

Q1 $S = \{a, b, c, d\}$.

Quiz

$$P(a) = 0.2$$

$$P(c) = 0.4$$

$$P(b) = 0.3$$

~~P(d) = 0.1~~

$$A = \{a, b\}$$

$$B = \{b, c, d\}$$

$$P(A) = ?$$

$$P(B) = ?$$

$$P(A \cup B) = ?$$

$$P(A \cap B) = ?$$

Sol. $P(d) = 1 - (0.2 + 0.3 + 0.4)$
 $= 0.1$

$$P(A) = P(\{a, b\}) = P(a) + P(b)$$
$$= 0.2 + 0.3 = 0.5$$

$$P(B) = P(\{b, c, d\}) = P(b) + P(c) + P(d)$$
$$= 0.3 + 0.4 + 0.1 = 0.8$$

$$P(A \cup B) = P(S) = 1.$$

$$P(A \cap B) = P(\{b\}) = 0.3$$

Q2: Consider a telegraph source generating two symbols (dots and dashes). We observed that the dots ~~are~~ ^{were} twice as likely to occur as the dashes. Find the probabilities of the dots and dashes.

$$\begin{cases} p = \text{prob. (dots)} \\ q = \text{prob. (dashes)} \end{cases} \Rightarrow \begin{cases} p + q = 1 \\ 2q + q = 1 \end{cases} \Rightarrow \begin{cases} q = \frac{1}{3} \\ p = \frac{2}{3} \end{cases}$$

Q3:

$$P(X, Y) = \begin{matrix} & \begin{matrix} 1 & 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \end{matrix} & \begin{bmatrix} K & 0.3 \\ 0.1 & 0 \end{bmatrix} \end{matrix} = \begin{matrix} & \begin{matrix} 1 & 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \end{matrix} & \begin{bmatrix} 0.6 & 0.3 \\ 0.1 & 0 \end{bmatrix} \end{matrix}$$

- ① Find K
- ② $P(X)$, $P(Y)$
- ③ $P(X|Y)$, $P(Y|X)$.

Sol. ① $K = 1 - (0.1 + 0 + 0.3)$
 $= 0.6$

② $P(X) = \begin{bmatrix} 0 & 1 \\ 0.9 & 0.1 \end{bmatrix}$

$P(Y) = \begin{bmatrix} 1 & 2 \\ 0.7 & 0.3 \end{bmatrix}$

③ $P(\cancel{X}/X) = \frac{\begin{bmatrix} 0.6 & 0.3 \\ 0.1 & 0 \end{bmatrix}}{\begin{bmatrix} 0.9 & 0.1 \end{bmatrix}} = \begin{bmatrix} \frac{0.6}{0.9} & \frac{0.3}{0.9} \\ \frac{0.1}{0.1} & 0 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ 1 & 0 \end{bmatrix}$

$P(\cancel{X}/\cancel{X}) = \frac{\begin{bmatrix} 0.6 & 0.3 \\ 0.1 & 0 \end{bmatrix}}{\begin{bmatrix} 0.7 & 0.3 \end{bmatrix}} = \begin{bmatrix} \frac{0.6}{0.7} & \frac{0.3}{0.3} \\ \frac{0.1}{0.7} & 0 \end{bmatrix}$

$= \begin{bmatrix} \frac{6}{7} & 1 \\ \frac{1}{7} & 0 \end{bmatrix}$