Q1: Suppose a discrete r.v. X has the following pmf (probability mass function):

x_i	1	2	3	4
$p(x_i)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

1. Find and sketch the CDF (**F(x)**) of the r.v. X.
2. Calculate $P(X \leq 1)$.
3. Compute $P(1 < X \leq 2)$.
4. Find the mean (μ), mean square ($E[X^2]$), and the variance (σ^2).

Q2: The joint pdf (probability density function) of the random variables X and Y is given by:

$$P(X,Y) = \begin{array}{cc} & \begin{array}{ccc} & \text{Y} & \\ 1 & 2 & 3 \end{array} \\ \text{X} \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 0.1 & 0.2 & 0.15 \\ 0.05 & 0.18 & k \end{bmatrix} \end{array}$$

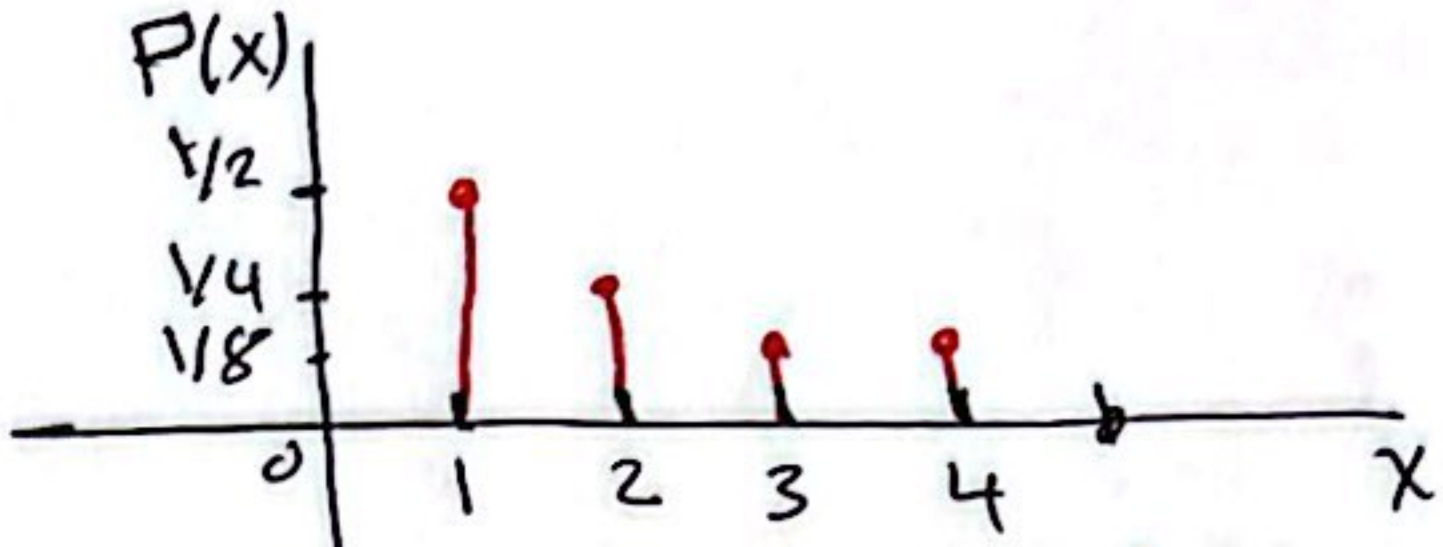
1. Find the constant **k**.
2. The marginal probability function for X and Y (**P(X)** and **P(Y)**).
3. The mean for each random variable (μ_X and μ_Y).
4. The conditional probabilities $P(Y|X)$ and $P(X|Y)$.

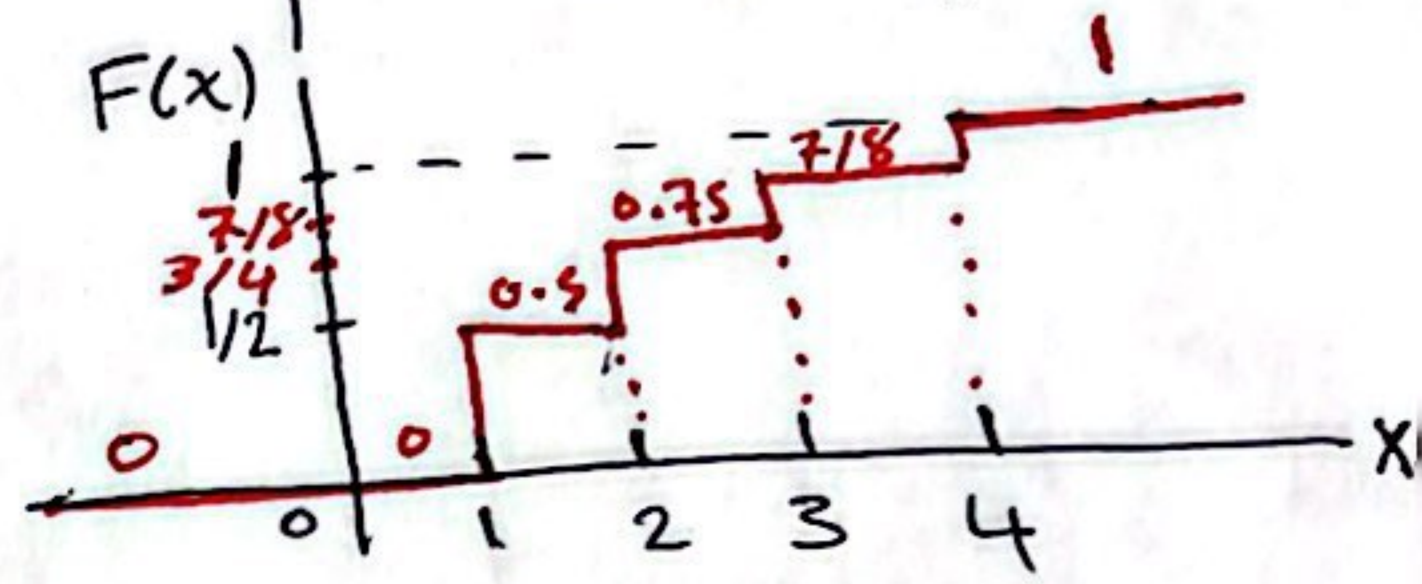
Bonus: Consider a telegraph source generating two symbols, '**0**', and '**1**'. We observed that the '**0**' were twice as likely to occur as the '**1**'. Find the probabilities of the '**0**' occurring and the '**1**' occurring.

Q1: $F(x) = \sum_{u \le x} p(u)$

x_i	1	2	3	4
$p(x_i)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

①
$$F(x) = \begin{cases} 0 & x < 1 \\ 0.5 & 1 \le x < 2 \\ 0.75 & 2 \le x < 3 \\ 0.875 & 3 \le x < 4 \\ 1 & x \ge 4 \end{cases}$$





② $p(X \le 1) = F(1) = \frac{1}{2}$.

③ $p(1 < X \le 2) = F(2) - F(1) = 0.75 - 0.5 = 0.25$.

Q2: $K = 1 - (0.1 + 0.2 + 0.15 + 0.05 + 0.18)$
$= 0.32.$

$$p(X,Y) = \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.1 & 0.2 & 0.15 \\ 0.05 & 0.18 & 0.32 \end{bmatrix} \end{array}$$

$$p(X) = \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 0.45 & 0.55 \end{bmatrix} \end{array}, \quad p(Y) = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.15 & 0.38 & 0.47 \end{bmatrix} \end{array}$$

$$p(Y/X) = \frac{\begin{bmatrix} 0.1 & 0.2 & 0.15 \\ 0.05 & 0.18 & 0.32 \end{bmatrix}}{\begin{bmatrix} 0.45 & 0.55 \end{bmatrix}} = \begin{bmatrix} \frac{0.1}{0.45} & \frac{0.2}{0.45} & \frac{0.15}{0.45} \\ \frac{0.05}{0.55} & \frac{0.18}{0.55} & \frac{0.32}{0.55} \end{bmatrix} = \frac{p(X,Y)}{p(X)}.$$

$$p(X/Y) = \frac{\begin{bmatrix} 0.1 & 0.2 & 0.15 \\ 0.05 & 0.18 & 0.32 \end{bmatrix}}{\begin{bmatrix} 0.15 & 0.38 & 0.47 \end{bmatrix}} = \begin{bmatrix} \frac{0.1}{0.15} & \frac{0.2}{0.38} & \frac{0.15}{0.47} \\ \frac{0.05}{0.15} & \frac{0.18}{0.38} & \frac{0.32}{0.47} \end{bmatrix} = \frac{p(X,Y)}{p(Y)}.$$

Q3 $p = \text{prob.}('0')$
$q = \text{prob.}('1')$
$p = 2q$

$\Rightarrow p + q = 1$
$\therefore 2q + q = 1$

$\Rightarrow q = \frac{1}{3} = p('1')$.
$p = \frac{2}{3} = p('0')$.