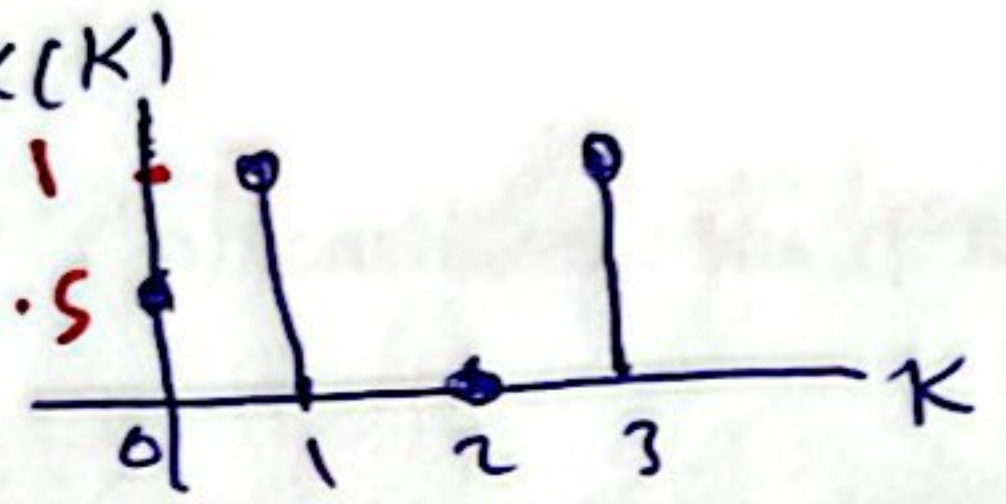
Q1: Find $y(n) = x(n) \circledast h(n)$ where

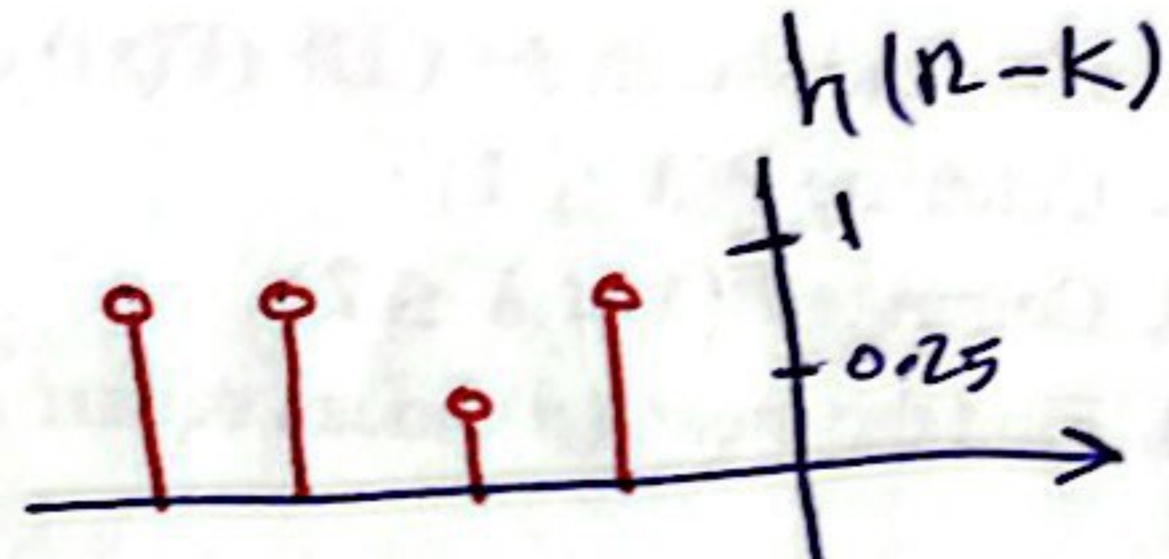
$$x(n) = \{\underset{\uparrow}{\tfrac{1}{2}}, 1, 0, 1\} \rightarrow n_x = 4$$

$$h(n) = \{\underset{\uparrow}{1}, 0.25, 1, 1\} \rightarrow n_h = 4$$

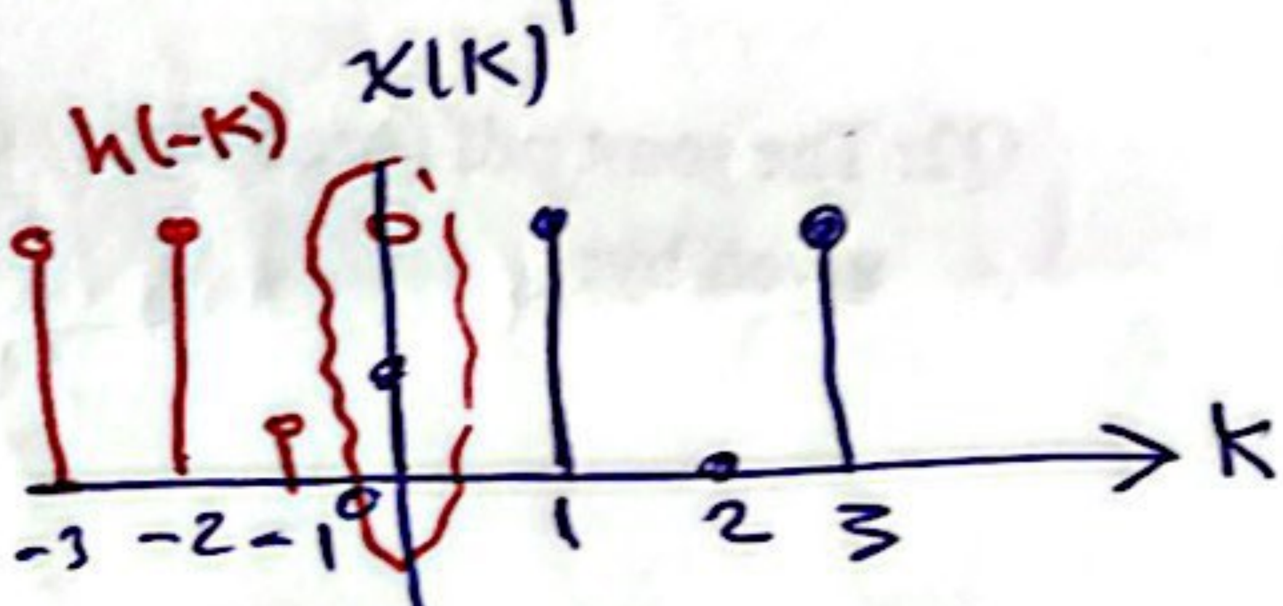
Sol.

$$y(n) = \sum_{k=-\infty}^{+\infty} x(k)\, h(n-k) \rightarrow n_y = 4+4-1 = 7$$



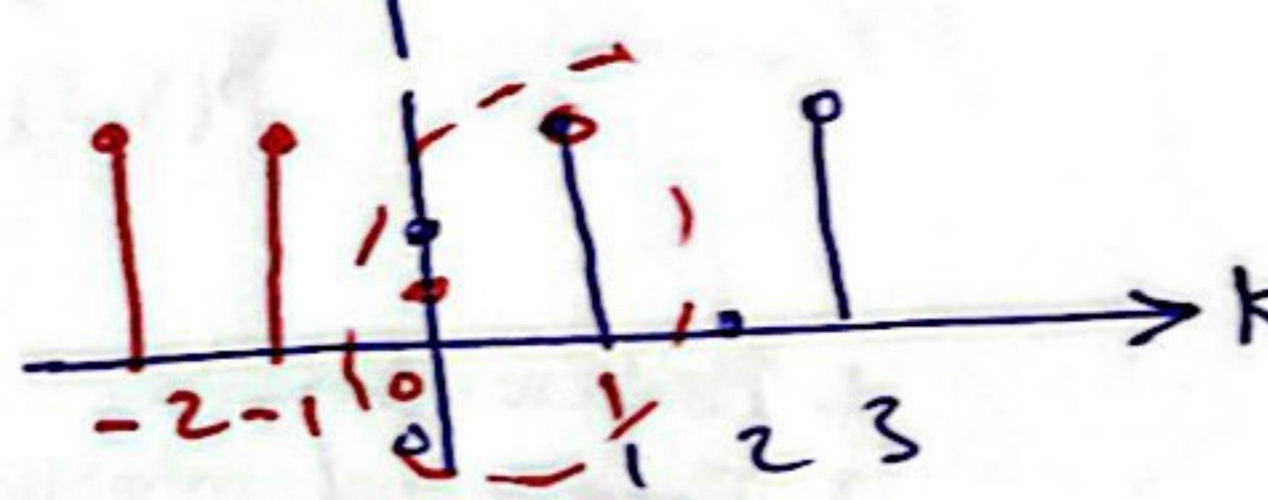


$n < 0 \rightarrow y(n) = 0$



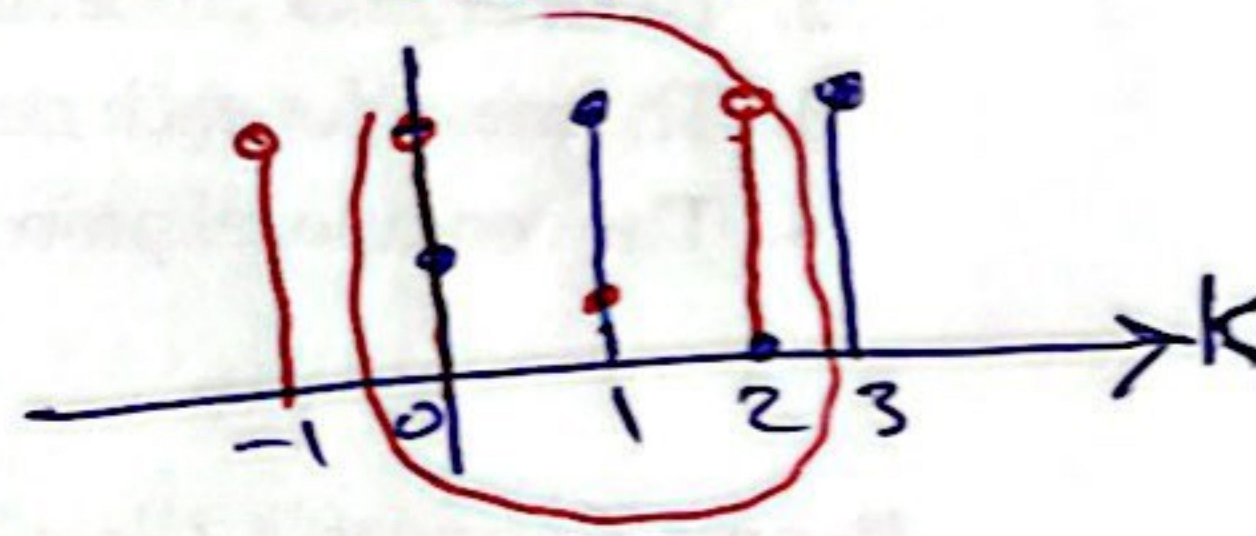
① $n=0 \rightarrow y(0) = \sum_{K=0}^{0} x(K)\, h(-K)$

$= \boxed{0.5}$



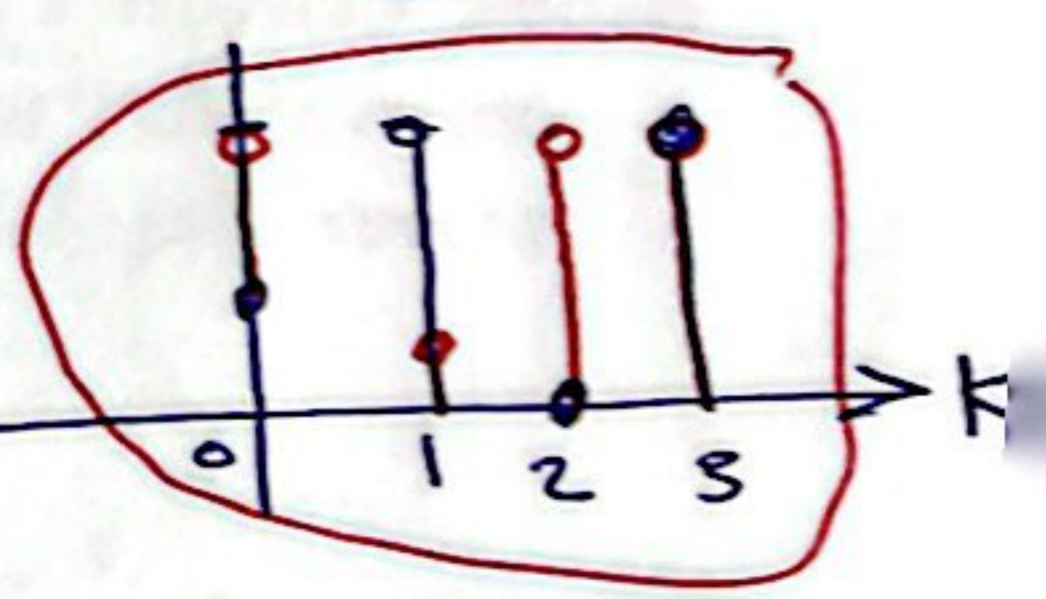
② $n=1 \rightarrow y(1) = \sum_{K=0}^{1} x(K)\, h(1-K)$

$= \frac{1}{2} * \frac{1}{4} + 1*1 = \boxed{1.125}$



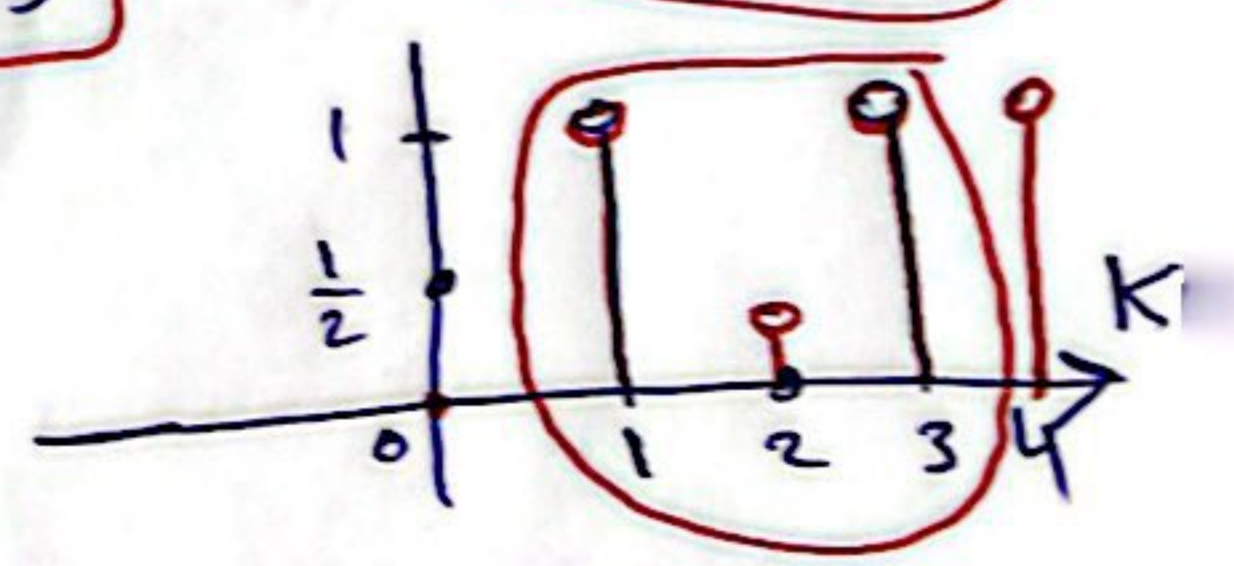
③ $n=2 \rightarrow y(2) = \sum_{K=0}^{2} x(K)\, h(2-K)$

$= \frac{1}{2}*1 + \frac{1}{4}*1 + 0*1 = \boxed{0.75}$



④ $n=3 \rightarrow y(3) = \sum_{K=0}^{3} x(K)\, h(3-K)$

$= \frac{1}{2}*1 + 1*\frac{1}{4} + 0*1 + 1*1 = \boxed{1.75}$

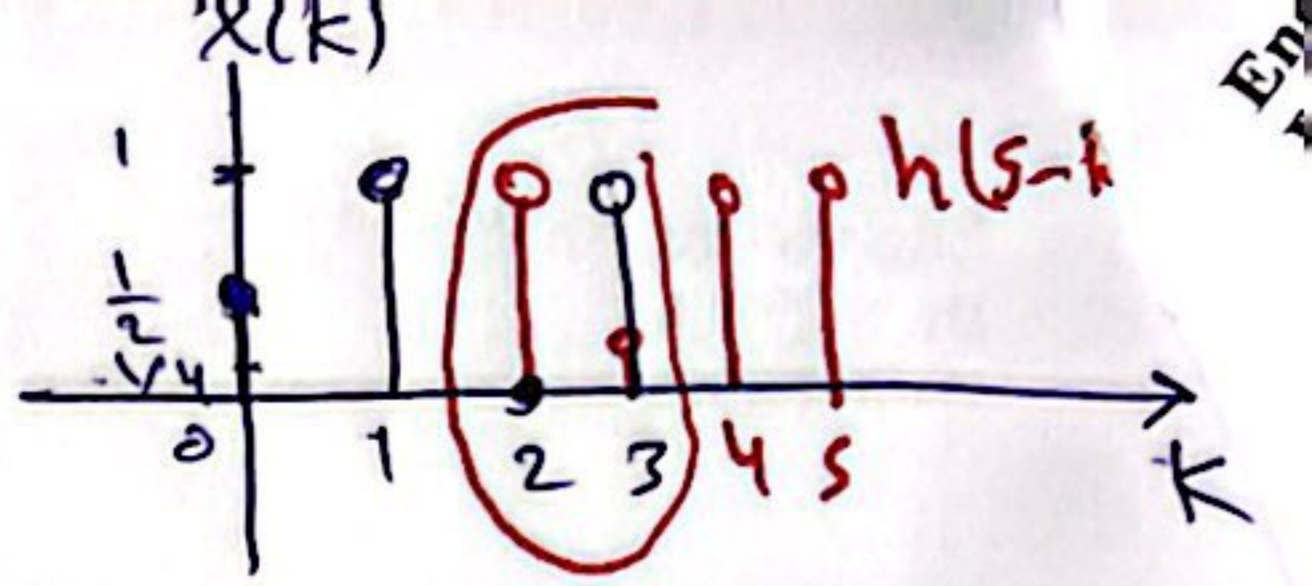


⑤ $n=4 \rightarrow y(4) = \sum_{K=1}^{3} x(K)\, h(4-K)$

$= 1*1 + 0*\frac{1}{4} + 1*1 = \boxed{2}$

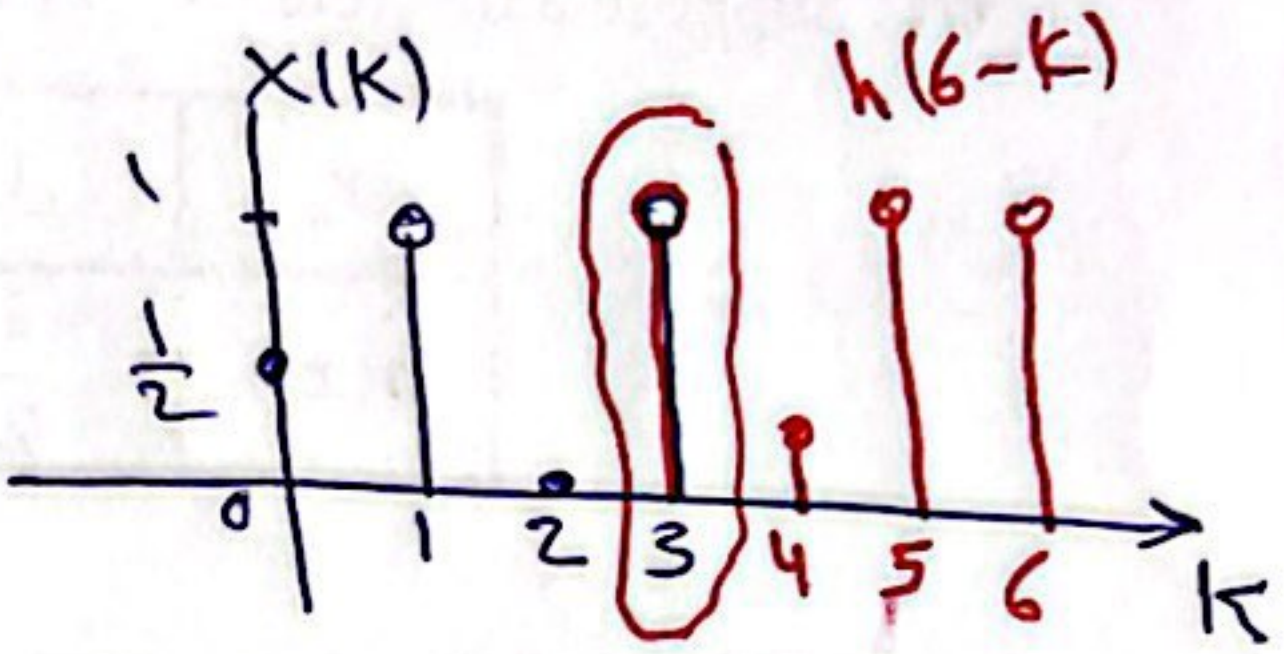
(6)
$n=5 \rightarrow y(5) = \sum_{k=2}^{3} x(k)h(5-k)$

$= 0*1 + 1*\frac{1}{4} = \boxed{0.25}$



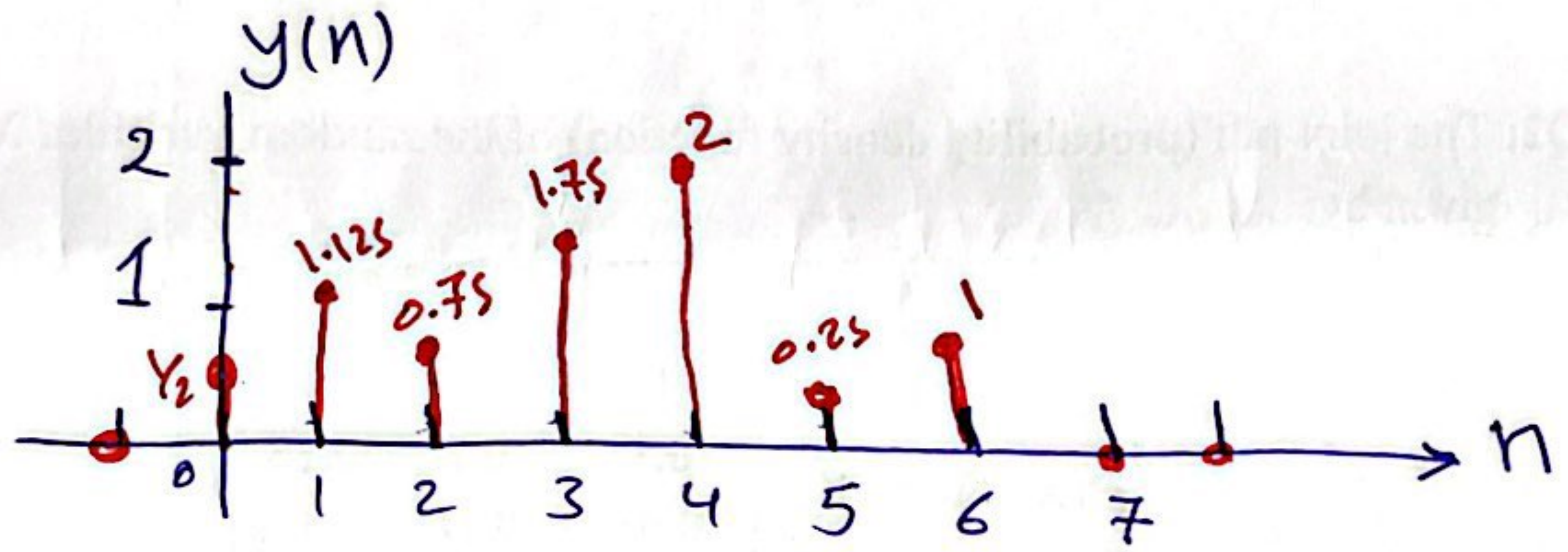
(7)
$n=6 \rightarrow y(6) = \sum_{k=3}^{3} x(k)h(6-k)$

$= 1*1 = \boxed{1}$



$n > 6 \rightarrow y(n) = 0.$

# of samples of $y(n) = 4 + 4 - 1 = \underline{\underline{7}}$



Q_2: Find $\mathcal{Z}\{(n+1)u(n)\} = ?$

Sol.

$$\mathcal{Z}\{(n+1)u(n)\} = \mathcal{Z}\{nu(n) + u(n)\}$$

$$= \mathcal{Z}\{nu(n)\} + \mathcal{Z}\{u(n)\} \quad \text{---} *$$

$$\mathcal{Z}\{u(n)\} = \frac{z}{z-1} \quad \text{---} ①$$

$$\mathcal{Z}\{nu(n)\} = \frac{z}{(z-1)^2} \quad \text{from table.}$$

OR $$\mathcal{Z}\{nu(n)\} = -z \cdot \frac{d}{dz}\left[\frac{z}{z-1}\right]$$

$$= -z \cdot \frac{(z-1)-z}{(z-1)^2} = \frac{z}{(z-1)^2} \quad \text{---} ②$$

Sub ① and ② into (*), we get:

$$\mathcal{Z}\{(n+1)u(n)\} = \frac{z}{(z-1)^2} + \frac{z}{z-1}$$

$$= \frac{z + z(z-1)}{(z-1)^2} = \frac{\cancel{z} + z^2 - \cancel{z}}{(z-1)^2}$$

$$= \frac{z^2}{(z-1)^2}$$