Basic Static Assignment to Transportation Networks

Supply Models

Introduction

Traffic assignment models simulate the interaction of demand and supply on a transportation network. These models allow the calculation of performance measures and user flows for each supply element (network link), resulting from origin-destination (O-D) demand flows, path choice behavior, and the mutual interactions between supply and demand.

Assignment models combine the supply and demand models; for this reason, they are also referred to as demand-supply interaction models. Figure 1 illustrates the general modeling framework. Assignment models play a central role in comprehensive transportation system models because their outputs describe the system states, or rather, the mean state and its variation. Assignment model outputs, in turn, are inputs required for the plan, design, and/or evaluation of transportation projects.

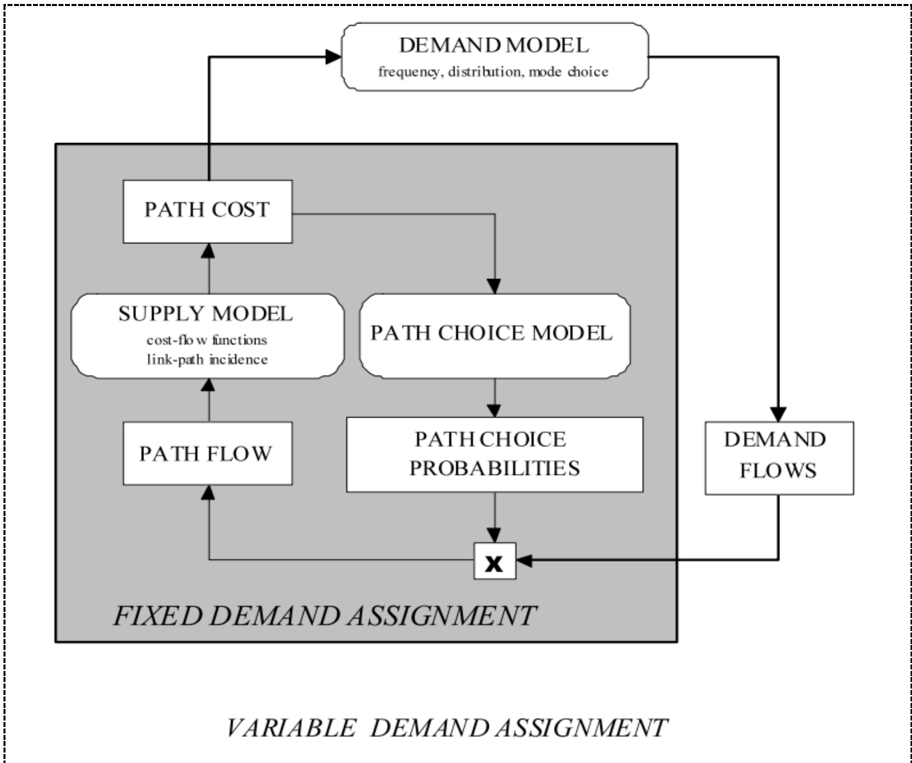


Fig. 1 Schematic representation of assignment models.

Assignment, represents equilibrium configurations of the system, that is, configurations in which demand, path, and link flows are consistent with the costs that they produce in the network. From a mathematical point of view, equilibrium assignment can be defined as the problem of finding a flow vector that reproduces itself based on the correspondence defined by the supply and demand models. This problem can be easily formulated with fixed point models, or else with variation inequality or optimization models, as shown in the following sections. The alternative approach for representing supply-demand interactions leads to between-period (or day-to-day) dynamic process assignment models. In this case, it is assumed that the system evolves (i.e., in successive reference periods), through possibly different feasible states, as a result of changes in the number of users undertaking trips, path choices, supply performance, and so on. Mechanisms that drive the changes from one state to another is the dependency between flows and costs. In a given reference period the system state – defined by the demand, path, and link flows and the corresponding costs – may be internally inconsistent, and this may cause a change towards a different state in the following reference periods. Dynamic process assignment models explicitly simulate the evolution of the system state based on the mechanisms underlying path choice and information acquisition, which in turn determine user choices in successive reference periods. By analogy, equilibrium assignment could be termed within-day static assignment. Dynamic process models can be further categorized as deterministic or stochastic, depending on whether the system state is modeled using deterministic or stochastic (random) variables.

The dependence of link performance variables on flows is the other main supply-based classification factor. When link costs are independent of flows (i.e., congestion effects are negligible), UNcongested network (UN) assignment models result. On the other hand, if link costs depend on flows, congested network assignment models are obtained.

Assignment models can be classified based on assumptions regarding supply characteristics. The first classification factor is the nature of the transportation service being represented; the service can be classified as either continuous or scheduled.

Assignment models can be distinguished based on their hypotheses regarding path choice behavior. In general, the particular path followed for a trip may result from a sequence of decisions made before and during the trip; these are referred to as pre-trip and en-route choices, respectively.

Pre-trip choice, which takes place at the origin before a journey is begun, considers as alternatives either single paths to be followed without deviation from origin to destination, or decision strategies for en-route choice among paths.

En-route choices involve a strategy for determining the path to follow as a result of decisions made during the journey in response to information received while traveling.

Many models consider only fully pre-trip behavior, where the pre-trip choice is between alternative O-D paths, and the chosen path is followed unswervingly to the destination. In all cases, user choice takes into account the cost attributes of the choices offered by the network. For example, the pre-trip path choice model represents the choice of single paths or hyper-paths as a function of the corresponding cost attributes. Models based on random utility theory are typically used to simulate these choices. In particular, deterministic choice models assume that the perceived utility of a path is deterministic and that users will only choose the alternative(s) having maximum average utility (minimum average cost). On the other hand, probabilistic or stochastic choice models assume that the perceived utility of a path is a random variable, and express the probability that users will choose each of the available alternatives.

With respect to demand segmentation, assignment models are called multiuser class models if users are subdivided into several classes. Users in different classes have distinct travel perceptions, behaviors, and/or impacts, whereas all users in a given class are considered sufficiently similar that they can be represented by a single model. In this way, different choice models might be applied to different trip purposes or user socioeconomic categories such as income. Similarly, different vehicle types (motorcycles, cars, commercial vehicles, etc.) might be represented in a road network model. Single-user class assignment is a special case where all users share the same choice model and have the same network effects, and are distinguished only in terms of their origins and destinations.

A demand-related classification factor is the dependence of O-D demands on path performance measures and costs. Fixed (or inelastic) demand assignment models assume that O-D demand flows are independent of changes in network costs that may occur as a result of congestion. Variable demand models, on the other hand, assume that demand flows vary with congestion costs; demand flows are therefore a function of path costs resulting from congestion, as well as of activity system attributes. Depending on the modeling context, demand might be an assumed variable in certain choice dimensions only. For example, it might be assumed that the total O-D matrix is cost-independent (meaning that frequency and destination choices are not influenced by cost variations), but that mode choice is affected by the relative costs of the available modes; in this way, multimode assignment models are obtained. Obviously, from a practical viewpoint, demand elasticity is relevant only for congested networks where costs depend on flows.

Transportation systems can be represented under two contrasting assumptions regarding the within-period variability of their characteristics. This lecture does not consider possible variations of demand and/or supply within the reference period considered for the network analysis (e.g., the morning peak period). The assignment models presented here are thus within-period (or within-day) static. This hypothesis is realistic only if travel demand and supply characteristics can reasonably be assumed constant over a reference period that is long compared to typical trip times in the system. Thus static assignment models are mainly adopted for planning applications. Otherwise, within-period (or within-day) dynamic assignment models should be adopted; these require extensions of the demand models and, to an even greater extent, the supply models. Dynamic assignment models can also be classified using the criteria discussed in this section.

Figure 2 summarizes the different assignment model classification factors discussed above. The technical literature does not usually refer to assignment models using such a complete taxonomy. Nonetheless, it is a useful exercise to classify an assignment model according to the full set of factors considered here, as the assumptions underlying the model are then clearly identified.

Supply factors	
Type of service	Continuous Scheduled
Congestion effects	Uncongested networks Congested networks
Demand factors	
Demand segmentation	Single user class Multiple user classes
Demand elasticity	Fixed demand Variable demand
Path choice behavior	Fully pre-trip Pre-trip/en-route
Path choice model	Deterministic Probabilistic
Dynamics factors	
Within-period variability	Within-period static Within-period dynamic
Demand–supply interaction	User equilibrium Deterministic dynamic process Stochastic dynamic process

Fig. 2 Assignment model classification factors.

Fields of Application of Assignment Models

Models described above may be adopted for several types of applications, as briefly discussed in the following.

Assignment models as estimators of the present state of the transportation system. In this monitoring application, the assignment model receives as inputs the present network and O-D demand flows, and is applied to estimate other quantities that would be too costly or complicated to measure directly. Typically the relevant variables are the flows using different supply elements (road sections, intersection turning movements, lines of public transport services, motorway toll barriers) represented by links in the network model, the congestion levels of these elements (usually expressed by flow/capacity ratios or load factors), the performance attributes (travel times, monetary costs, etc.) comprising the generalized cost of links and paths (used as inputs to demand models), and external impacts (emission and concentration of air pollutants, noise levels, fuel consumption, traffic revenues, etc.). Although costs and impacts were introduced and discussed in the presentation of supply models, in congested networks they depend on link flows and therefore cannot be calculated without the application of an assignment model and its estimated flows. The results of assignment models can complement direct observations such as link flow counts or path travel time measurements because such observations are usually not available for all elements of the system. The network variables listed can be used both in project design (identification of critical points, analyses of supply inefficiencies, levels of accessibility, etc.) and in monitoring the effects of planned actions. For this type of application, fixed (present) demand assignment models can be used.

Assignment models for simulating the effects of modifications to the transportation system. In this application, assignment models are used to estimate the changes in relevant network variables due to changes in supply and/or demand. This is the typical application of representing models as design tools. The relevant effects of different actions, or projects, are simulated to define the technical elements of the project (design) and/or compare alternative hypotheses (evaluation). In this application, the supply and demand models (or the input variables to demand functions) will correspond to the projects and the future demand scenarios. If the project network is congested, variable demand models should be adopted, at least for the demand dimensions that are expected to be affected by the planned actions.

Different assignment models can be adopted for the design and evaluation phases. Computationally efficient models such as DUE are often used for design, either through supply design models or successive trials (since several runs are usually required at this stage). Assignment models used to provide measures that allow the comparison of alternative projects should be able to simulate flows and other indicators as accurately as possible, even at the cost of a greater computational effort, such as stochastic assignment models.

Assignment models for the estimation of travel demand. Assignment models are seeing an increasing application for the estimation of O-D demand flows and/or for the calibration of demand models. This type of application, which is dealt with at length reverses the usual role of assignment models. When assignment models are used in this way, they provide relationships connecting present (unknown) O-D flows to the traffic flows measured on some network links, rather than predicting link traffic flows from known demand flows. For theoretical reasons regarding the uniqueness of path choice probabilities and flows, it is preferable to use probabilistic (stochastic) assignment models rather than deterministic ones for this purpose.

This lecture describes the theoretical foundations and the structure of some of the simplest algorithms for solving basic within-day static assignment models, say single-class single-mode equilibrium assignment with fixed demand and fully pre-trip path choice.

Extensions to combined pre-trip/en-route path choice behavior, assignment with variable demand and/or multimodal systems, assignment with multiple user classes, and a general introduction to dynamic process assignment (which is still mainly a research topic). Extensions of supply, demand, and demand/supply interaction models to within-period dynamic systems with continuous or scheduled services. Algorithms are based on simple and effective solution approaches that apply to assignment models for large-scale networks.

Definitions, Assumptions, and Basic Equations

This section summarizes the definitions and assumptions underlying the demand and supply models. A single mode is considered here (single-mode assignment), and it is assumed that the O-D demand flows for this mode are known and independent of the congested link costs (fixed-demand assignment). It follows that path choice is the only choice dimension explicitly simulated. Users are considered to be homogeneous; that is, they share common behavioral and cost characteristics regardless of trip purpose and differ only in terms of their origins and destinations (single-user class assignment). Also, path choice is considered to be a completely pre-trip decision. These assumptions are not uncommon in practical work, for example, in simple analyses of road networks.

The symbols are presented below for the convenience of the reader (to simplify notation, the underlying analysis time band h and mode m are omitted, and user category i and trip purpose s are not considered due to assumptions made above). Let:

o be a origin centroid node

d be a destination centroid node

od be an origin-destination pair

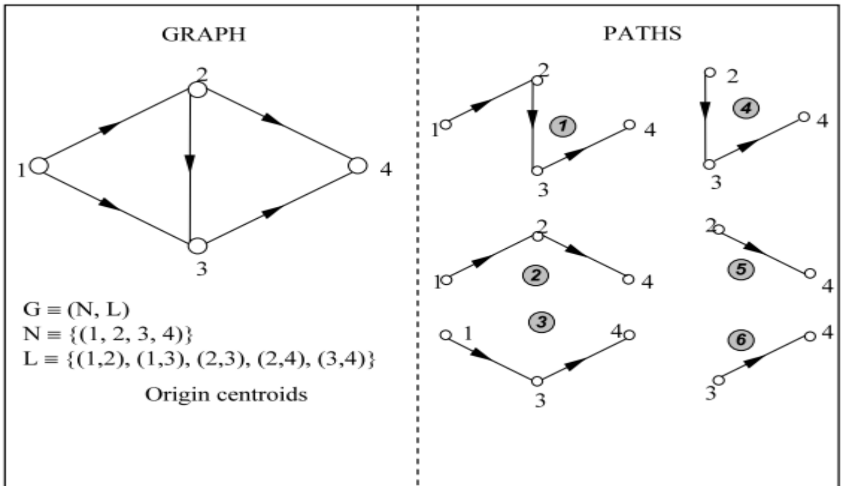
K_{od} be the set of paths for O-D pair od; each path k is uniquely associated with one and only one O-D pair od such that $k \in K_{od}$, assumed in the following nonempty (each O-D pair, say, is connected by at least one path) and finite

Δ_{od} be the link–path incidence matrix for the O-D pair od.

Δ be the overall link–path incidence matrix, obtained by placing side by side the blocks Δod corresponding to each O-D pair

An example is shown in Fig. 3.

In the following, it is assumed that the set of network links is nonempty and finite. Furthermore, for each O-D pair od, the set of available paths K_{od} is not empty if there is at least one path connecting o and d, and it is finite because we consider only elementary (loopless) paths. As a result, the link and path variables considered in this lecture are finite-dimensional, and analysis can take place in finite-dimensional vector spaces unless otherwise noted.



LINK-PATH INCIDENCE MATRIX

O-D Pairs		1-4			2-4		3-4
Links \ Paths		1	2	3	4	5	6
1		1	1	0	0	0	0
2		0	0	1	0	0	0
3		1	0	0	1	0	0
4		0	1	0	0	1	0
5		1	0	1	1	0	1

Fig. 3 Assignment model classification factors.

Supply Model

Transportation supply is simulated with a (congested) network model. A (generalized) cost c_a is associated with each link a; if travel time t_a is the only component of cost,

It yields: $c_a = \beta t_a$.

Furthermore, each path k is associated with a path cost g_k, consisting of two types of cost attributes:

Likewise additive (or generic) path costs are obtained by adding up the corresponding costs of the links on the path, regardless of the particular O-D pair and/or path (for instance travel time); these costs may depend on link flows in the case of congested networks;

Likewise nonadditive (or specific) path costs that are specific to the path and/or O-D pair, in the sense that they cannot be determined by adding up the generic costs of the links on the path (for instance, some types of tolls or fees). In the following analysis, these costs are assumed to be independent of congestion.

Therefore, we do not consider path costs that are simultaneously nonadditive and dependent on congestion. Let:

c be the link cost vector, with entries c_a

g_{od}^{ADD} be the vector of additive path costs for users of O-D pair od, consisting of elements g_k^{ADD} , $k \in K_{od}$

g_{od}^{NA}be the vector of nonadditive costs for users of O-D pair od, consisting of elements

g_{od}^{NA} , $k \in K_{od}$ be the vector of total path costs for users of O-D pair

g_{od}, consisting of elements g_k, $k \in K_{od}$

The relationship between link costs and path costs is given for each O-D pair od by the following equations (see Figs. 3 and 4):

$$g_{od}^{ADD} = \Delta_{od}^{T}\, c\ \forall od$$

$$g_{od} = g_{od}^{ADD} + g_{od}^{NA} = \Delta_{od}^{T} c + g_{od}^{NA} \forall od \qquad 1$$

The above relation can be expressed using matrix notation. Let:

$g^{ADD} = [g_{od}^{ADD}]_{od}$ be the overall vector of additive path costs, consisting of the vectors of additive path costs g_{od}^{ADD} for all O-D pairs.

$g^{NA} = [g_{od}^{NA}]_{od}$ be the overall vector of nonadditive path costs, consisting of the vectors of nonadditive path costs $g_{od}^{NA}$for all O-D pairs.

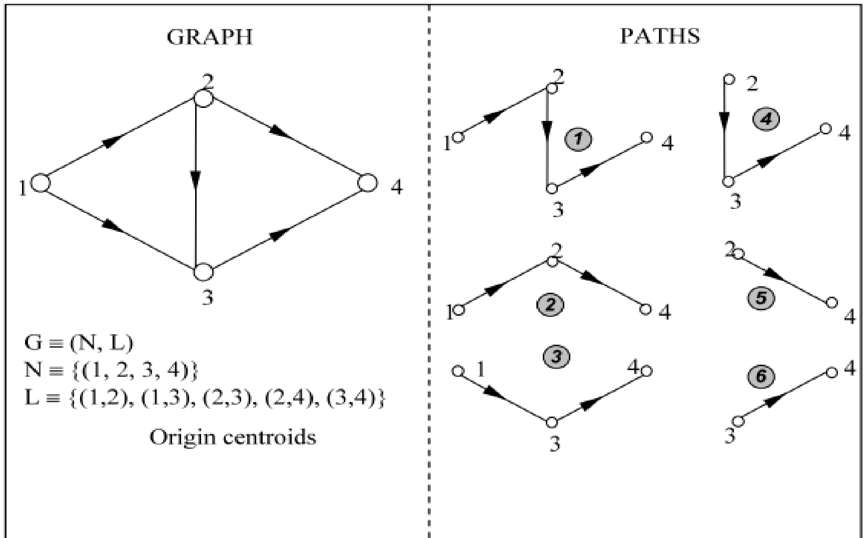
$g = [g_{od}]_{od}$ be the overall vector of the total path costs, consisting of the vectors of total path costs g_{od}] for all O-D pairs

A flow f_a is associated with each link a. Link flows are measured in units commensurate with demand flows. Let:

f be the link flow vector, with entries f_a.

In congested networks, link costs depend on link flows through the cost functions:

$$c = c(f) \qquad 2$$



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3		1	0	0	1	0	0
4		0	1	0	0	1	0
5		1	0	1	1	0	1

Fig. 4 Example of a graph and its link–path incidence matrix.

In turn, link flows depend, through the network flow propagation model, on the flow associated with each path. In particular, for a given O-D pair, the path flows induce the corresponding O-D specific link flows through the link-path incidence matrix. Furthermore, the total flow on a link is the sum of the flows induced by all paths and all O-D pairs. (Demand, path, and link flows are assumed to be expressed in consistent units.) Let:

h_{od} be the path flow vector for users of O-D pair od, the elements of which are the flows h_k for all $k \in K_{od}$

f^{od} be the vector of O-D specific link flows f_a^{od}, resulting from the trips for O-D pair od over available paths

The relationship between link flows and path flows is expressed by the following equations (Fig. 5):

$$f^{od} = \Delta_{od} h_{od} \ \forall od$$

From which

$$f = \sum_{od} f^{od} = \sum_{od} \Delta_{od} h_{od} \qquad 3$$

All the above relations can be expressed using matrix notation. Let:

h = $[h_{od}]_{od}$ be the overall vector of path flows, consisting of the vectors of path flows

h_{od} for all O-D pairs

The whole supply model is defined by (1) to (3) which combine to express the relationship between path costs and path flows:

$$g_{od} = \Delta_{od}^T c\left(\sum_{od} \Delta_{od} h_d\right) + g_{od}^{NA} \ \ \forall od \qquad 4$$

The above relations can be expressed using matrix notation.

$$g_{od} = \Delta^T c(\Delta h) + g^{NA}$$

$$\boldsymbol{g} = \boldsymbol{\Delta}^T \cdot \boldsymbol{c} + \boldsymbol{g}^{\mathrm{NA}}$$

$$\left[\begin{array}{c} 6 \\ 4 \\ 2 \\ \hline 4 \\ 2 \\ \hline 1 \end{array}\right] = \left[\begin{array}{ccccc} 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ \hline 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 \end{array}\right] \cdot \begin{bmatrix} 2 \\ 1 \\ 3 \\ 2 \\ 1 \end{bmatrix} + \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ \hline 0 \end{array}\right]$$

$$\boldsymbol{g} = \boldsymbol{g}^{\mathrm{ADD}} + \boldsymbol{g}^{\mathrm{NA}} = \boldsymbol{\Delta}^{\mathrm{T}} \boldsymbol{c} + \boldsymbol{g}^{\mathrm{NA}}$$

Fig. 5 Example of the relationship between link costs and path costs (nonadditive costs are zero for the sake of simplicity).