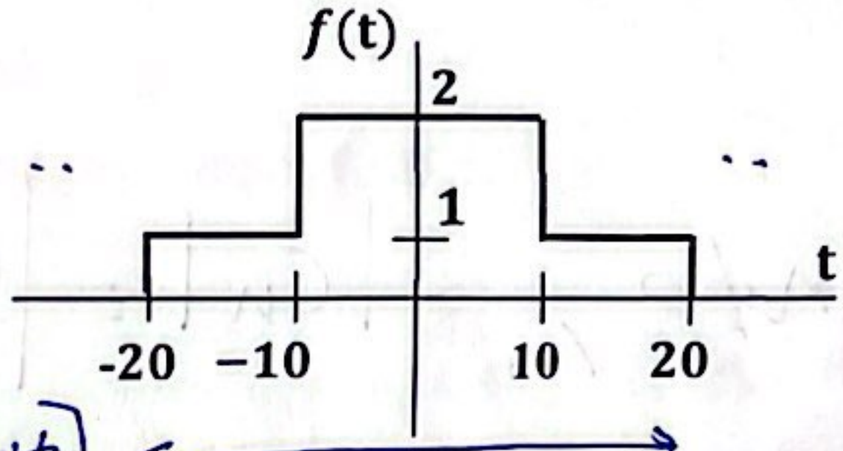
Engineering Analysis-EE301

الاسم:

Dr. Safaa A. Ali | Fourier Analysis Exam | 2022-12-15

Q1: Calculate the Complex Exponential Fourier coefficients (C_n) and sketch the frequency spectra considering to the following periodic signal $f(t)$.



$$C_n = \frac{1}{T}\int_0^T f(t)\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{40}\left[\int_{-20}^{-10} e^{-jn\omega_0 t}\, dt + \int_{-10}^{10} 2e^{-jn\omega_0 t}\, dt + \int_{10}^{20} e^{-jn\omega_0 t}\, dt\right]$$

$$\omega_0 = \frac{2\pi}{T} = \frac{\pi}{20}.$$

$$= \frac{1}{40}\left[\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Big|_{-20}^{-10} + 2\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Big|_{-10}^{10} + \frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Big|_{10}^{20}\right]$$

$$= \frac{1}{-j40n\omega_0}\left[e^{+j10n\omega_0} - e^{j20n\omega_0} + 2e^{-j10n\omega_0} - 2e^{j10n\omega_0} + e^{-j20n\omega_0} - e^{-j10n\omega_0}\right]$$

$$= \frac{1}{-j40n\omega_0}\left[-e^{j10n\omega_0} + e^{-j10n\omega_0} - e^{j20n\omega_0} + e^{-j20n\omega_0}\right]$$

$$= \frac{1}{-j40n\omega_0}\left[-2j\sin(10n\omega_0) - 2j\sin(20n\omega_0)\right]$$

$$= \frac{\sin(10n\omega_0) + \sin(20n\omega_0)}{20n\omega_0}.$$

$$C_n = \frac{\sin\left(n\frac{\pi}{2}\right) + \sin(n\pi)}{n\pi} = \frac{(-1)^n}{n\pi}. \quad \sin(n\pi) = 0.$$

h(t)

Q2: Find the impulse response of the system described by:

$$y(t) - y'(t) = 3x(t)$$ y(t)

using the Fourier transform. Also, find the output response if $x(t) = e^{4t}u(t)$.

① $y(t) - y'(t) = 3x(t)$.

$Y(w) - jwY(w) = 3X(w)$

$Y(w)[1 - jw] = 3X(w)$ — ⊛

$H(w) = \frac{Y(w)}{X(w)} = \frac{3}{1-jw}$.

$h(t) = \mathcal{F}^{-1}\{H(w)\} = 3e^{t}u(-t)$. ✓

$x(-t) \leftrightarrow X(-w)$

$e^{-at}u(t) \leftrightarrow \frac{1}{a+jw}$

$\frac{1}{a-jw} \leftrightarrow e^{at}u(-t)$

② from ⊛ and $X(w) = \frac{1}{-4+jw}$.

$Y(w) = \frac{3}{1-jw}X(w) = \frac{3}{1-jw} \cdot \frac{1}{-4+jw}$.

$= \frac{A}{1-jw} + \frac{B}{-4+jw} = -\frac{1}{1-jw} - \frac{1}{-4+jw}$.

A=-1 B=-1

$y(t) = \mathcal{F}^{-1}\{Y(w)\} = -e^{t}u(-t) - e^{4t}u(t)$. ✓

$A = \left[\frac{3}{-4+jw}\right]_{jw=1} = -1$, $B = \left[\frac{3}{1-jw}\right]_{jw=4} = -1$.

Q3: Given $\mathcal{F}\{e^{-|t|}\} = \frac{2}{w^2+1}$ find the Fourier transform of the following using properties:

1. $\frac{d}{dt}e^{-|t|}$.
2. $\frac{1}{t^2+1}$.

Sol. ① $\frac{d}{dt}x(t) \longleftrightarrow jw\,X(w)$

$\therefore \frac{d}{dt}e^{-|t|} \longleftrightarrow jw \cdot \frac{2}{w^2+1} = \frac{2jw}{w^2+1}$ ... ✓

② $x(t) \leftrightarrow X(w)$ **duality**

and $X(t) \longleftrightarrow 2\pi\, x(-w)$

$\therefore \frac{1}{t^2+1} \longleftrightarrow \left(\frac{1}{2} * 2\pi\, e^{-|-w|}\right)$.

$= \pi e^{-|w|}$. ✓