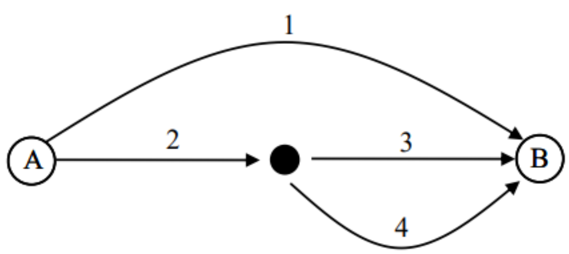
Question Problems

Problem 1:

Consider the following four-link transportation network, consisting of a single origin-destination pair (A, B), and three alternative routes from A to B. The link numbers a (=1, 2, 3, 4) are indicated in the network. The travel demand from A to B is 1000 veh/h.



The link travel times (in minutes), denoted by t_a, depend on the car flows q_a (veh/h), and are given by the following functions:

$$t_1(q_1) = 2 + \frac{3q_1}{500}$$

$$t_2(q_2) = 1 + \frac{q_2}{1000}$$

$$t_3(q_3) = 1 + \frac{q_3}{500}$$

$$t_4(q_4) = 1 + \frac{q_4}{500}$$

Determine analytically the link flows in a user equilibrium assignment.

Solution:

Let the route flows for routes 1, 2, and 3 be denoted by f_1, f_2, and f_3. The travel time on the first route is given by:

$$t_{route1} = 2 + \frac{3f_1}{500}$$

The travel time on the second route is given by

$t_{route2} = 1 + \frac{f_2+f_3}{1000} + 1 + \frac{f_2}{500}$

The travel time on the third route is given by:

$t_{route3} = 1 + \frac{f_2+f_3}{1000} + 1 + \frac{f_3}{500}$

The conversation flow equation is given by:

$f_1 + f_2 + f_3 = 1000$

$2 + \frac{3f_1}{500} = 1 + \frac{f_2+f_3}{1000} + 1 + \frac{f_2}{500}$ 2

$1 + \frac{f_2+f_3}{1000} + 1 + \frac{f_2}{500} = 1 + \frac{f_2+f_3}{1000} + 1 + \frac{f_3}{500}$

$f_1 + f_2 + f_3 = 1000$

The second equation implies $f_2 = f_3$

$2 + \frac{3f_1}{500} = 2 + \frac{2f_2}{1000} + \frac{f_2}{500}$

$f_2 + 2f_2 = 1000$

Which leads to

f_1= 250, f_2=375, f_3=375

The corresponding link flows:

q_1=250, q_2 =750, q_3=375, q_4=375

Problem 2:

Consider a road with a demand of:

$$q_{\text{in}} = \begin{cases} 3600\text{v/h} & \text{for } t < 1\text{h} \\ 5000\text{v/h} & \text{for } 1\text{h} < t < 1.5\text{h} \\ 2000\text{v/h} & \text{for t} > 1.5\text{h} \end{cases}$$

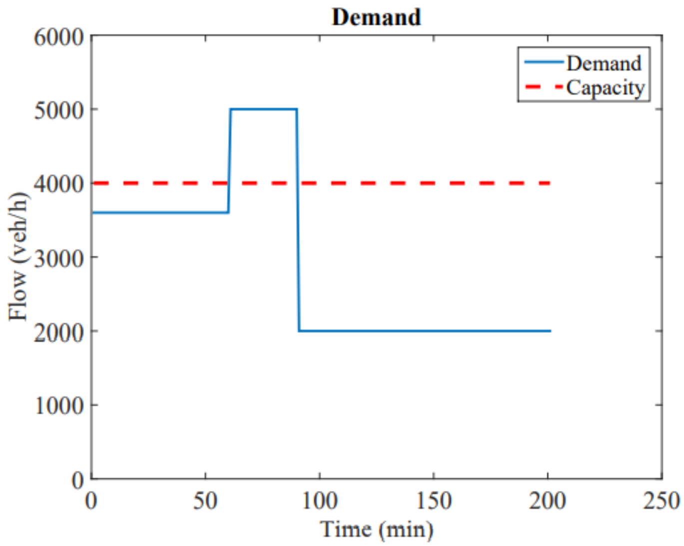


Figure 1: Demand and capacity.

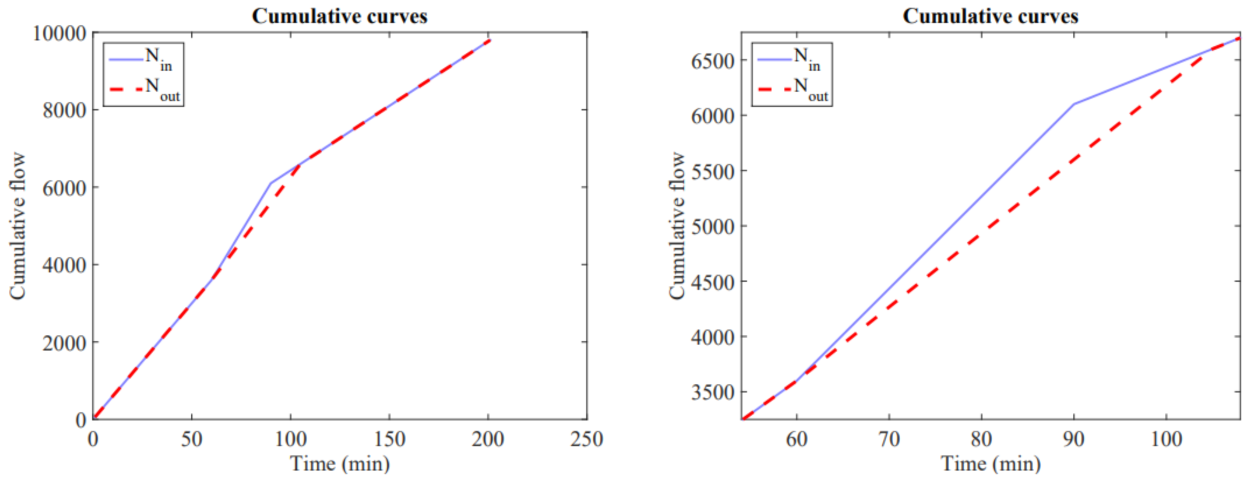
The capacity of the road is 4000 veh/h. A graph of the demand and capacity is shown in Figure 2.

1. Construct the (translated=moved) cumulative curves.
2. Calculate the first vehicle which encounters delay (N).
3. Calculate the time at which the delay is the largest.
4. Calculate the maximum number of vehicles in the queue.
5. Calculate the vehicle number (N) with the largest delay.
6. Calculate the delay this vehicle encounters (in h, or min).
7. Calculate the time the queue is solved.
8. Calculate the last vehicle (N) which encounters a delay.
9. Calculate the total delay (veh-h).
10. Calculate the average delay of the vehicles which are delayed (h).

Solutions:

1. For the cumulative curves, an inflow and an outflow curve needs to be constructed; both increase. For the inflow curve, the slope is equal to the demand. For the outflow curve, the slope is restricted to the capacity. During the first hour, the demand is lower than the capacity, hence the outflow is equal to the demand. From t=1h, the inflow exceeds the capacity and the outflow will be equal to the demand. The cumulative curve hence increases with a slope equal to the capacity. As long as there remains a queue, i.e. the cumulative inflow is higher than the outflow, the outflow remains at capacity. The outflow remains hence increasing with a slope equal to the capacity until

it intersects with the cumulative inflow. Then, the outflow follows the inflow: see Figure 2(a) and for a more detailed Figure 2(b).



(a) Overview of the cumulative curves (b) More detailed view of the cumulative curves

Figure 2: Cumulative curves for the example.

2. The first vehicle which encounters delay (N) Delays as soon as q>C: so after 1h at 3600 v/h = 3600 vehicles.
3. The time at which the delay is largest: A queue builds up as long as q>C, so up to 1.5 h. At that moment, the delay is largest.
4. The maximum number of vehicles in the queue: 0.5 h after the start of the queue, 0.5*5000=2500 veh entered the queue, and 0.5*4000=2000 left: so 500 vehicles are in the queue at t=0.5h.
5. The vehicle number (N) with the largest delay: N (1.5h) =3600+0.5*5000 = 6100.
6. The delay this vehicle encounters (in h, or min): It is the 2500th vehicle after t=1h. The delay is the horizontal delay between the entry and exit curve. It takes at capacity 2500/4000 = 37.5 min to serve 2500 vehicles. It entered 0.5 hours = 30 min after t=1, so the delay is 7.5 min.
7. The time the queue is solved: This is the time point that the inflow and outflow curves intersect again. 500 vehicles is the maximum queue length, and it reduces with 4000-2000=2000veh/h. So 500/2000=15 minutes after the time that q.
8. The last vehicle (N) which encounters delay. This is the vehicle number at the moment the inflow and outflow curves meet again. 15 minutes after the vehicle number with the largest delay: 6100+0.25*2000 = 6600 veh.

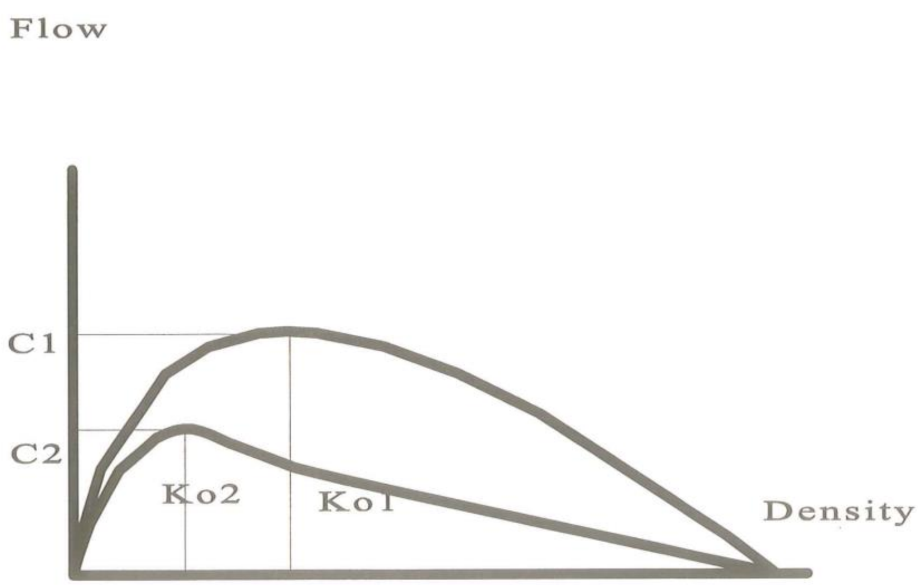
9. The total delay. This is the area of the triangle between inflow and outflow curve. This area is computed by 0.5 * height * base = 0.5 * 500 * (30+15)/60 = 187, 5 veh-h. Note that here we use a generalized equation for the area of a triangle. Indeed, we transform the triangle to a triangle with a base that has the same width, and height which is the same for all times (i.e., we skew it). The height of this triangle is 500 vehicles (the largest distance between the lines) and the width is 45 minutes.
10. The average delay of the vehicles which are delayed (h) 187,5 veh-h/ (6600-3600) veh = 0,0625 h = 3,75 min.

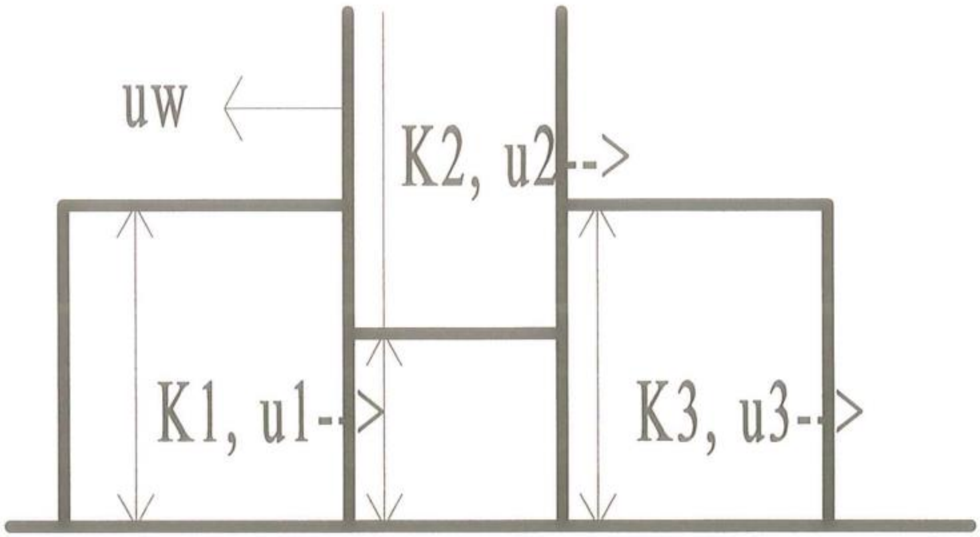
Problem 3:

Using appropriate diagrams, describe the effect of a sudden reduction in capacity (bottleneck) on a highway both in the upstream and downstream sections.

Solutions:

The diagrams below illustrate the impact of a bottleneck on traffic flow> in the first diagram, the reduction in capacity is shown (C_1 reduced C_2) and the corresponding density at capacity changes from k_0^1 to k_0^2. The second diagram illustrates the effect of the botteneck in terms of the shockwave that is formed. When the flow is reduced due to the botteneck, a queue is formed and continues to grow as long as the demand flow is greater than the service flow. The rate at which the queue grows is dependent on the speed of the shockwave, u_w.





Problem 4:

Given: Arrival rate of the vehicle at a ticket gate=30 veh/hr. Arrivals are Poisson. The average service rate is 1.5 min/veh. Determine:

a) Expected queue length, excluding vehicles being served.
b) Probability that no more than 5 cars (including the vehicle being served) are waiting.
c) Average waiting time per vehicle.

Solutions:

q= 30 veh/hr (arrival rate)
Q= 40 veh/ hr (service rate)

a) Expected queue length
   Using equation
   $E(m) = q^2/ [Q(Q-q)] = (30)^2/ [40(40-30)] = 2.25$ vehicles
b) Probability of no more than 5 cars
   $P(n>N) = (q/Q)^{N+1}$ (probability more than N

$P(n>5) = (30/40)^6 = 0.178$

For $P(n \le 5)$ 1- 0.178=0.822

c) Average waiting time per vehicle

E(v) = 1/(Q-q)=0.1 hr = 6 minutes wait time including queue time and service time

E(v) = q/Q(Q-q))=0.075 hr = 4.5 minutes wait time in the queue

Problem 5:

A tunnel has been constructed that connects two cities separated by a river. The cost to use the tunnel, excluding tolls, is expressed as C = 50 + 0.5V, where V is the number of veh/h and C is the out-of-pocket driving cost/vehicle trip. Units are in cents. The traffic demand for travel for a given time period can be expressed as Vt = 2500 - 10C.

Determine:

a) The volume of traffic across the bridge without a toll.
b) The volume of traffic across the bridge with a toll of 25 cents.
c) The toll chart would yield the highest revenue and the resulting travel demand.

Solution:

a) To determine the volume of traffic without a toll, substitute the cost function, C, into the demand function, V:
   a. V = 2500 - 10C = 2500 - 10(50 + 0.5V)
   b. V = 2500 - 500 - 5V = 2000 - 5V
   c. V = 2000
   d. V = 333 veh/h

b) For the volume of traffic if a toll of 25 cents is added, the supply function is
   a. C = 50 + 0.5V + 25. Again, substitute the cost function, C, into the demand function, V.
   b. V = 2500 - 10(75 + 0.5V)
   c. 6V = 1750
   d. V = 292 veh/h

c) To determine the toll to yield the highest revenue, let T = toll rate in cents. The supply function is:
   C = 50 + 0.5V + T. The demand function is
   V = 2500 - 10(50 + 0.5V + T)
   V = (2000 - 10T)/6
   Let R = revenue generated by the toll facility.

$R = VT$

Substitute V in the equation for R.

$R = \{(2000 - 10T)/6\}T = (2000T - 10T^2)/6$

Maximize R by setting $dR/dT = 0$.

$dR/dT = 2000 - 20T = 0$

T = 100 cents. Thus the toll charge maximizes revenues. If this toll is used in the supply function, the equilibrium demand is

$V = (2000 - 10T)/6 = \{2000 - 10(100)\}/6 = 167$ veh/h