Q1: Use PFM to find I.Z.T of:

$$X(z) = \frac{1-\frac{1}{3}z^{-1}}{(1-z^{-1})(1+2z^{-1})^2}, \quad |z|>2.$$

Sol.

$$X(z) = \frac{(1-\frac{1}{3}z^{-1})\cdot z^3}{(1-z^{-1})\cdot z\cdot(1+2z^{-1})^2\cdot z^2}.$$

$$\boxed{\frac{X(z)}{z} = \frac{z^2-\frac{1}{3}z}{(z-1)(z+2)^2}} \quad \text{---(*)}$$

$$= \frac{A}{z-1} + \frac{B}{(z+2)^2} + \frac{C}{z+2} \quad \text{---(*)}$$

$$A = \left.\frac{z^2-\frac{1}{3}z}{(z+2)^2}\right|_{z=1} = \frac{2}{27}.$$

$$B = \left.\frac{z^2-\frac{1}{3}z}{z-1}\right|_{z=-2} = -\frac{14}{9}.$$

$$C = \frac{1}{(2-1)!}\lim_{z\to-2}\frac{d^{2-1}}{dz^{2-1}}\left[\frac{z^2-\frac{1}{3}z}{z-1}\right] = \frac{25}{9\times3} = \frac{25}{27}$$

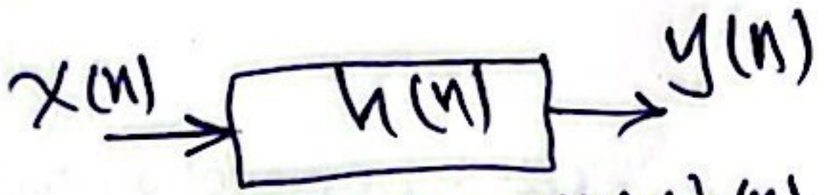
$$X(z) = \frac{\frac{2}{27}z}{z-1} - \frac{14}{9}\frac{z}{(z+2)^2} + \frac{25}{27}\frac{z}{z+2}.$$

$$\therefore x(n) = \mathcal{Z}^{-1}\{X(z)\} = \frac{2}{27}u(n) - \frac{14}{9}n(-2)^{n-1}u(n) + \frac{25}{9}(-2)^n u(n)$$

(2)

Q2: Difference eq: $y(n-1)+2y(n)=x(n)$

determine the zero-state response (initial values = 0.) of the o/p system for $n \geq 0$ (casual) when the input is $x(n)=\left(\frac{1}{4}\right)^n u(n)$.



$y(n) = x(n) * h(n)$

$Y(z) = X(z)\,H(z)$.

Sol.

$$x(n)=\left(\frac{1}{4}\right)^n u(n) \rightarrow X(z)=\frac{z}{z-\frac{1}{4}}.$$

$$y(n-1)+2y(n)=x(n)$$

$$z^{-1}Y(z)+2Y(z)=X(z).$$

$$Y(z)=\frac{1}{2+z^{-1}}X(z)=\frac{z}{2z+1}\cdot\frac{z}{z-\frac{1}{4}}$$

$$\therefore Y(z)=\frac{\frac{1}{2}z^2}{(z+\frac{1}{2})(z-\frac{1}{4})} \quad \text{---(*)}$$

$$\therefore \boxed{\frac{Y(z)}{z}=\frac{A}{z+\frac{1}{2}}+\frac{B}{z-\frac{1}{4}}=\frac{\frac{1}{2}z}{(z+\frac{1}{2})(z-\frac{1}{4})}} \quad \text{---(2)}$$

$$A=\left[\frac{\frac{1}{2}z}{z-\frac{1}{4}}\right]_{z=-\frac{1}{2}}=\frac{1}{3},\quad B=\left[\frac{\frac{1}{2}z}{z+\frac{1}{2}}\right]_{z=\frac{1}{4}}=\frac{1}{6}$$

$$\therefore Y(z)=\frac{1}{3}\frac{z}{z+\frac{1}{2}}+\frac{1}{6}\frac{z}{z-\frac{1}{4}}.$$

$$y(n)=\frac{1}{3}\left(-\frac{1}{2}\right)^n u(n)+\frac{1}{6}\left(\frac{1}{4}\right)^n u(n).$$

Bounus: $H(z) = \dfrac{z^2 - 2z}{z^2 + \frac{5}{2}z + 1}$

write the difference equation of the system.

Sol.

$$H(z) = \frac{z^2 - 2z}{z^2 + \frac{5}{2}z + 1} \quad \frac{\div z^2}{\div z^2}$$

but $H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{1 - 2z^{-1}}{1 + \frac{5}{2}z^{-1} + z^{-2}}$.

$$Y(z)\left[1 + \frac{5}{2}z^{-1} + z^{-2}\right] = \left[1 - 2z^{-1}\right] X(z).$$

$$\Rightarrow y(n) + \frac{5}{2}y(n-1) + y(n-2) = x(n) - 2x(n-1).$$