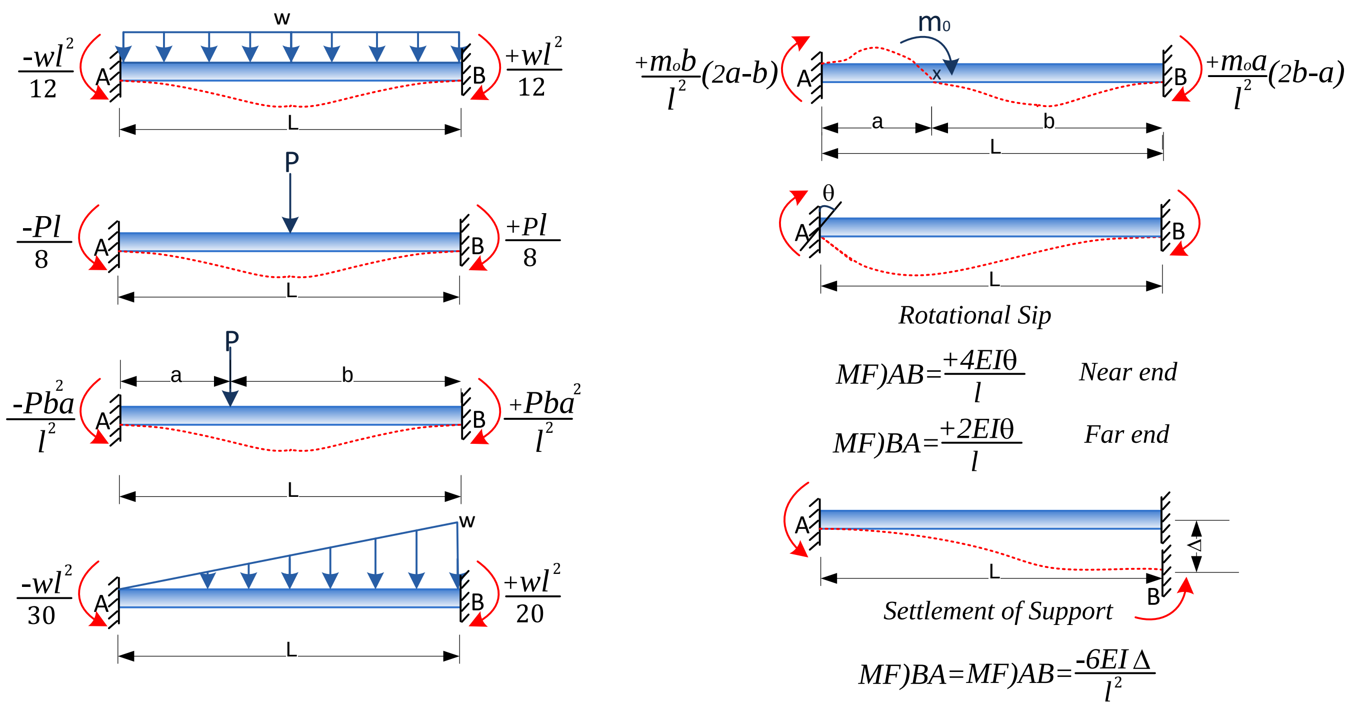
Analysis of Indeterminate Structures

Slope Deflection Method

This method related slope (θ) and deflection (Δ) with fixed end moments (FEM). The method is applicable for the analysis indeterminate structures including: Beams, Frames.

Fixed End Moment-Beams with Constant EI



Slope-Deflection Equations

When the structural member subjected to general load, the final member moments (M_{AB}) and (M_{BA}) can be calculated by using the concept of superposition, by sum the following cases:-

Case-1 Applied Load

($M1_{AB}$) and ($M1_{BA}$) are the moments due to applied load on the member taken as fixed on both ends, called (**Fixed End Moment**) →(use Tables !).

$M1_{AB}=MF_{AB}$ and $M1_{BA}=MF_{BA}$

Case-2 Rotation at the left Support (θ_A)

$(M2_{AB})=4EI/L.\ \theta_A$

$(M2_{BA})=1/2(M2_{AB})= 2EI/L.\ \theta_A$

Case-3 Rotation at the left Support (θ_B)

$(M3_{BA})=4EI/L.\ \theta_B$

$(M3_{AB})=1/2(M3_{BA})= 2EI/L.\ \theta_B$

Case-4 Settlement at the Support (Δ)

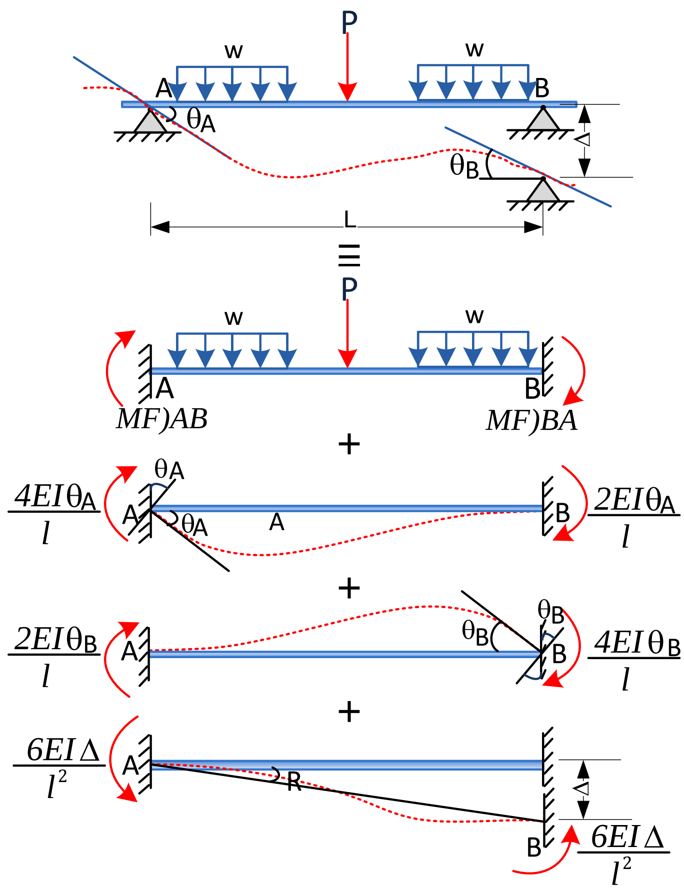
$(M4_{AB}) = (M4_{BA})= (6EI/L^2).\ \Delta$

Then;

$M_{AB}=M1_{AB} +M2_{AB} +M3_{AB} +M4_{AB}$

$M_{AB}= MF_{AB} +4EI/L.\ \theta_A+2EI/L.\ \theta_B - (6EI/L^2).\ \Delta$

$\mathbf{M_{AB} = MF_{AB} +2EI/L(2\ \theta_A+ \theta_B-3\ \Delta/L)}$



Also,

M_{BA} = MF_{BA} +2EI/L(2 θ_B+ θ_A-3 Δ/L)

Actual Slope-Deflection Equations

$$M_{AB} = M_{AB}^F + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

$$M_{BA} = M_{BA}^F + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\Delta}{L}\right)$$

Modified Slope-Deflection Equations

$$M_{AB} = M_{AB}^F + k_{rel)AB}\left(2\theta_A + \theta_B - 3R_{rel)AB}\right)$$

$$M_{BA} = M_{BA}^F + k_{rel)AB}\left(2\theta_B + \theta_A - 3R_{rel)AB}\right)$$

General Slope-Deflection Equations

$$M_{ij} = M_{ij}^F + \frac{2EI}{L}\left(2\theta_i + \theta_j - \frac{3\Delta}{L}\right)$$

Where:

M_{AB} is internal moment in the near end of the span

E, K modulus of elasticity of material and span stiffness

θ_A, θ_B near and far-end slopes or angular displacements of the span at the supports.

R is span rotation of its cord due to a linear displacement=Δ/L

MF_{AB} is fixed end moment at the near-end support

Δ is difference between the two ends of the member normal to the member

M, MF	**= +**	**if clockwise**
	-	**if counterclockwise**
θ	**= +**	**if clockwise**
	-	**if counterclockwise**
R	**= +**	**if clockwise**
	-	**if counterclockwise**

Procedure for Analysis

1. Label all the supports and joints (nodes) to identify the spans of the beam or frame between the nodes (members). Each node is either support, internal hinge, or where EI value changes, **the cantilever portion is not considered as a member**.
2. Identify the number of degrees of freedom (DOF). Each node can possibly have an angular displacement (θ) and a linear displacement (Δ).

No. of unknown = θ+Δ

Fixed	Hinge	Roller	Internal Hinge	Rigid Part
θ=0, Δ=0	θ=?, Δ=0	θ=?, Δv=0, Δh=0	Δ=?, θL=?, θR=?	Δ=?, θ=?

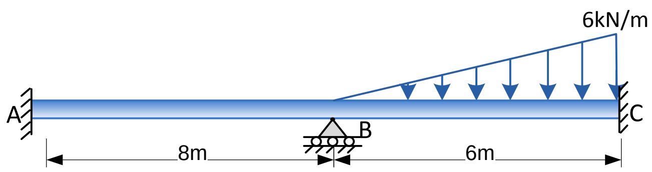
3. Compute the fixed end moment (MFE) for each member.
4. Apply the slope deflection Eq. to each end of the span that each member has two Eq's.
5. Write an equilibrium equation for each unknown degree of freedom for the structure. Each of these equations should be expressed in terms of unknown internal moments as specified by the slope-deflection equations. At each joint with θ, the equilibrium equation is $\Sigma M_j=0$.
6. If there is **sidesway**, shear equations related to the moments at the ends should be applied equal to the No. of unknown displacement Δ.
7. Solve the equations simultaneously to find θ and Δ.
8. Find the values of internal moment and hence the reactions at supports can be calculated.
9. Draw the Axial, Shear and Bending Moment diagrams.

Note

1-When calculating Δ, it must be normal to the member.
2-For internal hinge or roller, there is two rotation angles θ_{right} and θ_{left}.

Example-1: Draw the shear and bending moment diagrams for the beam shown below by Slope-Deflection method EI is constant.

Solution:



1. Degree of Freedom: θ_B, $\theta_A=\theta_C=0$
2. R values: R=0 since Δ =0, R=Δ/L
3. F.E.M's: since there is no load on span AB, $M^F_{AB}=M^F_{BA}=0$

$$M^F_{BC}=\frac{-wl^2}{30}=\frac{-6(6^2)}{30}=-7.2\ kN.m$$

$$M^F_{CB}=\frac{wl^2}{20}=\frac{6(6^2)}{20}=10.8\ kN.m$$

4. Slope-Deflection Equations:

$$M_{ij}=\pm M^F_{ij}+\frac{2EI}{L}\left(2\theta_i+\theta_j-\frac{3\Delta}{L}\right)$$

$$M_{AB}=\frac{2EI}{8}\left(2\theta_A+\theta_B-\frac{3\Delta}{L}\right)\rightarrow M_{AB}=\frac{EI}{4}(\theta_B) \quad \dots.(1)$$

$$M_{BA}=\frac{2EI}{8}\left(2\theta_B+\theta_A-\frac{3\Delta}{L}\right)\rightarrow M_{BA}=\frac{EI}{2}(\theta_B) \quad \dots.(2)$$

$$M_{BC}=\frac{2EI}{6}\left(2\theta_B+\theta_C-\frac{3\Delta}{L}\right)-7.2\rightarrow M_{BC}=\frac{2EI}{3}(\theta_B)-7.2 \quad \dots.(3)$$

$$M_{CB}=\frac{2EI}{6}\left(2\theta_C+\theta_B-\frac{3\Delta}{L}\right)+10.8\rightarrow M_{CB}=\frac{EI}{3}(\theta_B)+10.8 \quad \dots.(4)$$

5. <u>Additional Equations:</u>

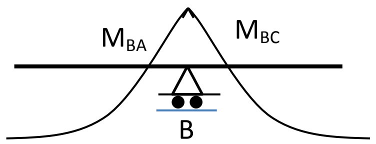
No. of additional Equation= No. of degree of freedom

In this Example, we need one additional Equation which comes from applying equilibrium Equations at joint (B).

$$\sum M_B = 0 \rightarrow M_{BA} + M_{BC} = 0 \qquad \dots.(5)$$

Substituting Eq's.(2) and (3) in (5) yields:

$$\boxed{\theta_B = 6.17/EI}$$



Substituting θ_B value into Eq's.(1-4) yields:

$$M_{AB} = 1.54\ kN.m \qquad M_{BA} = 3.09\ kN.m$$

$$M_{CB} = 12.86\ kN.m \qquad M_{BC} = -3.09\ kN.m$$

(+ve) sign of moment means that the moment acts clockwise, (-ve) sign mean it acts counterclockwise.

6. <u>Final Reactions:</u> To find the reactions at supports, make a section at a point just left and right the roller (B).

$$\sum M_A = 0 \rightarrow \frac{M_{AB} + M_{BA}}{L} = B_{y)L}$$

$$\frac{1.54 + 3.09}{8} = 0.578\ kN \uparrow$$

$$\sum F_y = 0 \rightarrow \quad A_y = 0.578\ kN \downarrow$$

$$\sum M_c = 0 \rightarrow B_{y)R} = \frac{12.86 - 3.09 - \left(0.5 * 6 * 6 * \frac{6}{3}\right)}{L} = 4.37kN \uparrow$$