Conjugate Beam Method

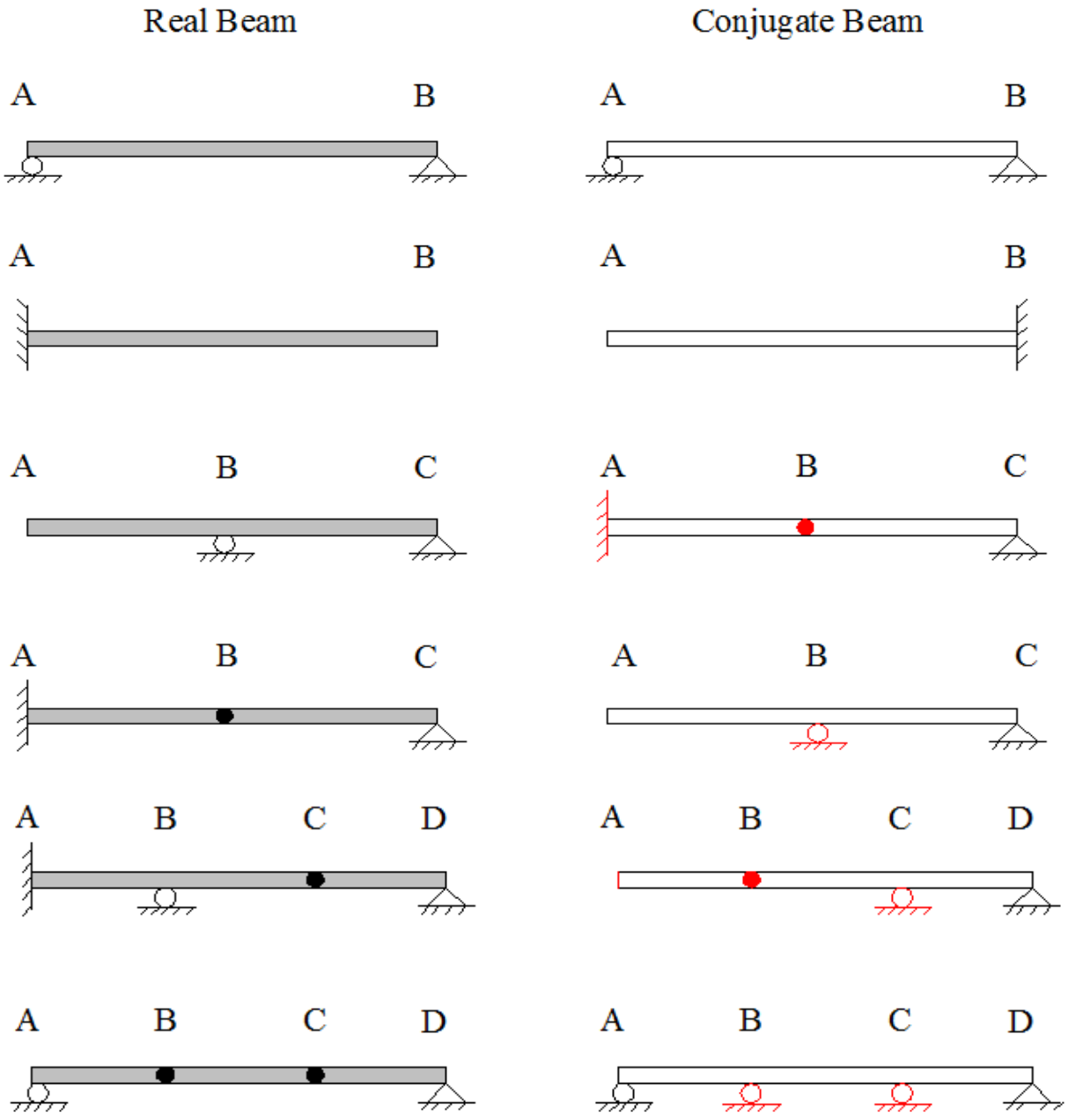
Conjugate Beam Method

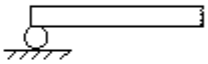
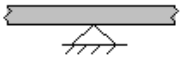
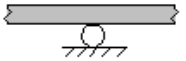
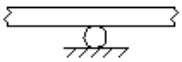
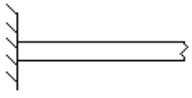
Theorem I: The Rotation in Beam is Equal to The Shear in Conjugate Beam.

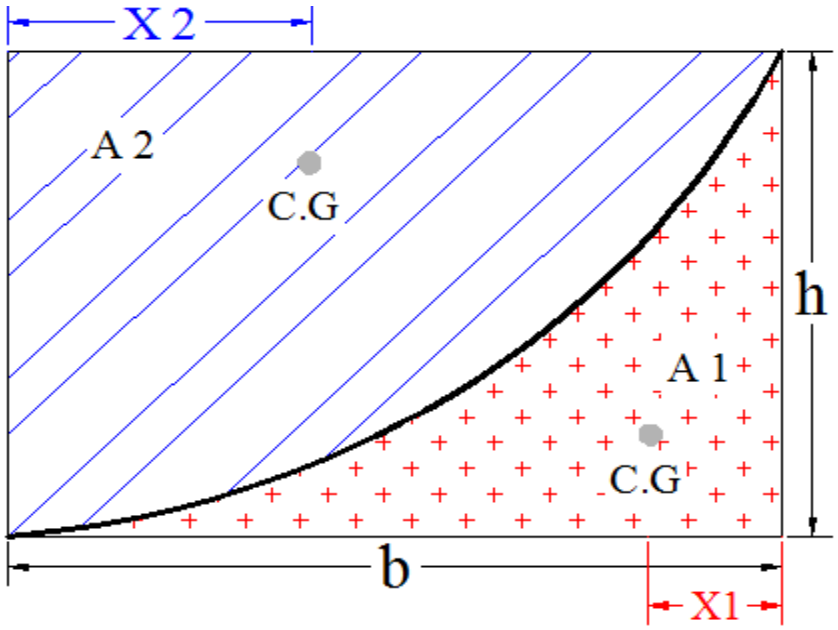
$$\theta_{\text{Real Beam}} = V_{\text{Conjugate Beam}}$$

Theorem I: The Deflection in Real Beam is The Moment in Conjugate Beam.

$$\Delta_{\text{Real Beam}} = M_{\text{Conjugate Beam}}$$



Real Beam		Conjugate Beam	
$\theta = 0$ $\Delta = 0$	 Fixed	$V = 0$ $M = 0$	 Free End
θ $\Delta = 0$	 Roller	V $M = 0$	 Roller
θ $\Delta = 0$	 Pin	A V $M = 0$	 Pin
θ $\Delta = 0$	 Internal Pin	V $M = 0$	 Internal Hinge
θ $\Delta = 0$	 Internal Roller	V $M = 0$	 Hinge
θ Δ	 Hinge	V M	 Internal Roller
θ Δ	 Free End	V M	 Fixed



A_1	X_1	A_2	X_2
$\frac{b\,h}{n+1}$	$\frac{b}{n+2}$	$\frac{n\,b\,h}{n+1}$	$(\frac{n+3}{2})$. X_1

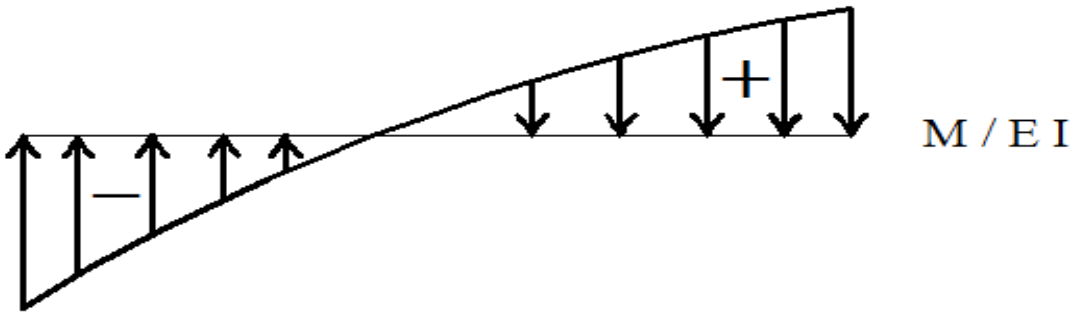
n = Equation Degree

If Applied Load is:

1. Concentrated, the Equation Degree for Moment , n =1

2. Distributed Load, the Equation Degree for Moment , n=2

3. Triangle Load, the Equation Degree for Moment , n=3

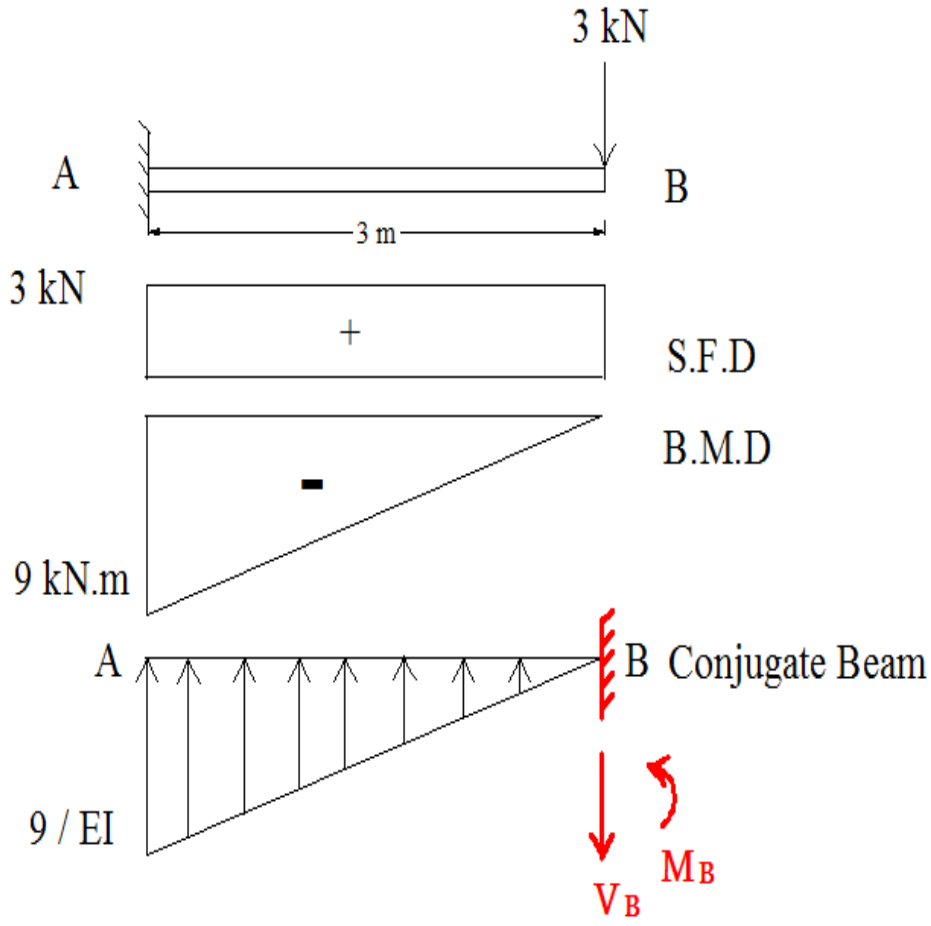


Example: Find the Rotation and Deflection at point B in Cantilever Beam Below by Using the Conjugate Beam Method.

Solution:

- $\sum F_y = 0 \uparrow$
  $Ay - 3 = 0$
  $Ay = 3\ kN \uparrow$

- $\sum M_A = 0 \circlearrowright$
  $3 * 3 - M_A = 0$
  $M_A = 9\ kN.m \circlearrowleft$



Finding V_B & M_B

- $\sum F_y = 0 \uparrow$

  $\frac{1}{2} * 3 * \frac{9}{E\,I} - V_B = 0$

  $V_B = \frac{13.5}{E\,I}$

  $\boldsymbol{\theta_B = \frac{13.5}{E\,I}}$

- $\sum M_B = 0 \circlearrowright$

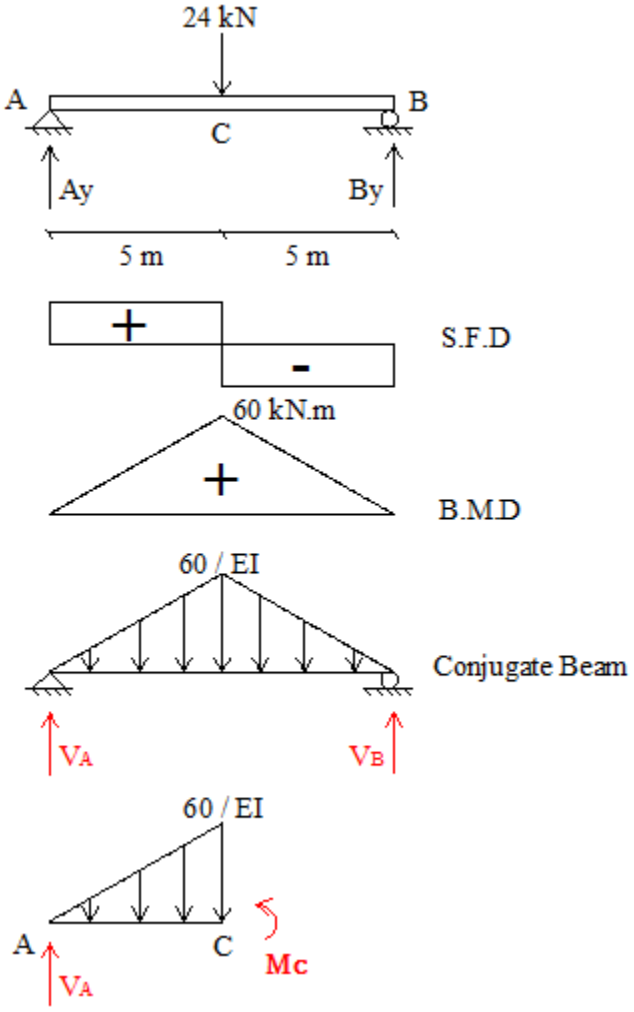
  $(\frac{1}{2} * 3 * \frac{9}{E\,I}) * (\frac{2}{3} * 3) - M_B = 0$

  $M_B = \frac{27}{E\,I}$

  $\boldsymbol{\Delta_B = \frac{27}{E\,I}}$

Example: Find the Rotation at supports, and Deflection at C in Simply supported Beam Below by Using the Conjugate Beam Method.

Solution :



- $\sum M_B = 0$ ↻

  $Ay * 10 - 24 * 5 = 0$

  $Ay = 12$ kN ↑

- $\sum F_y = 0$ ↑

  $12 - 24 + By = 0$

  $By = 12$ kN ↑

Finding V_A & V_B

- $\sum M_B = 0$ ↻

$$V_A * 10 - \left(\frac{1}{2} * 5 * \frac{60}{E\,I}\right) * \left(5 + \frac{1}{3} * 5\right) - \left(\frac{1}{2} * 5 * \frac{60}{E\,I}\right) * \left(\frac{2}{3} * 5\right) = 0$$

$$V_A = \frac{150}{E\,I}$$

$$\theta_A = \frac{150}{E\,I}$$

- $\sum F_y = 0$ ↑

$$\frac{150}{E\,I} - \left(\frac{1}{2} * 5 * \frac{60}{E\,I}\right) - \left(\frac{1}{2} * 5 * \frac{60}{E\,I}\right) + V_B = 0$$

$$V_B = \frac{150}{E\,I}$$

$$\theta_B = \frac{150}{E\,I}$$

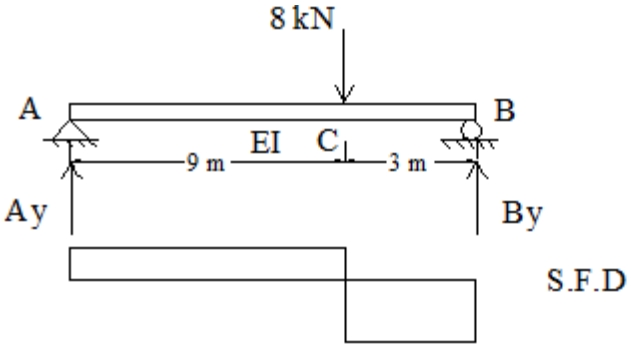
- $\sum M_C = 0$ ↻

$$\frac{150}{E\,I} * 5 - \left(\frac{1}{2} * 5 * \frac{60}{E\,I}\right) * \left(\frac{1}{3} * 5\right) - M_C = 0$$

$$M_C = \frac{500}{E\,I}$$

$$\Delta_C = -\frac{500}{E\,I}$$

Example: Find the Maximum Deflection at Beam AB Below by Using the Conjugate Beam Method.



Solution :

- $\sum M_A = 0 \circlearrowleft$

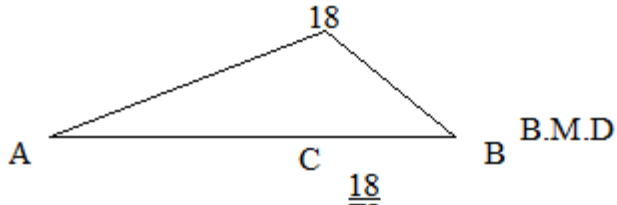
  $8 * 9 - B_y * 12 = 0$

  $B_y = 6\ kN \uparrow$

- $\sum F_y = 0 \uparrow$

  $A_y - 8 + 6 = 0$

  $A_y = 2\ kN \uparrow$

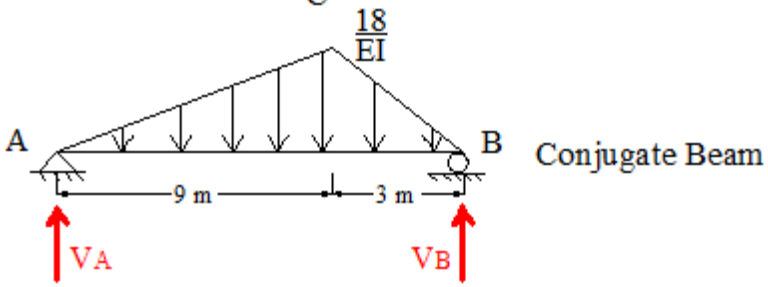


$$P_1 = \frac{1}{2} * 9 * \frac{18}{E\,I}$$

$$P_1 = \frac{81}{E\,I}$$

$$P_2 = \frac{1}{2} * 3 * \frac{18}{E\,I}$$

$$P_2 = \frac{27}{E\,I}$$

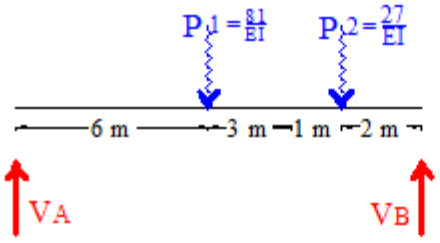


- $\sum M_A = 0 \circlearrowleft$

  $\frac{81}{E\,I} * 6 + \frac{27}{E\,I} * (6 + 3 + 1) - V_B * 12 = 0$

$$V_B = \frac{63}{E\,I}$$

$$\theta_B = \frac{63}{E\,I}$$

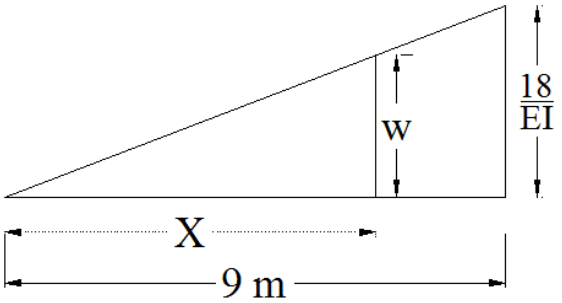


- $\sum F_y = 0 \uparrow$

$$V_A - \frac{81}{E\,I} - \frac{27}{E\,I} + \frac{63}{E\,I} = 0$$

$$V_A = \frac{45}{E\,I}$$

$$\theta_A = \frac{45}{E\,I}$$



$$\frac{w}{x} = \frac{\frac{18}{E\,I}}{9}$$

$$w = \frac{2\,x}{E\,I}$$

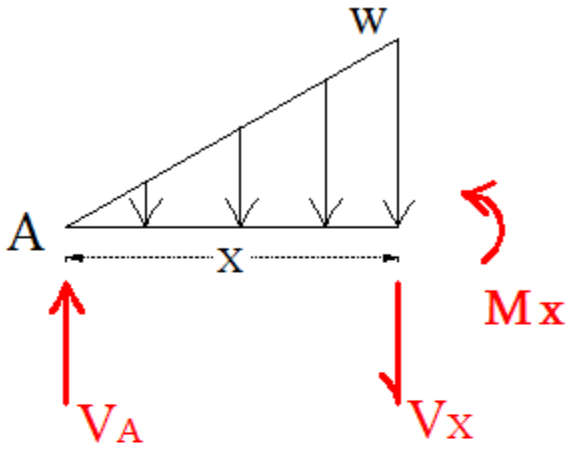
$P_3 = \frac{1}{2} * x * w$

$P_3 = \frac{1}{2} * x * \frac{2x}{EI}$

$P_3 = \frac{x^2}{EI}$

∵ x @ Maximum Deflection ,

∴ $\theta_x = 0$, $V_X = 0$



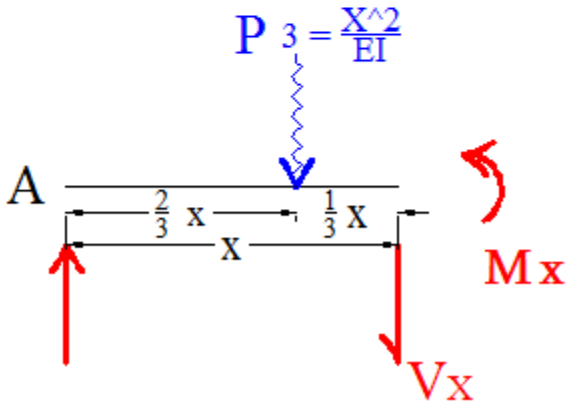
○ $\sum F_y = 0 \uparrow$

$V_A - \frac{x^2}{EI} - V_X = 0$

$\frac{45}{EI} - \frac{x^2}{EI} - 0 = 0$

$x = 6.71$ m , $0 \leq x \leq 9$, ∴ O.K

∵ $\Delta_{max} = M_X$



○ $\sum M_x = 0$ ↺

$\frac{45}{EI} * x - \frac{x^2}{EI} * (\frac{1}{3} * x) - M_X = 0$

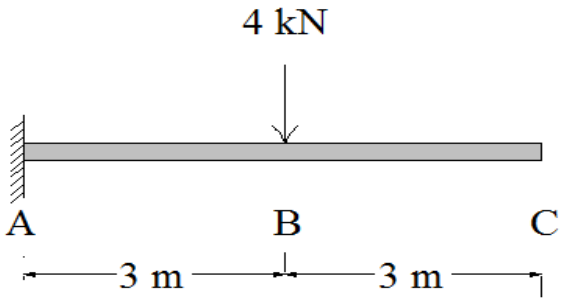
$\frac{45}{EI} * 6.71 - \frac{6.71^2}{EI} * (\frac{1}{3} * 6.71) - M_X = 0$

$M_X = \frac{201.2}{EI}$

$\Delta_{max} = -\frac{201.2}{EI}$

Example: By using Conjugate Beam for AC beam Determine

1. Deflection and Slope at B
2. Maximum Deflection
3. Slope at C.



Solution

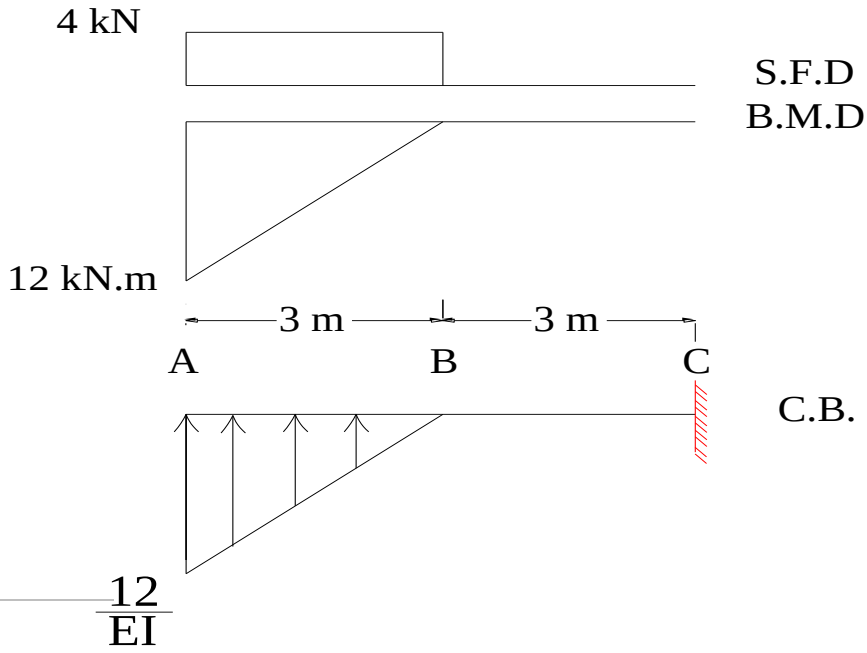
- $\sum Fy = 0 \uparrow$

  $Ay = 4\ kN \uparrow$

- $\sum M_A = 0 \circlearrowright$

  $4 * 3 - M_A = 0$

  $M_A = 12\ kN.m \circlearrowleft$



$\frac{12}{EI}$

1.

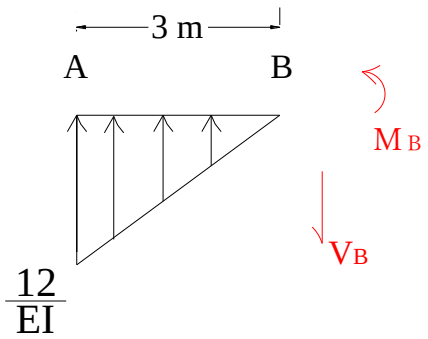
✓ θ_B

- $\sum Fy = 0 \uparrow$

  $\frac{1}{2} * 3 * \frac{12}{E\,I} - V_B = 0$

  $V_B = \frac{18}{E\,I}$

  $\theta_B = \frac{18}{E\,I}$



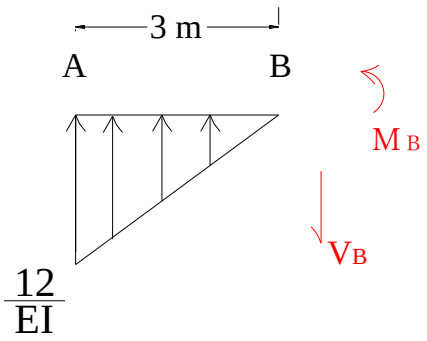
✓ Δ_B

○ $\sum M_B = 0 \circlearrowleft$

$\frac{1}{2} * 3 * \frac{12}{E\,I} * (\frac{2}{3} * 3) - M_B = 0$

$M_B = \frac{36}{E\,I}$

$\Delta_B = \frac{36}{E\,I}$



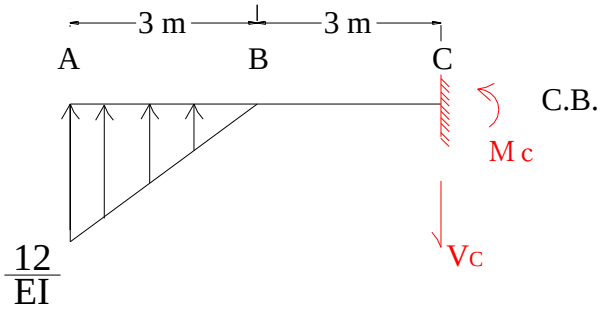
2.Δ_{MAX}

✓ $\sum M_C = 0 \circlearrowleft$

$\frac{1}{2} * 3 * \frac{12}{E\,I} * (\frac{2}{3} * 3 + 3) - M_C = 0$

$M_C = \frac{90}{E\,I}$

$\Delta_{MAX} = \frac{90}{E\,I}$



---

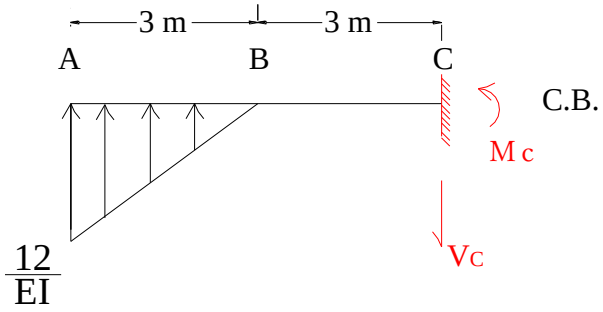
3. θ_C

✓ $\sum Fy = 0 \uparrow$

$\frac{1}{2} * 3 * \frac{12}{E\,I} - V_C = 0$

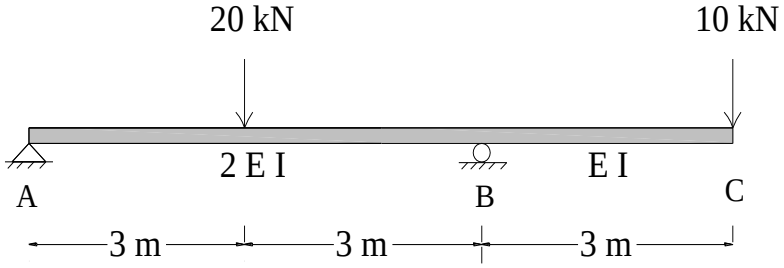
$V_C = \frac{18}{E\,I}$

$\theta_C = \frac{18}{E\,I}$



Example : Determine the Deflection and Rotation at C for Beam Below :

Solution



$\sum M_A = 0 \circlearrowleft$

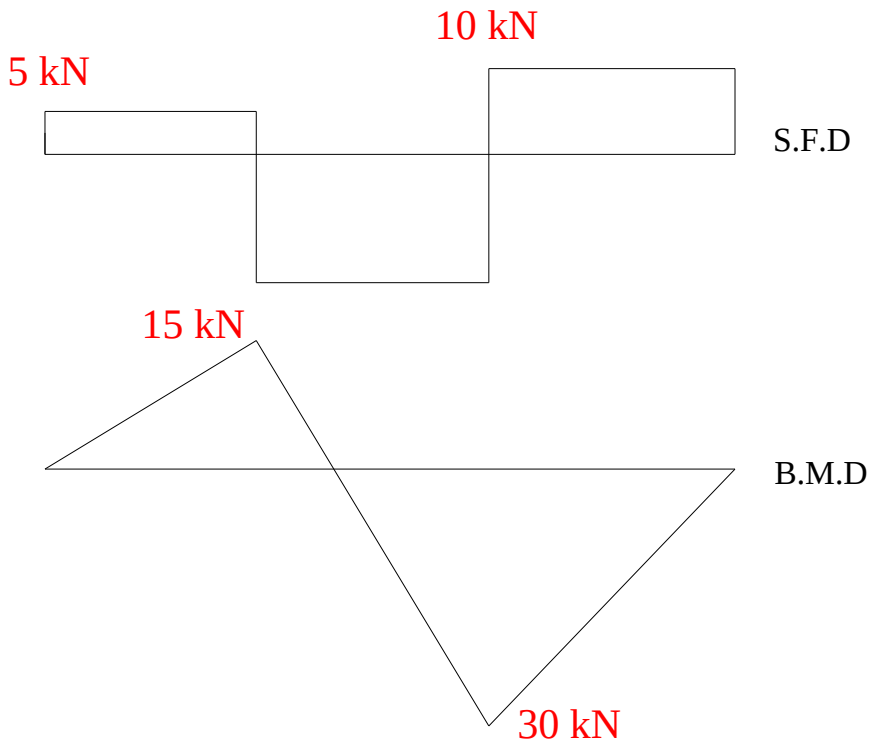
$20*3 - B_y*6 + 10*9=0$

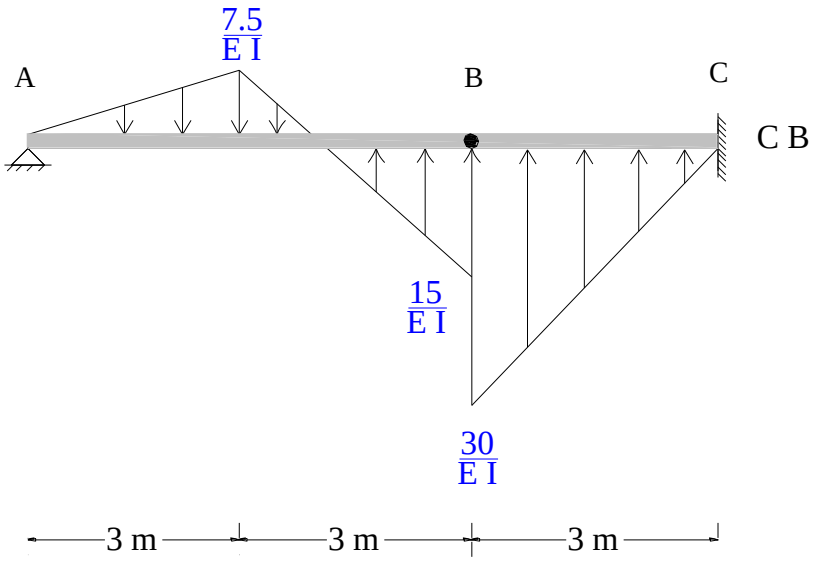
$B_y = 25\ kN \uparrow$

$\sum Fy = 0 \uparrow$

$25 + A_y - 20 - 10 = 0$

$A_y = 5\ kN \uparrow$



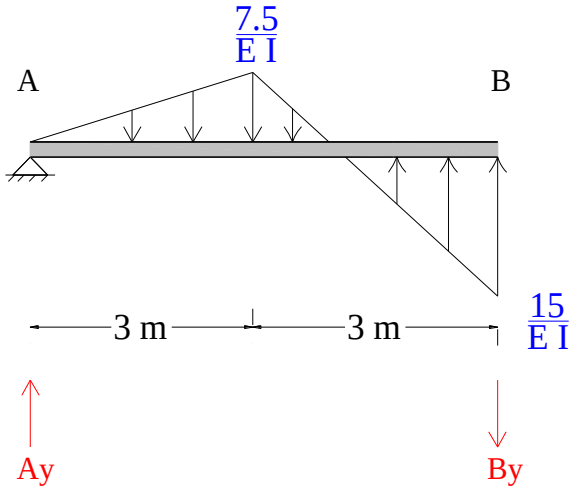


Member AB

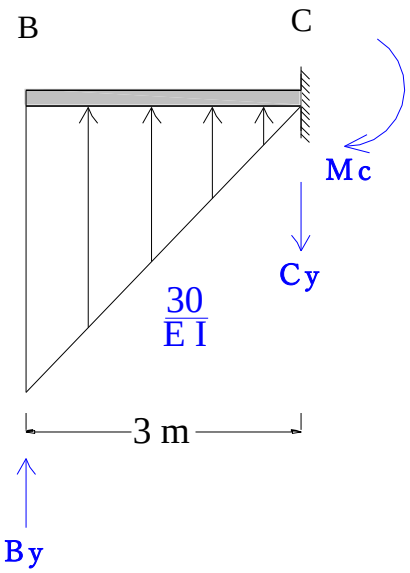
- $\sum M_A = 0 \circlearrowright$

$\{0.5 * 3 * \frac{7.5}{E\,I}\} * 2 + \{0.5 * 1 * \frac{7.5}{E\,I}\} * (3 + \frac{1}{3} * 1) - 0.5 * 2 * \frac{15}{E\,I} * (4 + \frac{2}{3} * 2) + B_y * 6 = 0$

$B_y = \frac{7.5}{E\,I}$ kN ↓



Member BC



- $\sum F_y = 0 \uparrow$

$\frac{7.5}{E\,I} + \{0.5 * 3 * \frac{30}{E\,I}\} - C_y = 0$

$C_y = \frac{52.5}{E\,I}$

$\theta_C = \frac{52.5}{E\,I}$

- $\sum M_C = 0 \circlearrowright$

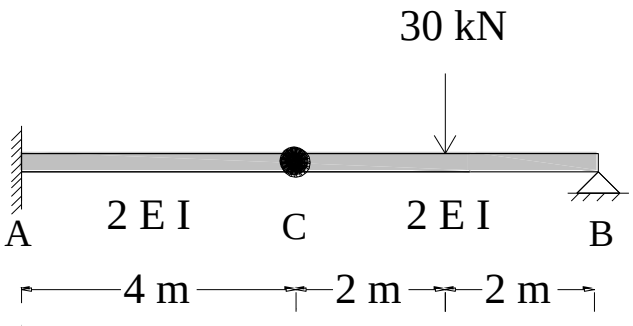
$\frac{7.5}{E\,I} * 3 + \{0.5 * 3 * \frac{30}{E\,I}\} * 2 - M_C = 0$

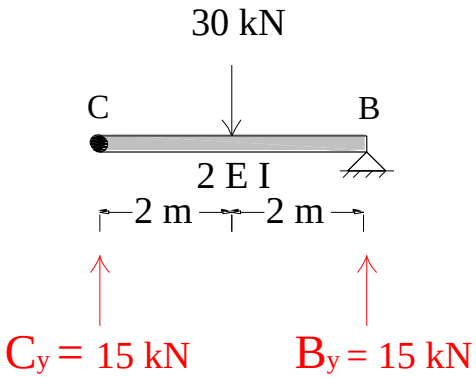
$M_C = \frac{112.5}{E\,I}$

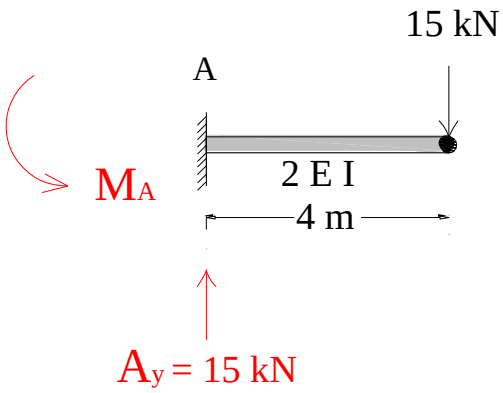
$\Delta_C = \frac{112.5}{E\,I}$

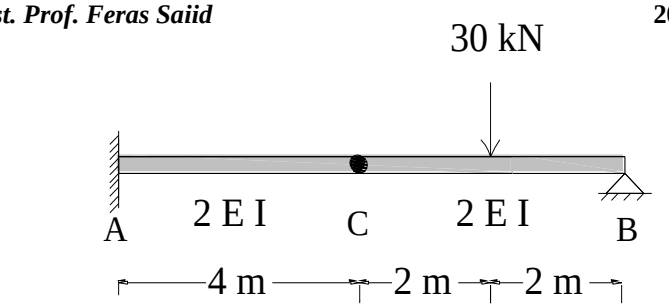
Example : Determine the Deflection and Rotation at C for Beam Below by using Conjugate Beam:

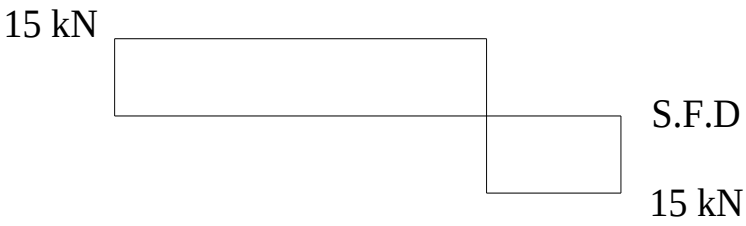
Solution

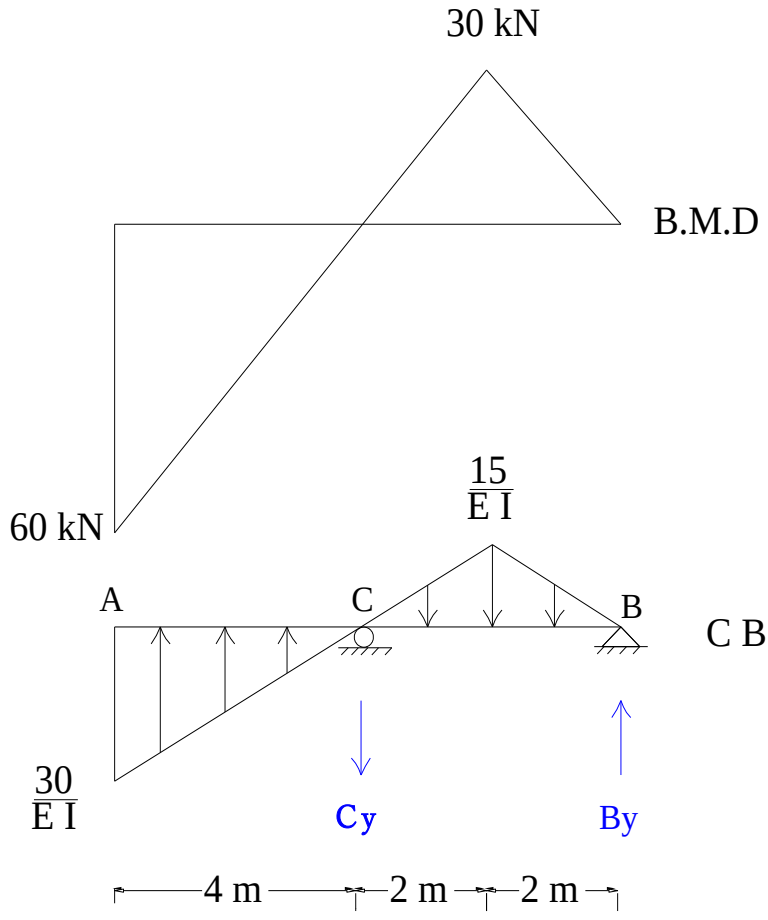












- $\sum M_C = 0$ ↻

  $\frac{1}{2} * 4 * \frac{30}{E\,I} * (\frac{2}{3} * 4) +$

  $(\frac{1}{2} * 4 * \frac{15}{E\,I} * (2)) - By\,(4) = 0$

  $By = \frac{55}{E\,I}$

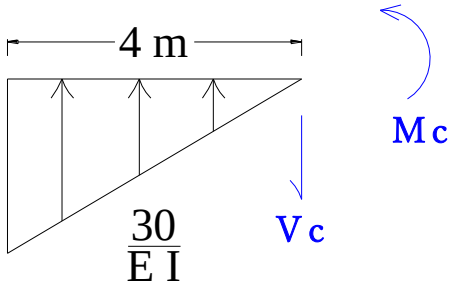
  $\theta_B = \frac{55}{E\,I}$

- $\sum Fy = 0$ ↑

  $- C_y + \frac{55}{E\,I} + (\frac{1}{2} * 4 * \frac{30}{E\,I}) - (\frac{1}{2} * 4 * \frac{15}{E\,I}) = 0$

  $C_y = \frac{85}{E\,I}$

Member AC (Before Support C)



- $\sum M_C = 0 \circlearrowleft$

  $\{ \frac{1}{2} * 4 * \frac{30}{E\,I} \} * (\frac{2}{3} * 4) - M_C = 0$

  $M_C = \frac{160}{E\,I}$

  $\Delta_C = \frac{160}{E\,I}$

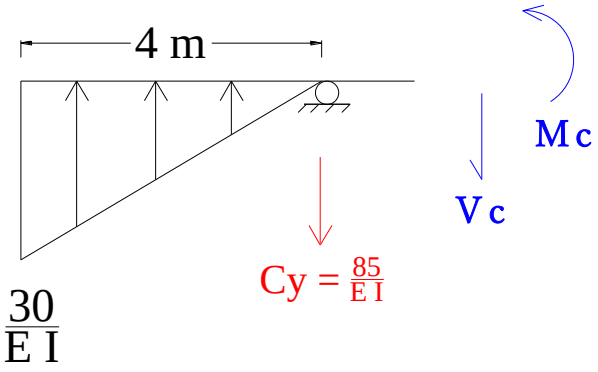
- $\sum Fy = 0 \uparrow$

  $\frac{1}{2} * 4 * \frac{30}{E\,I} - V_C = 0$

  $V_C = \frac{60}{E\,I}$

  $\theta_{C)}L = \frac{60}{E\,I}$

Member ACB (After Support C)



- $\sum Fy = 0 \uparrow$

  $\frac{1}{2} * 4 * \frac{30}{E\,I} - \frac{85}{E\,I} - V_C = 0$

  $V_C = -\frac{25}{E\,I}$

  $V_C = \frac{25}{E\,I} \uparrow$

  $\theta_{C)}R = \frac{25}{E\,I}$