Engineering Analysis 3ed **Probability Quiz-2**
Dr. Safaa A. **2023-2022**

Q1: Let X be a continuous r.v. with following pdf: **(6 scores)**

$$p(x) = \begin{cases} cx & 1 < x < 3 \\ 0 & o.w. \end{cases}$$

1. Determine the value of **c** and sketch p(x).
2. Find and sketch the corresponding CDF F(x).
3. Calculate $P(\frac{3}{2} < x \leq 2)$.
4. Find the mean, variance, and standard deviation.

Q2: Given a Gaussian distribution with mean $\boldsymbol{\mu = 3}$ and variance $\boldsymbol{\sigma^2 = 1}$.

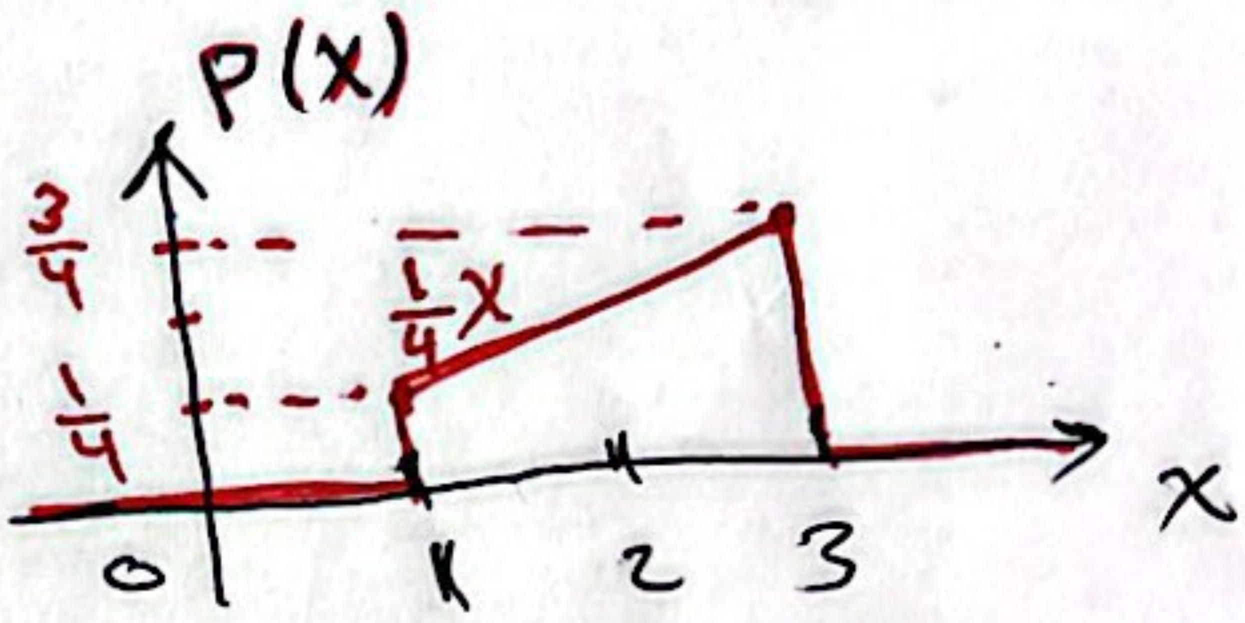
(4 scores)

1. Find the $\boldsymbol{P(0.75 < X \leq 4.52)}$.
2. The value of **K** such that $\boldsymbol{P(X \leq K) = 0.99224}$

Q_1: $p(x) = \begin{cases} cx & 1 < x < 3 \\ 0 & o.w \end{cases}$ ①

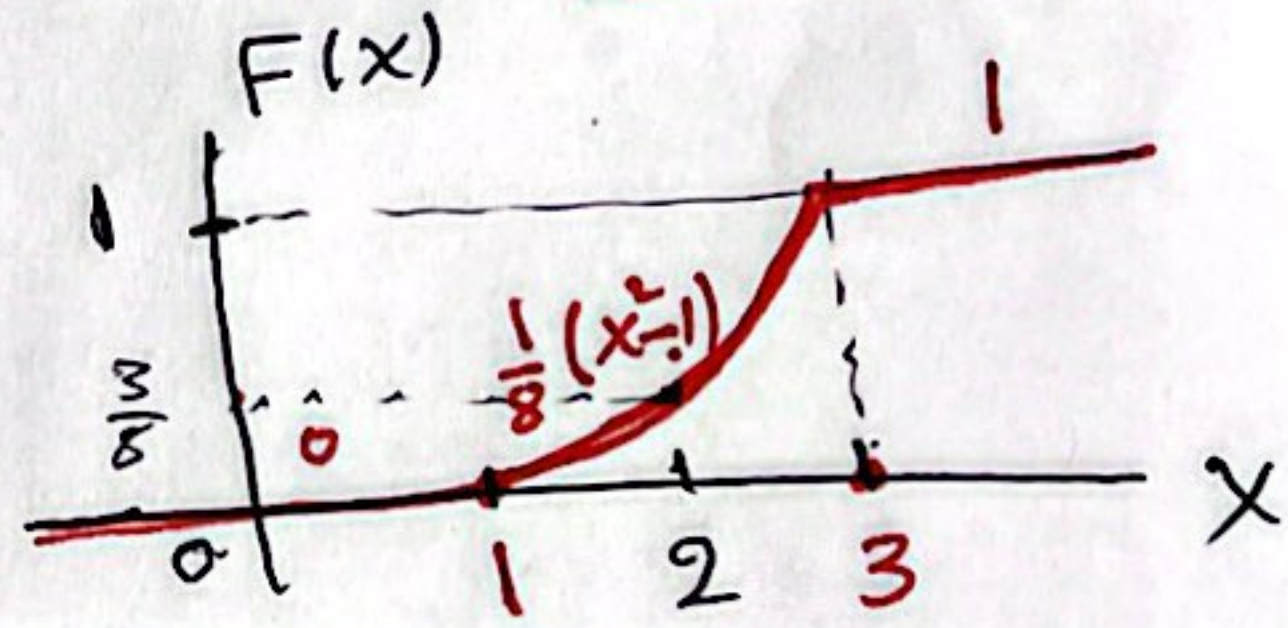
Sol. ① $\int_{-\infty}^{+\infty} p(x)\,dx = 1 \Rightarrow \int_1^3 cx\,dx = 1.$

$$c\left[\frac{x^2}{2}\right]_1^3 = 1 \Rightarrow c\left(\frac{9-1}{2}\right) = 1 \Rightarrow \boxed{c = \frac{1}{4}}$$



② CDF ($F(x) = \int_{-\infty}^{x} p(u)\,du$)

$$\therefore F(x) = \int_1^x \frac{1}{4} u\,du = \frac{1}{4}\left[\frac{u^2}{2}\right]_1^x = \boxed{\frac{1}{8}(x^2-1)}.$$



③ $P(\frac{3}{2} < X \leq 2) = F(2) - F(\frac{3}{2})$

$$= \frac{1}{8}(2^2-1) - \frac{1}{8}\left(\left(\frac{3}{2}\right)^2-1\right) = \frac{1}{8}[3] - \frac{1}{8}\left[\frac{9}{4}-1\right]$$

$$= \frac{1}{8}\left[3 - \frac{5}{4}\right] = \frac{1}{8} * \frac{7}{4} = \boxed{\frac{7}{32}}$$

OR $P(\frac{3}{2} < X \leq 2) = \int_{\frac{3}{2}}^{2} p(x)\,dx = \int_{3/2}^{2} \frac{1}{4} x\,dx = \boxed{\frac{7}{32}}$

(2)

(4) $\text{mean} \equiv \mu = \int_{-\infty}^{+\infty} x p(x) dx = \int_{1}^{3} x * \frac{1}{4} x dx = \frac{1}{4}\int_{1}^{3} x^2 dx$

$$= \frac{1}{4} * \frac{x^3}{3}\Big|_1^3 = \frac{1}{12}(3^3 - 1^3) = \boxed{\frac{26}{12}}.$$

$\text{mean square} \equiv \overline{X^2} = E[X^2] = \int_{-\infty}^{+\infty} x^2 p(x) dx.$

$$= \int_{1}^{3} \frac{1}{4} x^3 dx = \frac{1}{4} \frac{x^4}{4}\Big|_1^3 = \frac{1}{16}(3^4 - 1^4)$$

$$= \frac{80}{16} = 5.$$

$\text{Variance} \equiv \sigma_X^2 = \overline{X^2} - \mu^2 = \frac{80}{16} - \left(\frac{26}{12}\right)^2$

$$= 5 - 4.69 \cong \boxed{0.31}$$

$$SD = \sigma_X = \sqrt{\sigma_X^2} = \sqrt{0.31} \cong \boxed{0.557}.$$

Q2: $X \sim N(\overset{\mu}{3}, \overset{\sigma}{1})$ ③

Sol.

① $P(0.75 < X \leq 4.52) = F(4.52) - F(0.75)$

$F(4.52) = \text{erf}\left(\frac{4.52-3}{1}\right) = \text{erf}(1.52)$

$= 0.93574.$ from table

$F(0.75) = \text{erf}\left(\frac{0.75-3}{1}\right) = \text{erf}(-2.25)$

$= 0.01222.$ from table.

$\therefore P(0.75 < X \leq 4.52) = 0.93574 - 0.01222$

$= \boxed{0.92352}.$

② $P(X \leq K) = F(K) = 0.99224.$

$\therefore \text{erf}\left(\frac{K-3}{1}\right) = 0.99224.$

$\Rightarrow K-3 = 2.42$ from the table

$\therefore \boxed{K = 5.42}$