Fluid dynamics

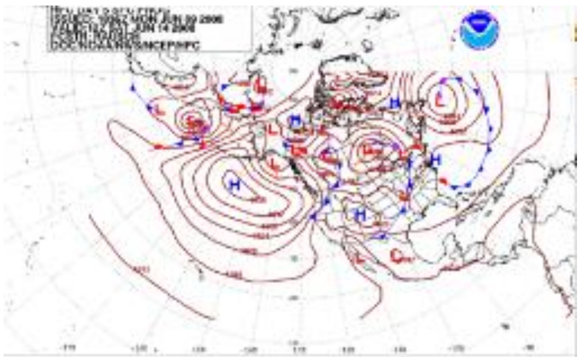
In physics and engineering, fluid dynamics is a subdiscipline of fluid mechanics that describes the flow of fluids—liquids and gases. It has several subdisciplines, including **aerodynamics** (the study of air and other gases in motion) and hydrodynamics (the study of liquids in motion).

Fluid dynamics has a wide range of applications, including calculating forces and moments on aircraft, determining the mass flow rate of **petroleum** through **pipelines**, and predicting weather patterns.

Aerodynamics, from Greek ἀήρ *aero* (air) + δυναμική (dynamics), is the study of motion of air, particularly when affected by a solid object, such as an airplane wing. It is a sub-field of fluid dynamics and gas dynamics, and many aspects of aerodynamics theory are common to these fields.

Petroleum, also known as **crude oil** and **oil**, is a naturally occurring, yellowish-black liquid found in geological formations beneath the Earth's surface. It is commonly refined into various types of fuels.

Pipeline transport is the long-distance transportation of a liquid or gas through a system of pipes—a pipeline—typically to a market area for consumption.

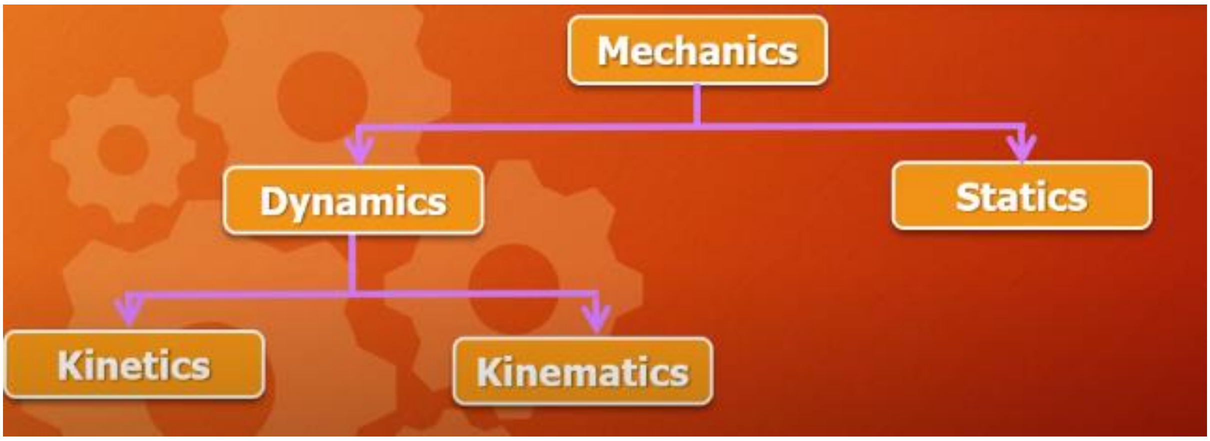


Weather forecasting is the application of science and technology to predict the conditions of the atmosphere for a given location and time.

Kinetics and Kinematics:

Kinetics is focused on understanding the cause of different types of motions of an object such as rotational motion in which the object experiences force or torque.

Kinematics explains the terms such as acceleration, velocity, and position of objects. The mass of the object is not considered while studying the kinematics.



BERNOULLI EQUATION:

The **Bernoulli equation** is *an approximate relation between pressure, velocity, and elevation,* and is valid in *regions of steady, incompressible flow where net frictional forces are negligible* (Fig. 12–1). Despite its simplicity, it has proven to be a very powerful tool in fluid mechanics. In this

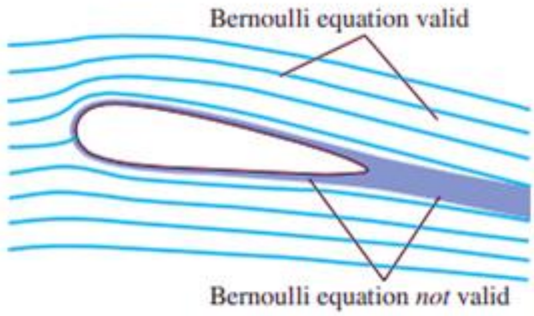


FIGURE 12–1 The *Bernoulli equation* is an *approximate* equation that is valid only in *inviscid regions of flow* where net viscous forces are negligibly small compared to inertial, gravitational, or pressure forces. Such regions occur outside of *boundary layers* and *wakes*.

Derivation of the Bernoulli Equation

Consider the motion of a fluid particle in a flow field in steady flow. Applying Newton's second law in the s-direction on a particle moving along a streamline gives

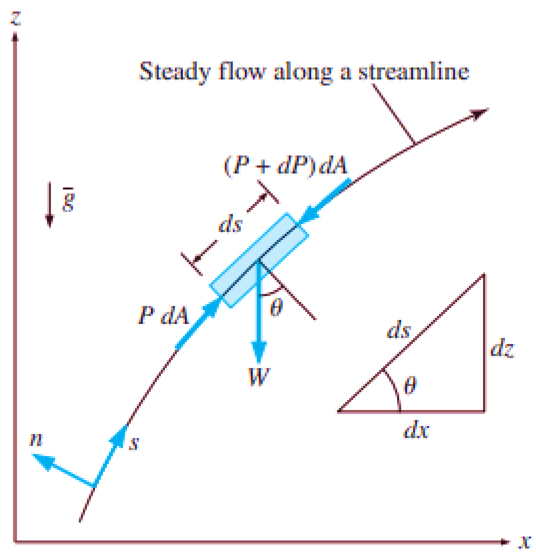


FIGURE 12–3 The forces acting on a fluid particle along a streamline.

$$\sum F_s = ma_s \tag{12–3}$$

In regions of flow where net frictional forces are negligible, there is no pump or turbine, and there is no heat transfer along the streamline, the significant forces acting in the s-direction are the pressure (acting on both sides) and the component of the weight of the particle in the s-direction (Fig. 12–3). Therefore, Eq. 12–3 becomes

$$P\,dA - (P + dP)\,dA - W \sin\theta = mV\frac{dV}{ds} \tag{12–4}$$

$$a_s = \frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} = \frac{\partial V}{\partial s}V = V\frac{dV}{ds}$$

where θ is the angle between the normal of the streamline and the vertical z-axis at that point, $m = \rho V = \rho\,dA\,ds$ is the mass, $W = mg = \rho g\,dA\,ds$ is the weight of the fluid particle, and $\sin\theta = dz/ds$. Substituting,

$$-dP\,dA - \rho g\,dA\,ds\frac{dz}{ds} = \rho\,dA\,ds\,V\frac{dV}{ds} \tag{12–5}$$

Canceling dA from each term and simplifying,

$$-dP - \rho g\,dz = \rho V\,dV \tag{12–6}$$

Noting that $V\,dV = \frac{1}{2}d(V^2)$ and dividing each term by ρ gives

$$\frac{dP}{\rho} + \frac{1}{2}d(V^2) + g\,dz = 0 \tag{12–7}$$

Integrating,

Steady flow: $$\int \frac{dP}{\rho} + \frac{V^2}{2} + gz = \text{constant (along a streamline)} \tag{12–8}$$

since the last two terms are exact differentials. In the case of incompressible flow, the first term also becomes an exact differential, and integration gives

Steady, incompressible flow: $$\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant (along a streamline)} \tag{12–9}$$

This is the famous **Bernoulli equation** (Fig. 12–4), which is commonly used in fluid mechanics for steady, incompressible flow along a streamline in inviscid regions of flow.

The value of the constant in Eq. 12–9 can be evaluated at any point on the streamline where the pressure, density, velocity, and elevation are known. The Bernoulli equation can also be written between any two points on the same streamline as

Steady, incompressible flow:
$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2 \qquad \textbf{(12–10)}$$

We recognize $V^2/2$ as *kinetic energy, gz* as *potential energy*, and P/ρ as *flow energy*, all per unit mass. Therefore, the Bernoulli equation can be viewed as an expression of *mechanical energy balance* and can be stated as follows (Fig. 12–5):

> The sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow when compressibility and frictional effects are negligible.

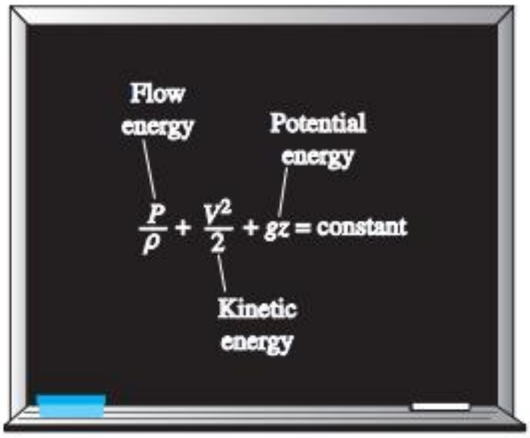


FIGURE 12–5 The Bernoulli equation states that the sum of the kinetic, potential, and flow energies (all per unit mass) of a fluid particle is constant along a streamline during steady flow.

It is often convenient to represent the level of mechanical energy graphically using *heights* to facilitate visualization of the various terms of the Bernoulli equation. This is done by dividing each term of the Bernoulli equation by g to give

$$\frac{P}{\rho g} + \frac{V^2}{2g} + z = H = \text{constant} \qquad \text{(along a streamline)} \qquad (12\text{–}16)$$

Each term in this equation has the dimension of length and represents some kind of "head" of a flowing fluid as follows:

- $P/\rho g$ is the **pressure head**; it represents the height of a fluid column that produces the static pressure P.
- $V^2/2g$ is the **velocity head**; it represents the elevation needed for a fluid to reach the velocity V during frictionless free fall.
- z is the **elevation head**; it represents the potential energy of the fluid.

Also, H is the **total head** for the flow. Therefore, the Bernoulli equation is expressed in terms of heads as: *The sum of the pressure, velocity, and elevation heads along a streamline is constant during steady flow when compressibility and frictional effects are negligible* (Fig. 12–13).

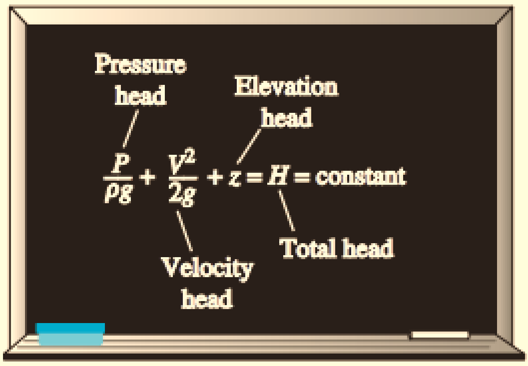


FIGURE 12–13 An alternative form of the Bernoulli equation is expressed in terms of heads as: *The sum of the pressure, velocity, and elevation heads is constant along a streamline.*

Energy grade line (EGL) & Hydraulic grade line (HGL):

If a piezometer (which measures static pressure) is tapped into a pressurized pipe, as shown in Fig. 12–14, the liquid would rise to a height of $P/\rho g$ above the pipe center.

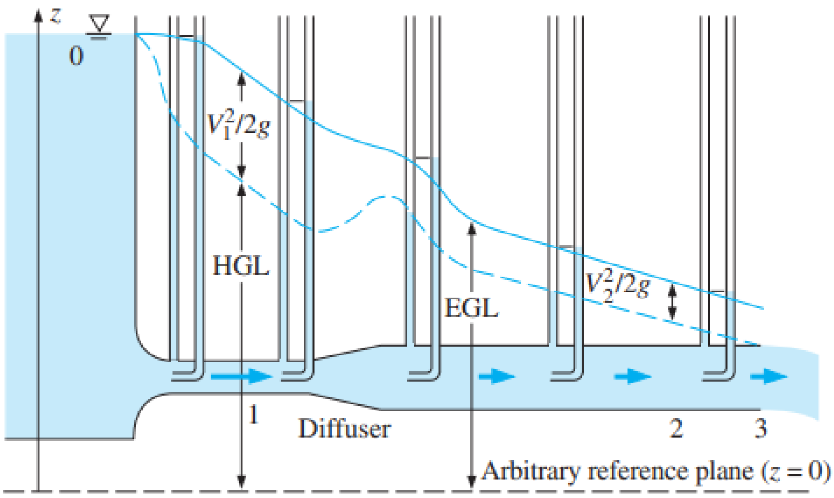


FIGURE 12–14

The *hydraulic grade line* (HGL) is obtained by doing this at several locations along the pipe and drawing a curve through the liquid levels in the piezometers. The vertical distance above the pipe center is a measure of pressure within the pipe.

Similarly, if a Pitot tube (measures static + dynamic pressure) is tapped into a pipe, the liquid would rise to a height of $P/\rho g + V^2/2g$ above the pipe center, or a distance of $V^2/2g$ above the HGL.

The *energy grade line* (EGL) is obtained by doing this at several locations along the pipe and drawing a curve through the liquid levels in the Pitot tubes.

Noting that the fluid also has elevation head z (unless the reference level is taken to be the centerline of the pipe), the HGL and EGL are defined as follows:

The line that represents the sum of the static pressure and the elevation heads, $P/\rho g + z$, is called the **hydraulic grade line**.

The line that represents the total head of the fluid, $P/\rho g + V^2/2g + z$, is called the **energy grade line**. The difference between the heights of EGL and HGL is equal to the dynamic head, $V^2/2g$.

In an *idealized Bernoulli-type flow,* EGL is horizontal and its height remains constant. This would also be the case for HGL when the flow velocity is constant (Fig. 12–15).

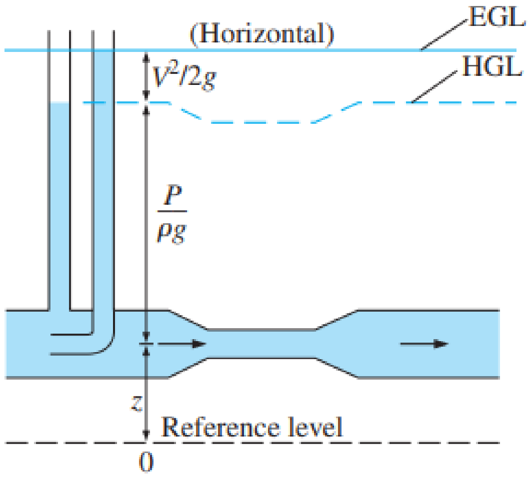


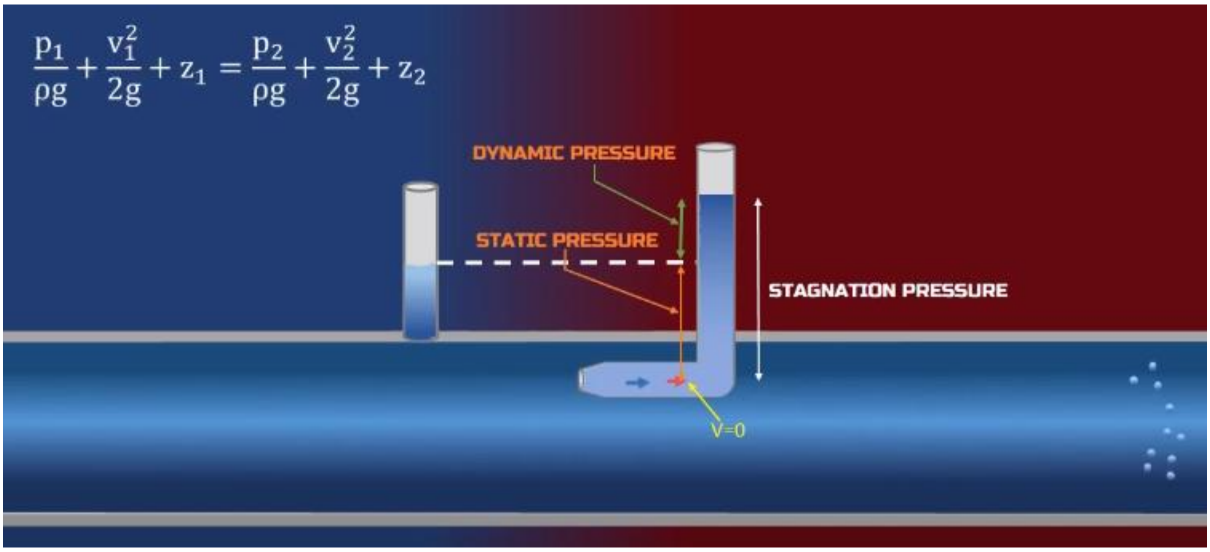
FIGURE 12–15

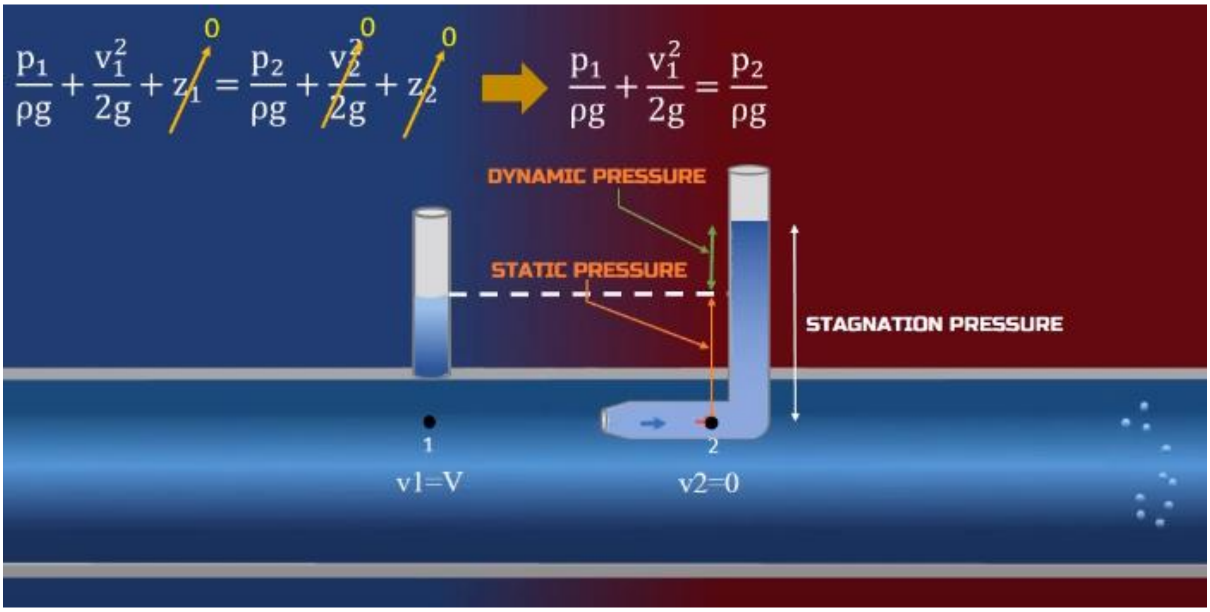
https://www.youtube.com/watch?v=-oecDDrYfyY

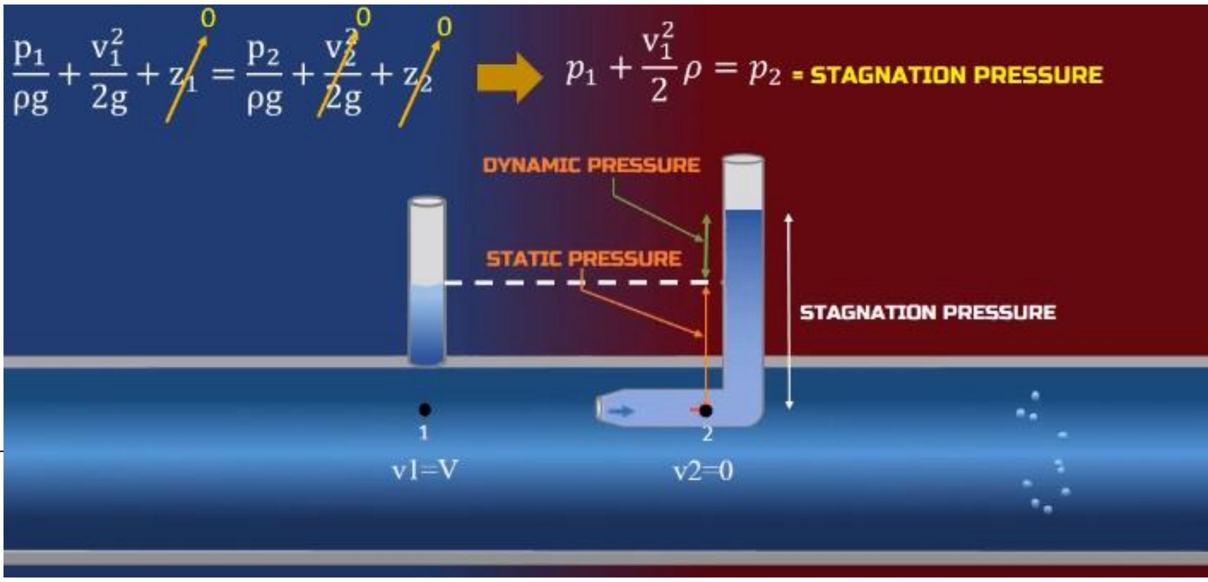
Flow measuring devices:

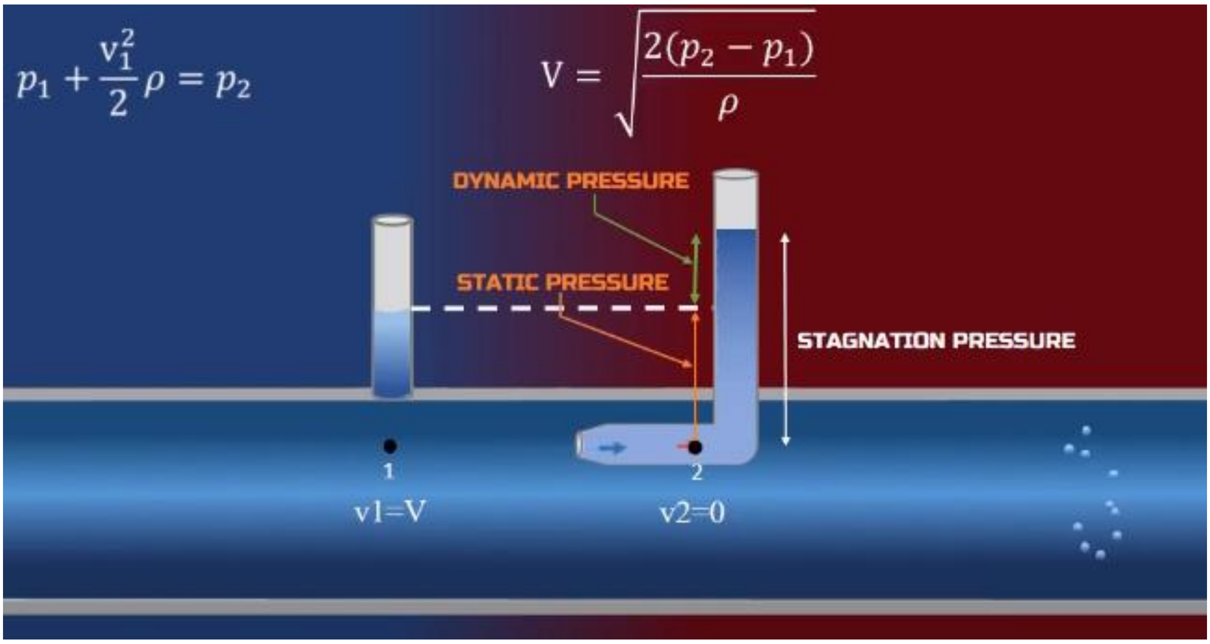
1. Pitot Tube:

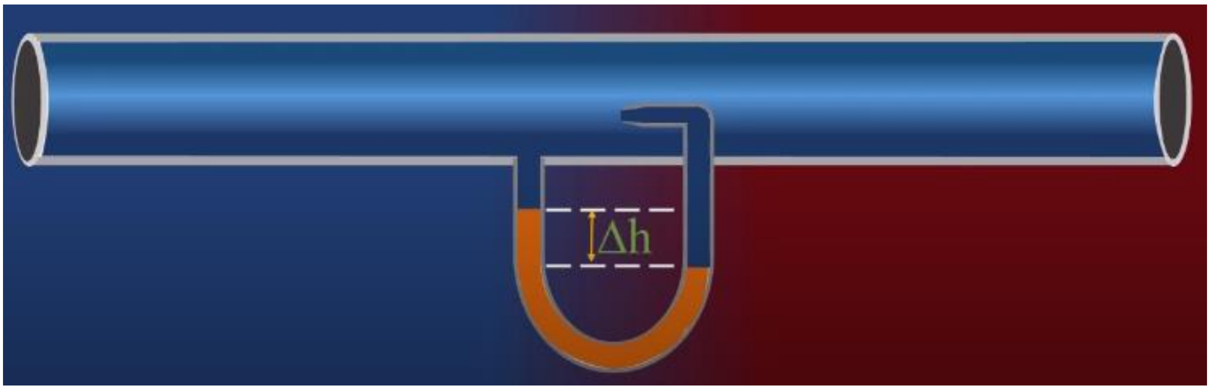
Example 1:

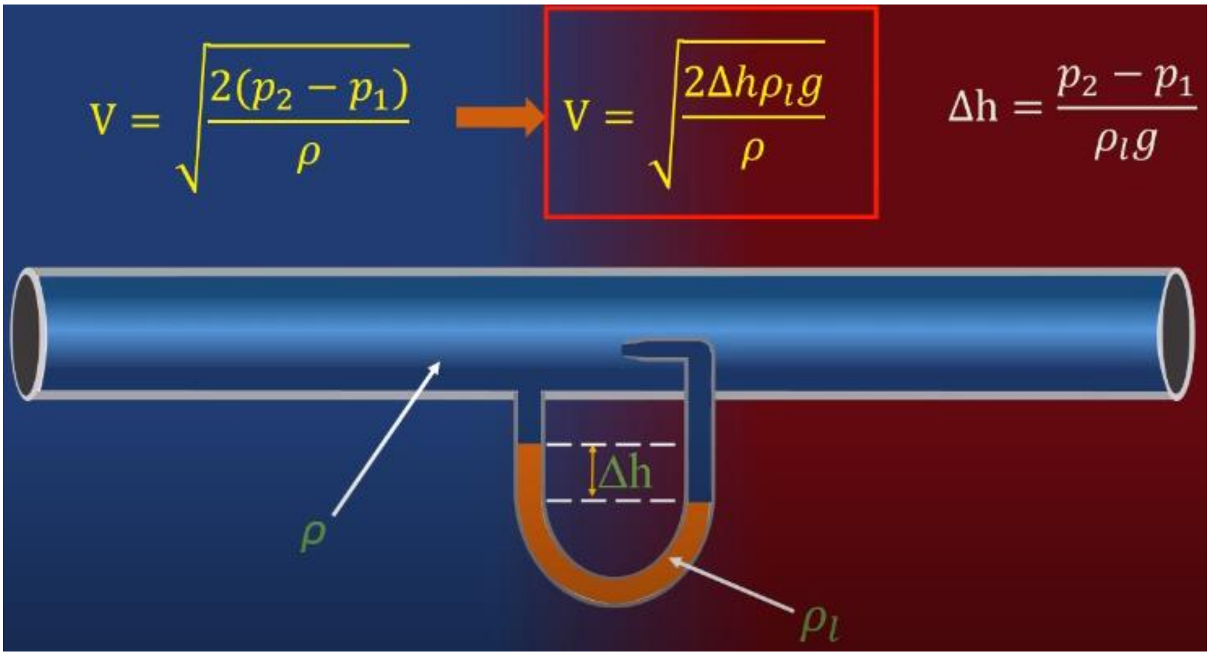






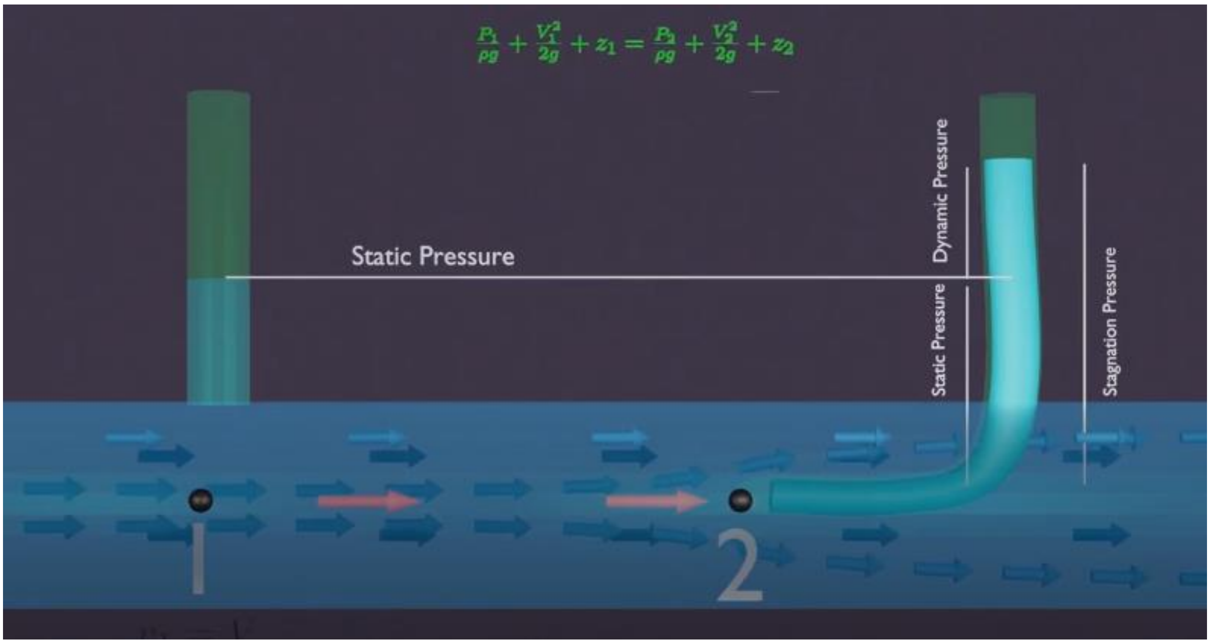


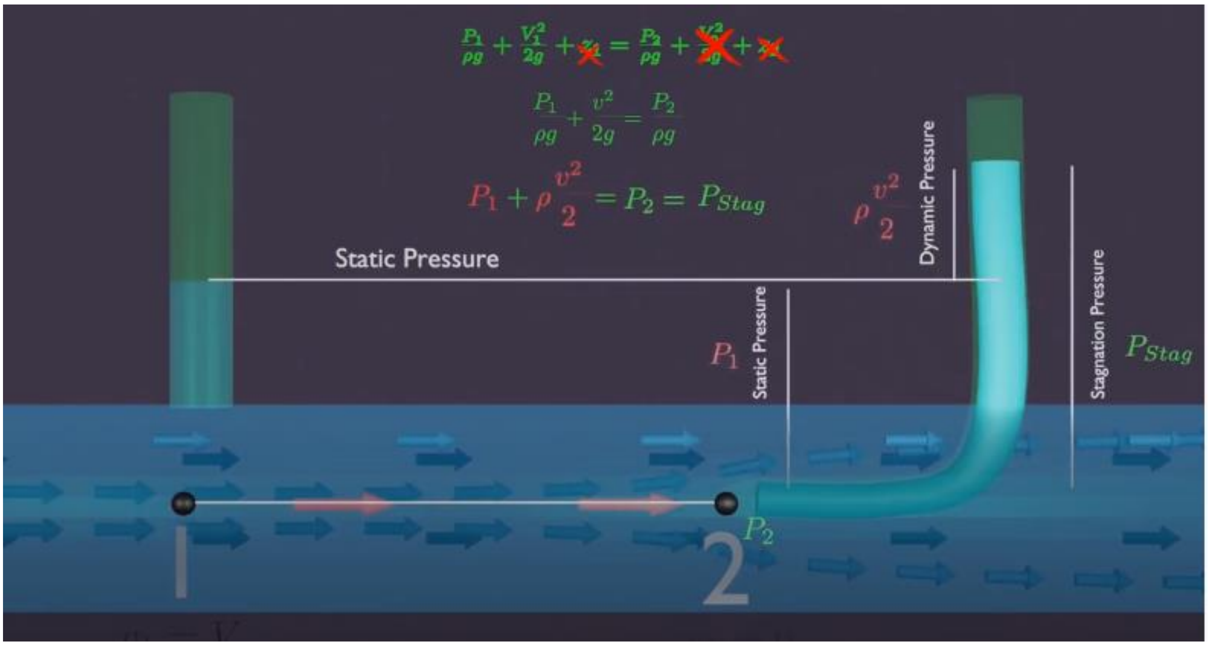




https://www.youtube.com/watch?v=wBXqF2Z3L7g

Example 2:

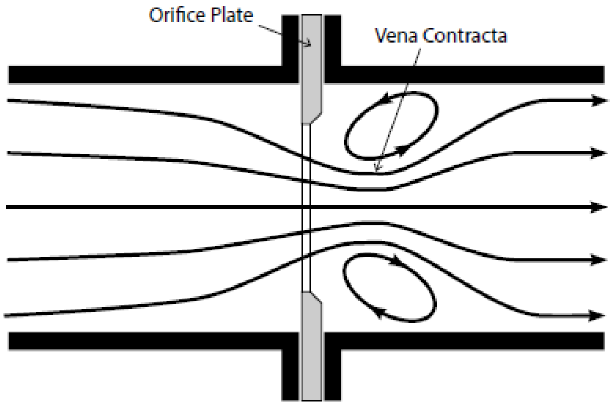




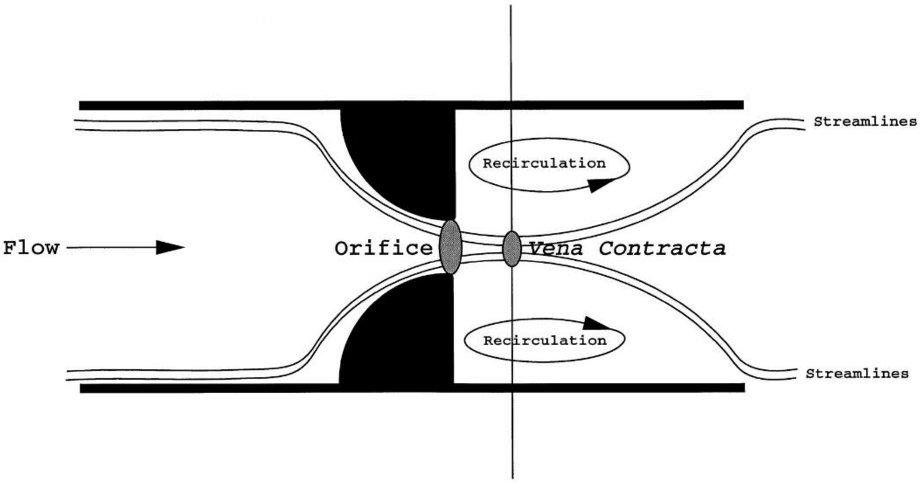
https://www.youtube.com/watch?v=3zEdtkuNYLU

2. Orifice and vena contracta:

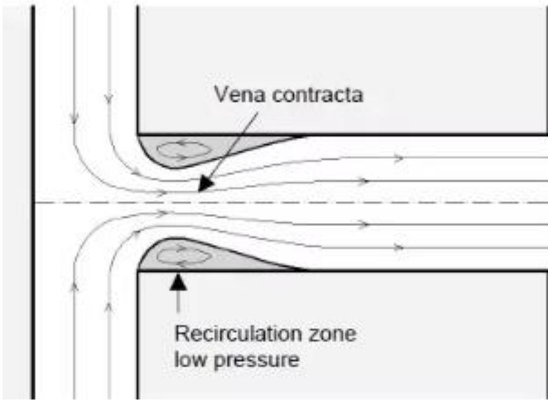
Orifice Meter is an instrument which is used to measure average velocity of a flowing fluid or flow rate of a flowing fluid.



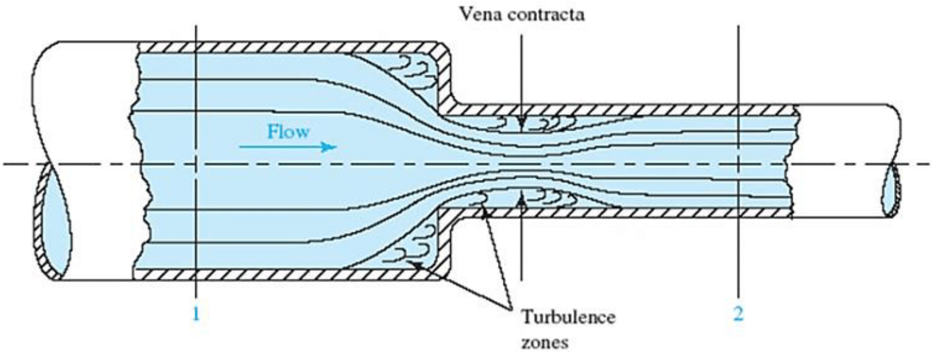
It measures the average velocity and the flow rate of the flowing fluid by introducing a restriction in the direction of the flowing fluid.



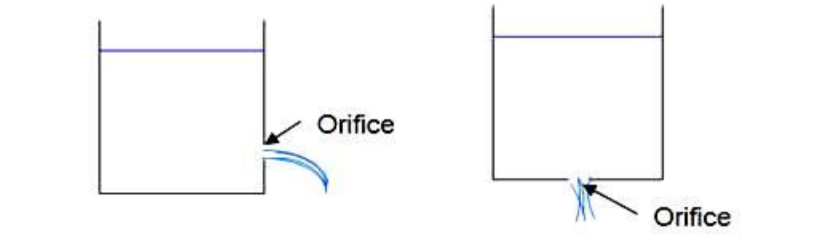
Vena contracta is the point in a fluid stream where the diameter of the stream is the least, and fluid velocity is at its maximum, such as in the case of a stream issuing out of a nozzle (orifice).



Vena-contract is a section in orifice which has a minimum cross-section area and has maximum velocity. At this, all streamlines are parallel, straight, and uniform, so have laminar flow.



An orifice may be an opening in the side or bottom of a vessel/tank to measure the discharge.



a. Orifice at the side of vessel wall b. Orifice at the bottom of vessel

Figure: Orifice

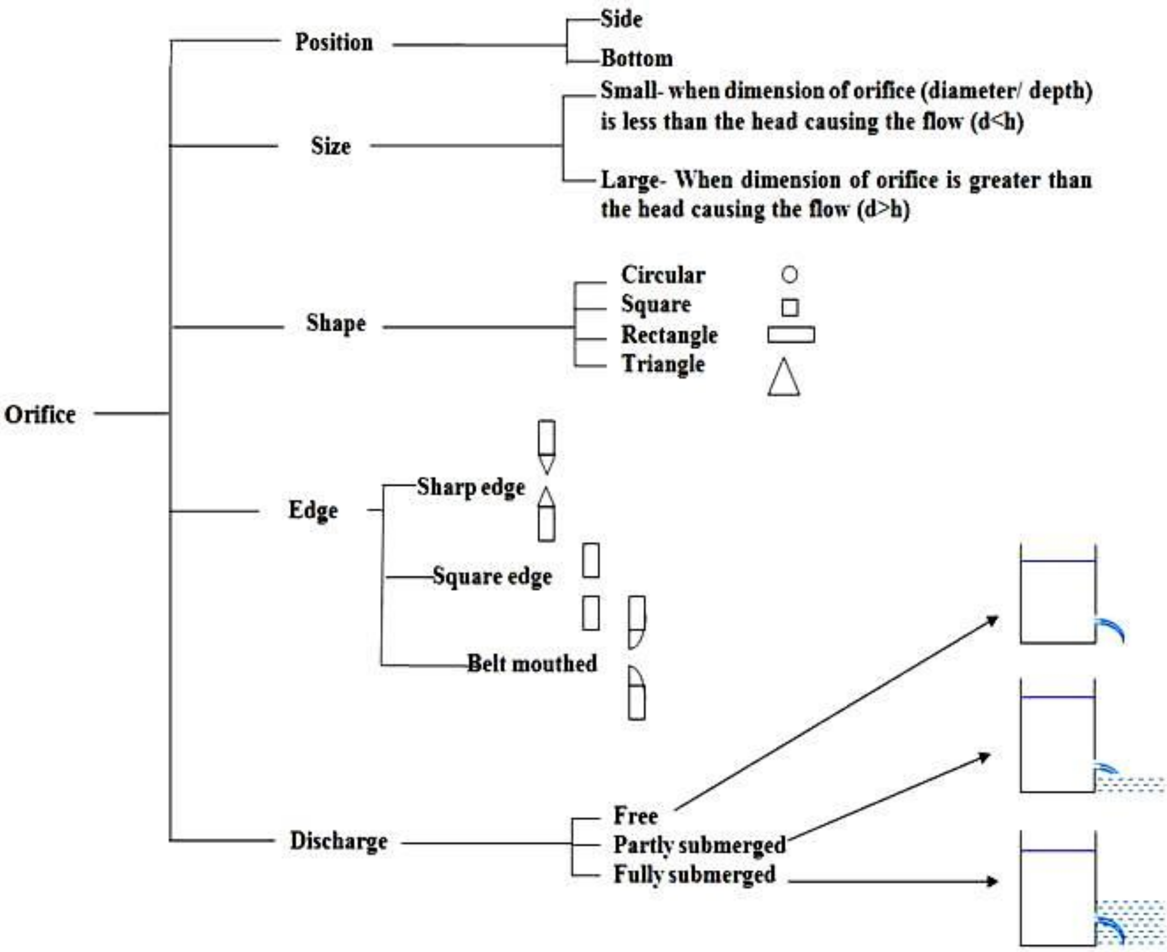


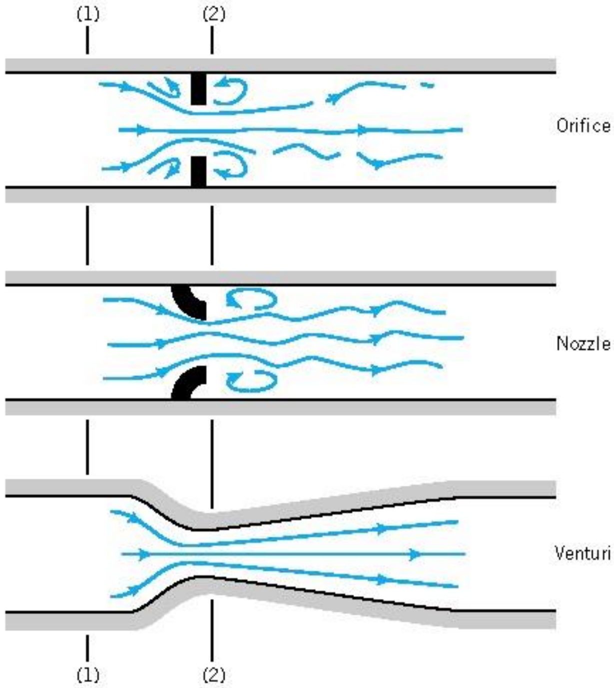
Figure: Classification of orifice

3. Venturi, nozzle and orifice meters:

The Venturi meter is consisted of a rapidly converging section, which increases the flow velocity and reduces the pressure. After the converging section, it returns to the original pipe diameter by a gently diverging diffuser-like section. The discharge is calculated by measuring the pressure differences.

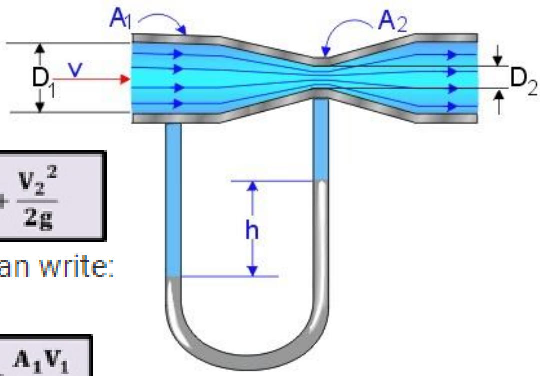
The nozzle meter is similar to Venturi meter, but, instead of converging section, a nozzle is installed inside the pipe, while there is no divergent section.

Finally, **the orifice meter** is a simpler and cheaper arrangement, in which a sharp-edged orifice is fitted concentrically in the pipe.



Equations:

All three instruments use the same principles – mathematical equations. Taking the Venturi meter of the above figure as an example, we apply the Bernoulli equation along the streamline from point 1 to point 2 (throat):



$$\boxed{\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g}}$$

If we consider the fundamental continuity equation, we can write:

$$\boxed{Q = A_1 V_1 = A_1 V_1 \text{ or } V_2 = \frac{A_1 V_1}{A_2}}$$

Substituting the V2 in Bernoulli equation, we take:

$$\boxed{V_1 = \sqrt{\frac{2g\left[\frac{P_1 - P_2}{\rho g} + z_1 - z_2\right]}{\left(\frac{A_1}{A_2}\right)^2 - 1}}}$$