Q1: Find the L.T using properties:

$$\mathcal{L}\left[\int_0^t e^{j2}\frac{d}{d\tau}[\cos(3\tau)u(\tau)]d\tau\right]$$

Sol.

$$= e^{j2}\cdot\mathcal{L}\left\{\int_0^t\frac{d}{d\tau}[\cos(3\tau)u(\tau)]d\tau\right]$$

$$= e^{j2}\cdot\mathcal{L}\{\cos(3t)u(t)\}$$

$$= e^{j2}\cdot\frac{s}{s^2+3^2} = \frac{se^{j2}}{s^2+9}.$$

or

$$\cos(3t)u(t)\xrightarrow{L.T}\frac{s}{s^2+9}.$$

$$\frac{d}{dt}\{\cos(3t)u(t)\}\longrightarrow s\cdot\frac{s}{s^2+9}$$

$$\int_0^t\frac{d}{d\tau}\{\cos(3\tau)u(\tau)]d\tau\rightarrow\cancel{\frac{1}{s}}\cdot\cancel{s}\cdot\frac{s}{s^2+9}$$

$$= \frac{s}{s^2+9}.$$

Q_2: Find the I.L.T

$$F(S) = \frac{12S}{(S-\frac{1}{2})(S+\frac{1}{2})(S+1)}.$$

Sol.

$$F(S) = \frac{A}{S-\frac{1}{2}} + \frac{B}{S+\frac{1}{2}} + \frac{C}{S+1}.$$

$$A = \left[\frac{12S}{(S+\frac{1}{2})(S+1)}\right]_{S=\frac{1}{2}} = \frac{6}{1*\frac{3}{2}} = 4.$$

$$B = \left[\frac{12S}{(S-\frac{1}{2})(S+1)}\right]_{S=-\frac{1}{2}} = \frac{-6}{-1*\frac{1}{2}} = 12$$

$$C = \left[\frac{12S}{(S-\frac{1}{2})(S+\frac{1}{2})}\right]_{S=1} = \frac{-12}{-\frac{3}{2}*-\frac{1}{2}} = -16.$$

$$\therefore F(S) = 4\frac{1}{S-\frac{1}{2}} + 12\frac{1}{S+\frac{1}{2}} - 16\frac{1}{S+1}.$$

$$\therefore f(t) = 4e^{\frac{1}{2}t}u(t) + 12e^{-\frac{1}{2}t}u(t) - 16e^{-t}u(t).$$

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