Matrix operations:

MATLAB is short for MATrix LABoratory, and is designed to be a tool for quick and easy manipulation

of matrix forms of data. We've seen the matrix before in Lecture 1 as a 2-D array. That is, many pieces of

information are stored under a single name. Different pieces of information are then retrieved by pointing

to different parts of the matrix by row and column. Here we will learn some basic matrix operations:

Adding and Subtracting, Transpose, Multiplication.

Adding matrices

Add two matrices together is just the addition of each of their respective elements.

If A and B are both matrices of the same dimensions (size), then

C = A + B

Produces C, where the i^{th} row and j^{th} column are just the addition of the elements (numbers) in the i^{th} row and j***th*** column of A and B

Let's say that:

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 7 & 9 & 11 \end{bmatrix}, \text{ and } B = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \end{bmatrix}$$

so that the addition is :

$$C = A + B = \begin{bmatrix} 3 & 7 & 11 \\ 15 & 19 & 23 \end{bmatrix}$$

The MATLAB commands to perform these matrix assignments and the addition are:

A = [1 3 5 ; 7 9 11]

B = [2 4 6 ; 8 10 12]

C = A + B

Rule: A, B, and C must all have the same dimensions

Transpose

Transposing a matrix means swapping rows and columns of a matrix. **No matrix dimension restrictions**

Some examples:

1-D $A = \begin{bmatrix} 5 & 2 & 9 \end{bmatrix}$, $A^T = \begin{bmatrix} 5 \\ 2 \\ 9 \end{bmatrix}$ 1x3 becomes ==> 3x1

7 9 11

2-D $B = \begin{bmatrix} 8.1 & -4.5 & -7.6 \\ 3.2 & 3.1 & 3.9 \end{bmatrix}$, $B^T = \begin{bmatrix} 8.1 & 3.2 \\ -4.5 & 3.1 \\ -7.6 & 3.9 \end{bmatrix}$ 2x3 becomes ==> 3x2

In general

$$B(i,j) = B^T(j,i)$$

In MATLAB, The transpose is indicated by a single quote after the array

```
>> B = [5 3 6 2; 9 8 4 7]
```

$$B = \begin{bmatrix} 5 & 3 & 6 & 2 \\ 9 & 8 & 4 & 7 \end{bmatrix}$$

```
>> B'
```

$$ans = \begin{bmatrix} 5 & 9 \\ 3 & 8 \\ 6 & 4 \\ 2 & 7 \end{bmatrix}$$

Multiplication

Multiplication of matrices is not as simple as addition / subtraction. It is not an element by element multiplication as you might suspect it would be. Rather, matrix multiplication is the result of the dot products of rows in one matrix with columns of another. Consider:

$$C = A * B$$

Matrix multiplication gives the i^{th} row and k^{th} column spot in C as the scalar results of the dot product of the i^{th} row in A with the k^{th} column in B. In equation form, this looks like:

$$C_{i,k} = \sum_{j=1}^{\text{\# or columns in A}} A_{i,j} * B_{j,k}$$

Let's break this down in a step-by-step example:

Step 1: Dot Product (a 1-row matrix times a 1-column matrix)
The Dot product is the **scalar** result of multiplying one row by one column

$$\underset{1\text{x}3}{\begin{bmatrix} 2 & 5 & 3 \end{bmatrix}} * \begin{bmatrix} 6 \\ 8 \\ 7 \end{bmatrix}_{3\text{x}1} = 2*6 + 5*8 + 3*7 = 73_{1\text{x}1}$$

DOT PRODUCT OF ROW AND COLUMN

Rule:

1) # of elements in the row and column must be the same
2) must be a row times a column, not a column times a row

Step 2: general matrix multiplication is taking a series of dot products
each row in pre-matrix by each column in post-matrix

$$\underset{2\text{x}3}{\begin{bmatrix} 1 & 4 & 2 \\ 9 & 3 & 7 \end{bmatrix}} * \begin{bmatrix} 5 & 6 \\ 8 & 12 \\ 10 & 11 \end{bmatrix}_{3\text{x}2} = \begin{bmatrix} 1*5+4*8+2*10 & 1+6*4*12+2*11 \\ 9*5+3*8+7*10 & 9*6+3*12+7*11 \end{bmatrix} = \begin{bmatrix} 57 & 76 \\ 139 & 167 \end{bmatrix}_{2\text{x}2}$$

C(i,k) is the result of the dot product of row i in A with column k in B

Matrix Multiplication Rules:

1) The # of columns in the pre-matrix must equal # of rows in post-matrix
   inner matrix dimensions must agree
2) The result of the multiplication will have the outer dimensions
   # rows in pre-matrix by # columns in post-matrix

For this example, apply rules

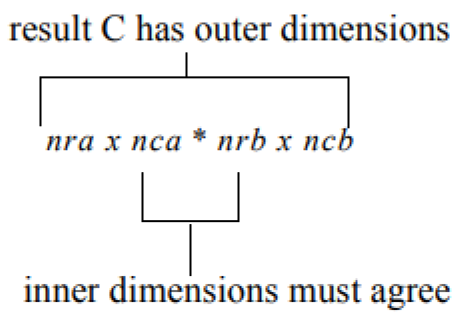
```
>> C = A * B
```

A is nra x nca   (# rows in a by # columns in a)
B is nrb x ncb

Rule 1 says:

nca = nrb   or else we can't multiply (can't take dot products with different number of terms in row and column)

Rule 2 says:
C will be nra x ncb



How to perform matrix multiplication in MATLAB???
Easy

```
>> A = [4 5; 2 1];
>> B = [9 1; 6 12];
>> C = A*B
```

if inner matrix dimensions don't match, you'll get an error
example: Let's try to multiply a 2x3 by another 2x3 (rules say we can't do this)

```
>> A = [3 4 1 ; 0 4 9];
>> B = [2 9 5 ; 9 4 5];
>> C = A * B
```

```
MATLAB will tell you:

??? Error using ==> *
Inner matrix dimensions must agree.
```

Since the # of columns in A was not equal to # of rows in B, we can't multiply A * B

IMPORTANT:
Another example: Say we create a 1-D vector x with the following:

```
>> x = [2 4 9 5 2];
```

Now say we want to square each number in x. It would seem natral to do this:

But MATLAB tells us:

??? Error using ==> ^
Matrix must be square.

Note that y = x^2 is the same as saying y = x*x

MATLAB by default will always interpret any multiplication as a standard dot product type matrix multiplication, thus we can't take a dot product of two row vectors, since rules of matrix multiplication are violated in this case. If we just want to square the numbers in x, we can do this:

```
>> y = x.^2
```

The period after the vector x tells MATLAB <u>DO NOT</u> follow multiplication rules, just operate on the individual elements within the matrix. So y = x . ^2 is NOT the same as y = x ^2 to MATLAB

Practice matrix operations on the following examples.
List the size of the resulting matrix first. then perform the operations by hands. Use MATLAB to confirm each of your answers.

$$\begin{bmatrix} 9 & 12 & 5 \end{bmatrix} * \begin{bmatrix} 2 \\ 8 \\ 3 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

$$\begin{bmatrix} 9 & 7 \\ 3 & 6 \\ 7 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

$$\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} * \begin{bmatrix} 8 & 7 \\ 9 & 6 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ 9 \\ 7 \end{bmatrix} * \begin{bmatrix} 2 & 1 & 8 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 9 \\ 8 & 6 & 4 \end{bmatrix} * \begin{bmatrix} 3 & 0 \\ 9 & 3 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

$$\begin{bmatrix} 4 & 9 & 8 \end{bmatrix}^T * \begin{bmatrix} 2 & 7 & 1 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

$$\left(\begin{bmatrix} 4 & 7 \\ 8 & 9 \end{bmatrix} + \begin{bmatrix} 5 & 3 \\ 0 & 1 \end{bmatrix} \right)^T * \begin{bmatrix} 10 & 2 & 7 \\ 6 & 3 & 5 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

Fundamental Program Structure

Labeling the program using comments
program title
student information
program summary

executable statements
program input (load command, assignment statements, etc.)
perform operations needed (sequential execution, loops, etc.)
display program output (graphs, numbers, output files, etc.)
intersperse comments to explain program

```
>> % Example program #1
>> % K. Gurley, CGN 3421, 1/11/99
>> % This program does very little
>> %
>> %   INPUT SECTION
>> %   enter x vector, then calc. y and z
>> x=[1, 2, 4, 8];   %input constants
>> %
>> %   PERFORM OPERATIONS NEEDED
>> %create a scalar y as a function of x(2) and x(3)
>> y= (x(2) + x(3))^2   %leave off ; to show result
>> z= x.^2 + y;          %create z vector from x and y
>> %
>> %   DISPLAY OUTPUT GRAPH
>> plot(x,z)
>> %end of program
```