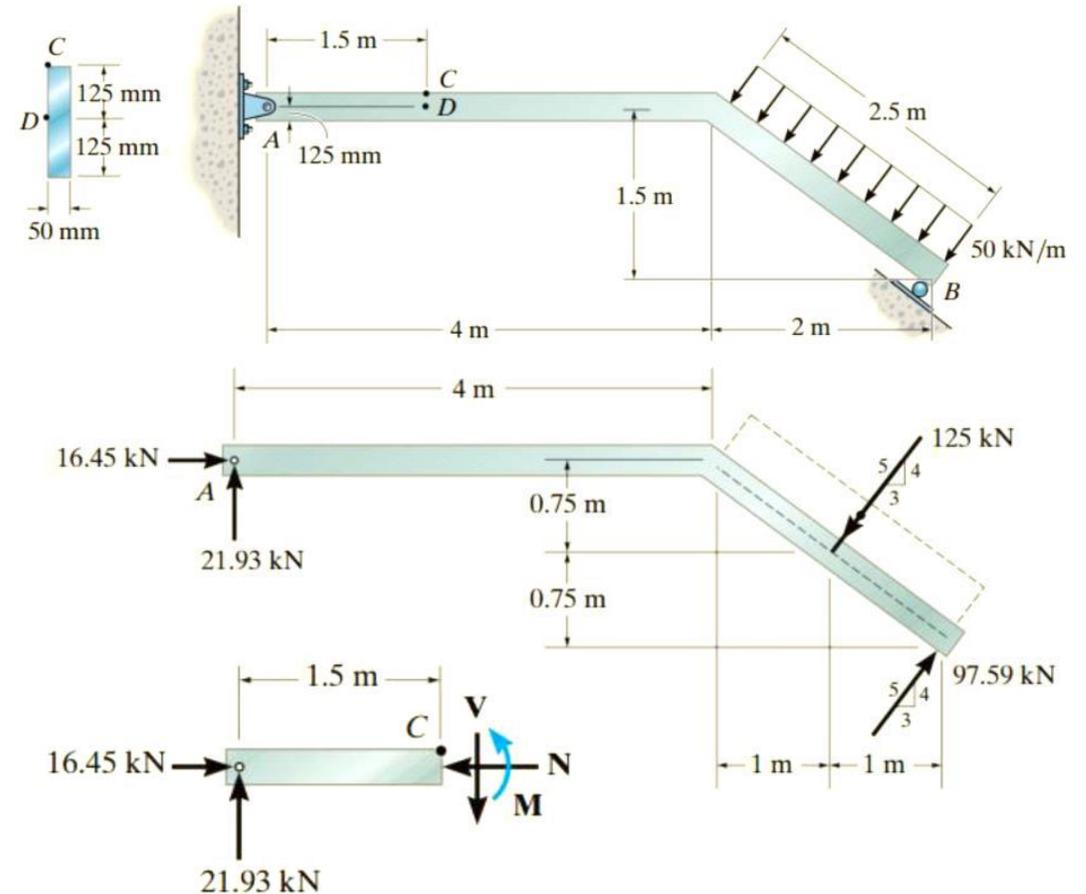
Normal Force:

$$\sigma_C = \frac{P}{A} = \frac{16.45(10^3) \text{ N}}{(0.050 \text{ m})(0.250 \text{ m})} = 1.32 \text{ MPa}$$

Shear Force:

$$Q = \bar{y}' A' = 0$$

$$\tau_C = 0$$



# Chapter 9 Combined Loads

## Bending Moment:

Point  $C$  is located at  $y = c = 0.125$  m from the neutral axis, so the normal stress at  $C$  is:

$$\sigma_C = \frac{M_c}{I} = \frac{(32.89(10^3) \text{ N} \cdot \text{m})(0.125 \text{ m})}{\left[\frac{1}{12}(0.050 \text{ m})(0.250 \text{ m})^3\right]} = 63.16 \text{ MPa}$$

Superposition. The shear stress is zero. Adding the normal stresses determined above gives a compressive stress at  $C$  having a value of

$$\sigma_c = 1.32 \text{ MPa} + 63.16 \text{ MPa} = 64.5 \text{ MPa}$$

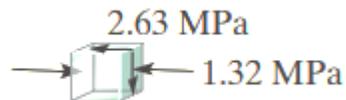
## Stress Components at $D$ .

**Normal Force.** This is the same as at  $C$ ,  $\sigma_D = 1.32$  MPa.

**Shear Force.**  $\tau_D = 1.5 \frac{V}{A} = 1.5 \frac{21.93(10^3)}{(250)(50)} = 2.63$  MPa

**Bending Moment.** Here  $D$  is on the neutral axis and so  $\sigma_D = 0$ .

## Superposition.



# Chapter 9 Combined Loads

**Example 9.5:** Find the maximum shearing stress due to the applied forces in the plane AB of the 10mm diameter circular shaft.

Solution:

a. Shear stress due to the shear force

$$\tau @ A \text{ and } B = 0;$$

$$\tau @ C, D \text{ and } E = \tau_{\max}.$$

$$\tau = \frac{VQ}{Ib}$$

$$\tau_{\max} = \frac{250 * \left[ \frac{\pi(5^2)}{2} * \left( \frac{4 * 5}{3\pi} \right) \right]}{\frac{\pi(5)^4}{4} * 10} = 4.24 \text{ MPa}$$

b. Shear stress due to the torque

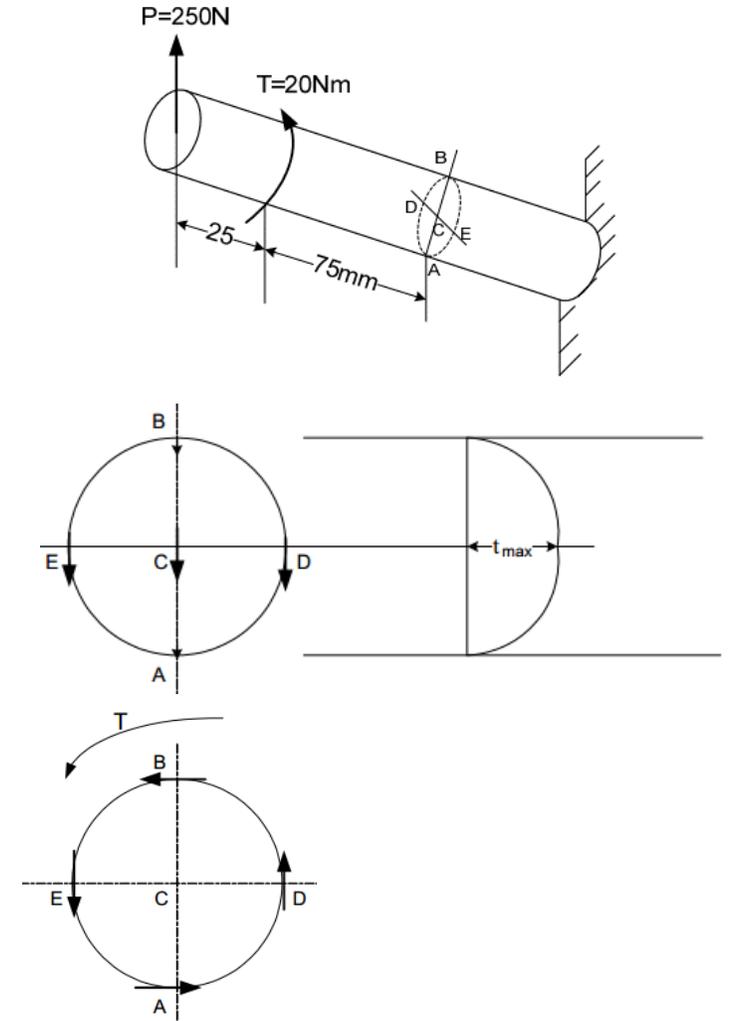
$$\tau @ C = 0;$$

$$\tau @ A, B, D \text{ and } E = \tau_{\max}.$$

$$\tau_{\max} = \frac{TR}{J}$$

$$\tau_{\max} = \frac{20000 * 5}{\frac{\pi}{2} (5^4)} = 101.86 \text{ MPa}$$

$$\tau_{\max} \text{ occurs @ point } E = 4.24 + 101.86 = 106.1 \text{ MPa}$$



# Chapter 9 Combined Loads

**H.W:** A steel pipe with an outside diameter of 4.500 in. and an inside diameter of 4.026 in. supports the loadings shown in the figure. Determine:

- The normal and shear stresses on the top of the pipe at point  $H$ .
- The normal and shear stresses on the side of the pipe at point  $K$ .
- The principal stresses and the magnitude of the maximum in-plane shear stress at point  $H$ , and show the orientation of these stresses on an appropriate sketch.
- The principal stresses and the magnitude of the maximum in-plane shear stress at point  $K$ , and show the orientation of these stresses on an appropriate sketch.

