2- Stability and Determinacy of Structures
2-1- Stability and Determinacy of Beams

The **Beams** is a horizontal member which transfer loads to supports (columns or walls). Figure (10) shows the common types of beams.

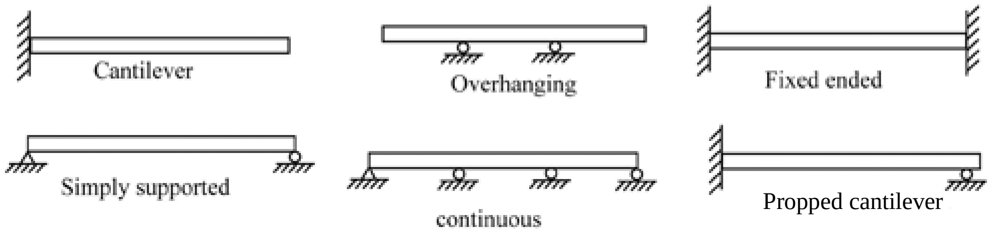


Figure (10) Common types of beams

The total **Equations of Equilibrium** for any beam are three:-

$$\Sigma Fx=0$$
$$\Sigma Fy=0$$
$$\Sigma M=0$$

In addition to the above equations, there are **Equations of Conditions** (C). Therefore, the total equations of equilibrium = **C+3**

To check the determinacy and stability of beams, the general law is: $\boxed{r \lesseqgtr 3+C}$

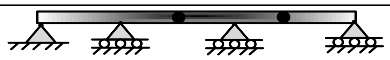
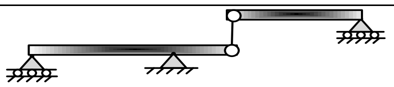
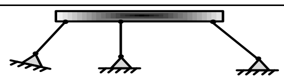
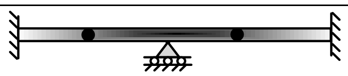
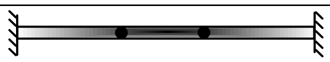
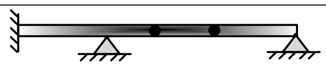
Where

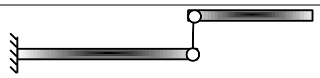
r =Total number of reaction in beam (unknowns).
C= Equations of conditions (if any).

If:

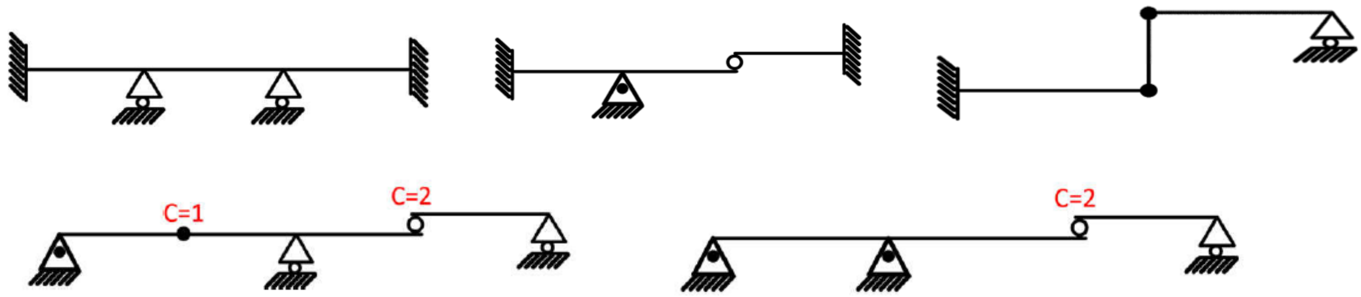
1- **r < 3+C → The beam is unstable.**
2- **r = 3+C→ The beam is stable and determinate.**
3- **r > 3+C→ The beam is stable and indeterminate to (D=r-(3+C)) Degree.**

Examples

Beam	**r**	**C**	$r \lesseqgtr 3+C$
	4	1	4=1+3, Stable Determinate
	5	2	5=2+3, stable determinate
	4	2	4<2+3, Unstable Internally
	3	-	Unstable externally (Reactions are concurrent)
	7	2	7>2+3, Stable indeterminate, 2nd degree
	6	2	6>2+3, Stable indeterminate, 1st degree
	7	2	7>2+3, Unstable internally

	3	2	3<2+3, Unstable

<u>Homework No. (1)</u>
Discuss the stability and determinacy of the following structures.



2-2- Stability and Determinacy of Frames

The frames are structures often used in buildings, industrial establishments, warehouses,….etc.; and are composed of beams and columns that are either pin or fixed connected. Generally, the loading on frames causes three types of forces on its members; **Axial Force** (**N**) works parallel to the member axis, **Shear Force** (**V**) works perpendicular to the member axis, and **Bending Moment** (**M**).

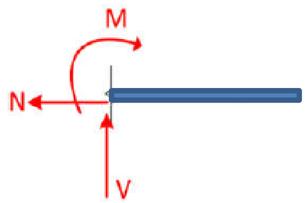
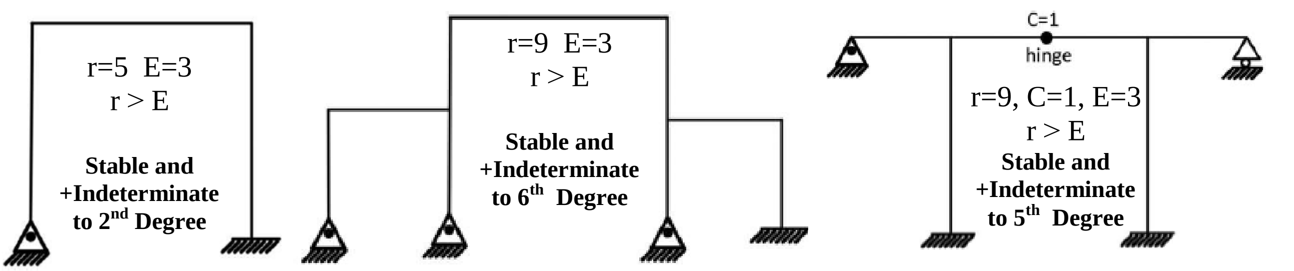


Figure (11) Axial Force (N), Shear Force (V) and Bending Moment (M).

There are two types of frames, **Opened Frames** (opened bay frame) and **Closed Frames** (closed bays frame).

2-2-1- Stability and Determinacy of Opened Frames

In this case, the frames is treated same as a beams. **To check the determinacy and stability of the opened frame the general law is**: $\boxed{r \lesseqgtr 3+C}$



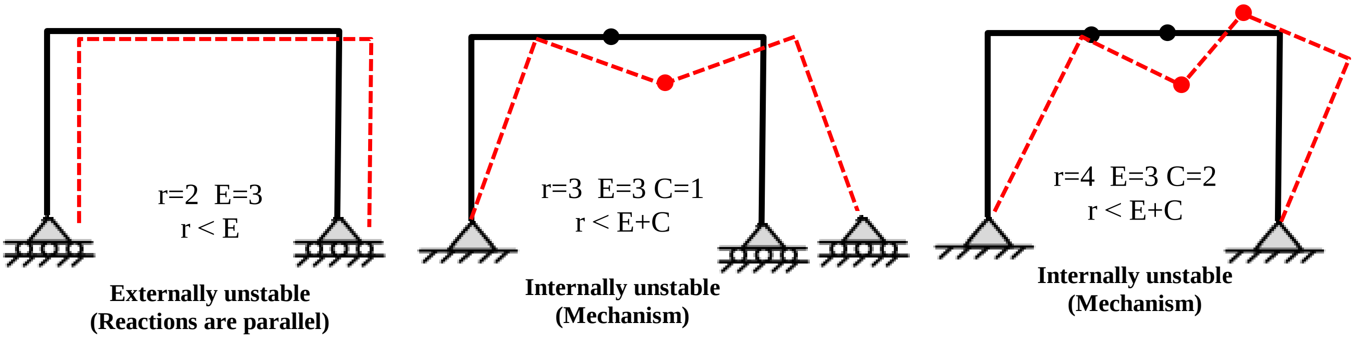


Figure (11) Common types of opened frames

<u>2-2-2- Stability and Determinacy of Closed Frames</u>

To check the determinacy and stability of the closed frame the general law is:

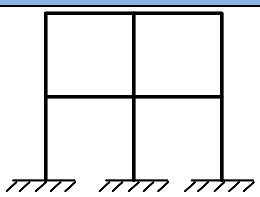
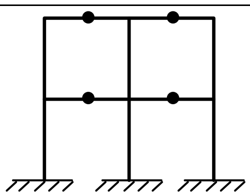
$$\boxed{3b+r \lesseqgtr 3j+C}$$

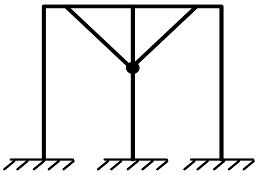
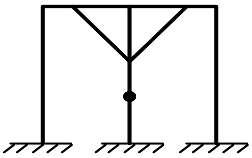
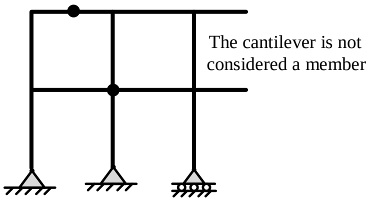
<u>Where:</u>
b = No. of members
r = No. of reactions
j = No. of joint
C = Equations of conditions (if any).

If:

1. **$3b+r < 3j+C \rightarrow$ The frame is unstable.**
2. **$3b+r = 3j+C \rightarrow$ The frame is stable determinate.**
3. **$3b+r > 3j+C \rightarrow$ The frame is stable indeterminate to (D=3b+r-(3j+C)) Degree.**

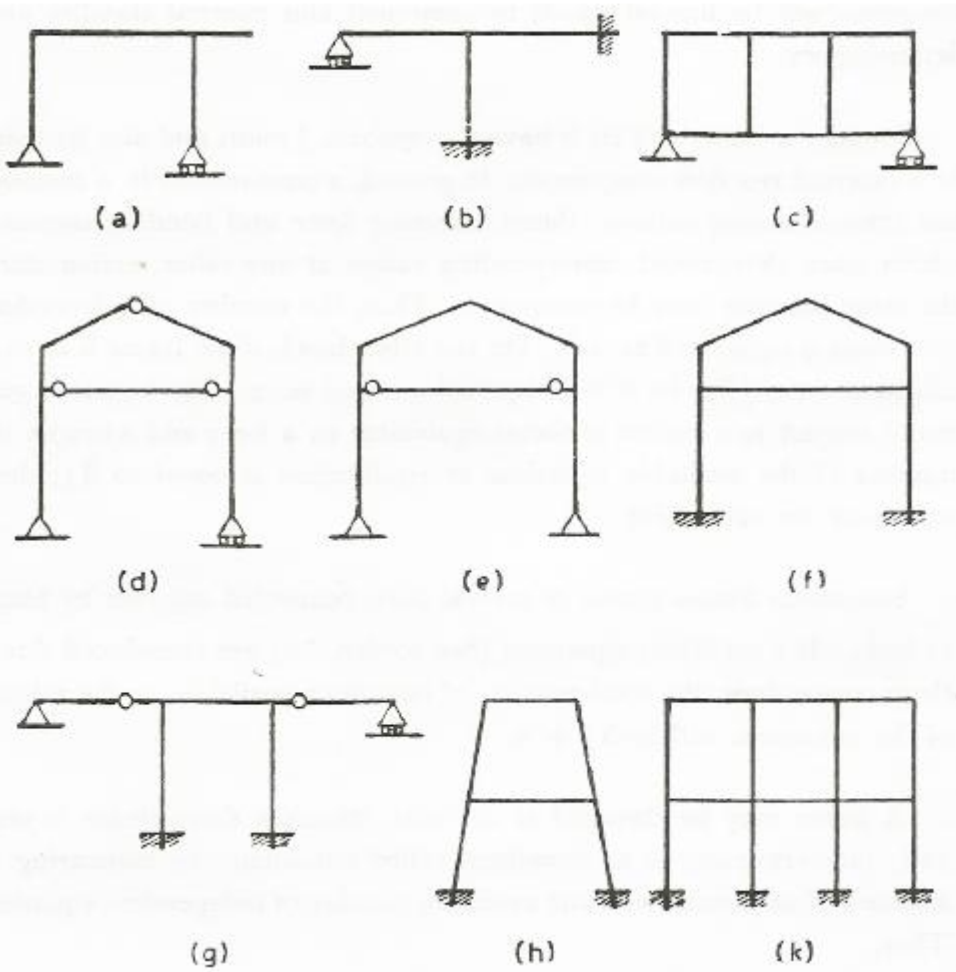
<u>Examples</u>

Frame	**b**	**r**	**j**	**c**	$3b+r \lesseqgtr 3j+C$
	10	9	9	0	39>27 Stable Indeterminate, 12th Degree
	14	9	13	4	51>43 Stable Indeterminate, 8th Degree

	10	9	9	3*	39>30 Stable Indeterminate, 9th Degree *See page (6)
	11	9	10	1	42>31 Stable Indeterminate, 11th Degree
	11	5	10	4*	38>34 Stable Indeterminate, 4th Degree *See page (6)

Homework No. (2)

Discuss the stability and determinacy of the following structures.



2-3- Stability and Determinacy of Plane Trusses

A truss may be defined as a structure which consists of a number of straight members pin-connected together at their ends so as to form a rigid structure. In practice, the truss members are bolted, riveted or welded at their ends; but, for structural analysis, the truss member assumed to be pin-connected. The trusses are often used in bridges, buildings, industrial establishments, warehouses, hangars,….etc.

Assumption for ideal Truss

1-All joints are frictionless pins.
2-External load and reactions are only applied at the pin joints.
3-All the truss members are straight and will be link member subjected to either axial tension (**Ties**) or axial compression (**Struts**).

In trusses, at each joint there are two equations of equilibrium:-

$$\Sigma Fx=0$$
$$\Sigma Fy=0$$

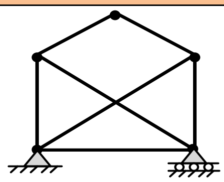
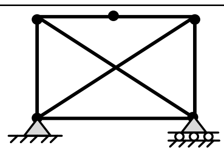
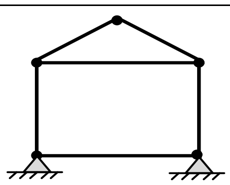
To check the determinacy and stability of truss, the general law is: $\boxed{r+b \lesseqgtr 2J}$

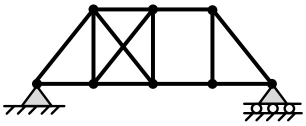
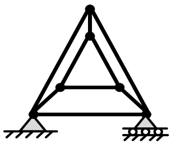
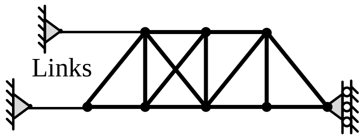
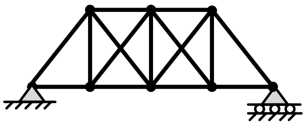
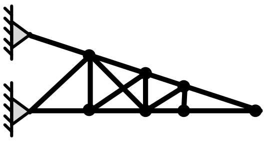
Where
r =Total number of reactions.
b=Total number of bars (members).
J= Total number of joints.

If

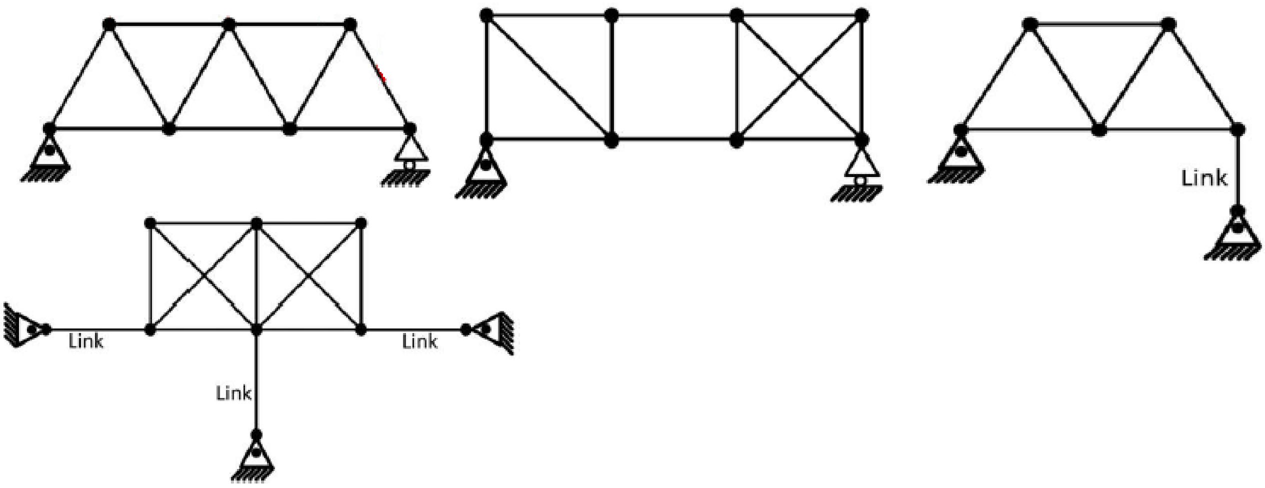
1- **b+r < 2j →** **The truss is unstable**
2- **b+r = 2j→** **The truss is stable determinate**
3- **b+r > 2j →** **The truss is stable indeterminate**

Important Note: It may be noted that, the above mentioned equations is not always sufficient to decide whether the truss is stable or not!!

Truss	b	r	j	$r+b \lesseqgtr 2J$
	7	3	5	10=10 Stable Determinate
	7	3	5	10=10 Unstable Internally
	6	4	5	10=10 Unstable Internally

	13	3	8	16=16 Unstable Internally
	9	3	6	12=12 Unstable Internally
	14	3	8	17>16 Unstable Externally (Parallel Reactions)
	15	3	8	18>16 Stable Indeterminate, 2^{nd} Degree
	15	4	9	19>18 Stable Indeterminate, 1^{st} Degree

<u>Homework No. (3)</u>
Discuss the stability and determinacy of the following structures.



<u>2-4- Stability and Determinacy of Arches</u>

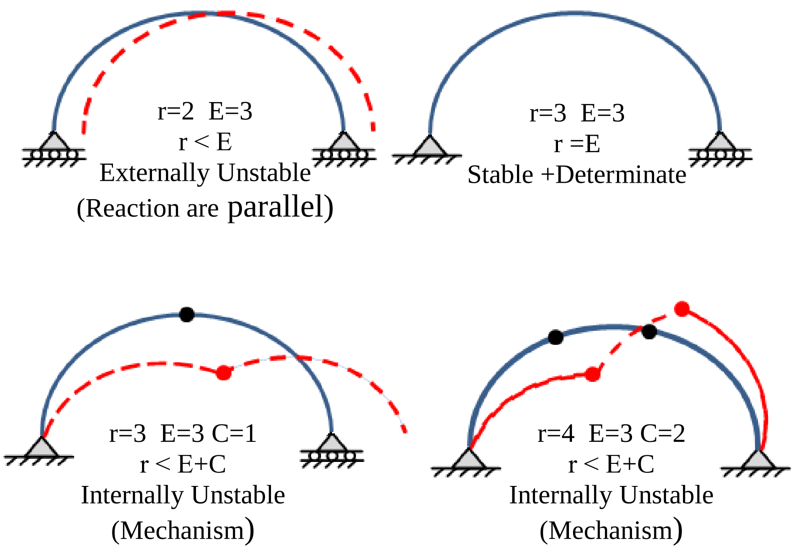
An arch is a curved beam or structure subjected to loads act on the convex side of the curve and re-sights the external loads by the force of thrust. It is subjected to three restraining forces, **Axial Thrust Force** (N) acting with the arch axis; **Shear Force** (V) and **Bending Moment** (M). The arches are often used in bridges, industrial establishments, hangars,….etc. The arches can be classified based on their boundary conditions (supports).

To check the determinacy and stability of the arches, the general law is: $r \lesseqgtr 3+C$

Where

r =Total number of reaction in beam (unknowns).

C= Equations of conditions (if any).



The common types of the arches are:-

1- Fixed at both ends with no hinges present at its crown.
2- Fixed at both ends with a hinge at its crown.
3- Two-hinged arches.
4- Three-hinged arches

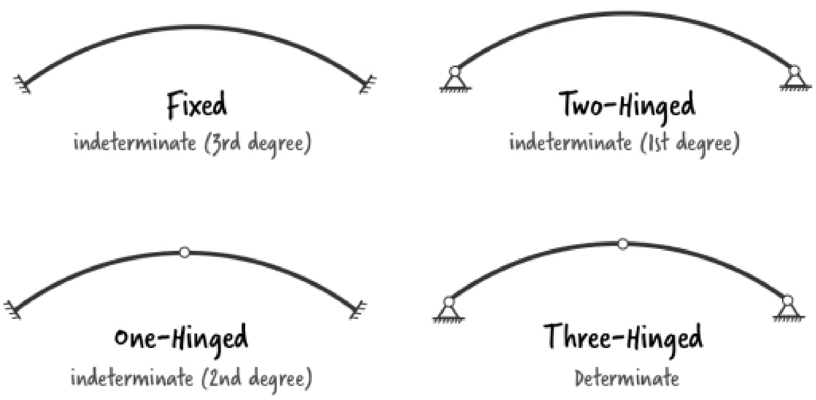


Figure (12) Common types of arches