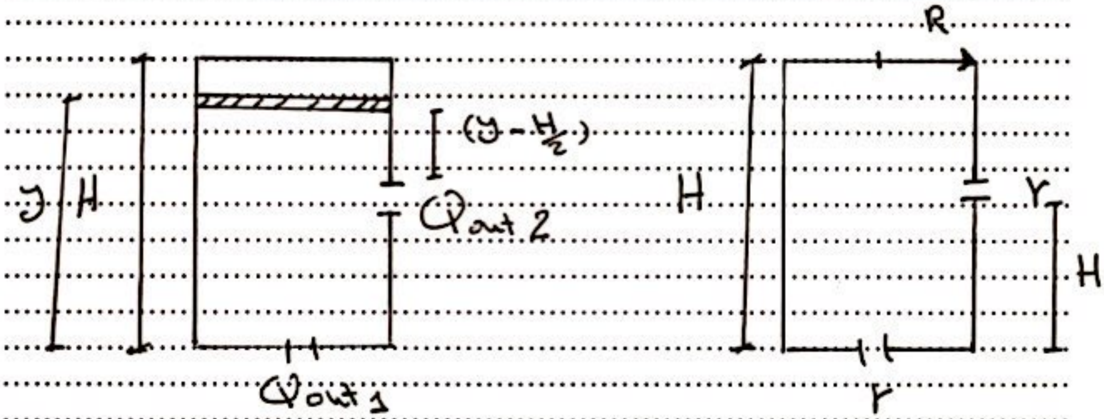
$$\frac{4y^{1.5}}{1.5} - \frac{y^{2.5}}{2.5} = -4.427 * 10^{-3} t + c$$

B.C.S.:- at $t = 1$ hr , $y = 0.5$ m

**Example 4:-** A cylinderical tank with radius R and hight H have two holes, one at the side at distance $\frac{H}{2}$ from bottom with radius r and the other is at the bottom with radius r too. Find the depth of water with respect to time?




Solution:-

Case 1: $(y > H/2)$

$$(Q_{out})_1 = Cd * \pi * r^2 * \sqrt{2gy}$$

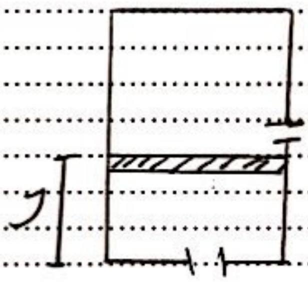
$$(Q_{out})_2 = Cd * \pi * r * \sqrt{2g\,(y - H/2)}$$

$$\frac{dV}{dt} = Q_{in} - Q_{out}$$

$$= 0 - \sqrt{2gy}\ \pi r^2 + \sqrt{2g(y - H/2)}\ \pi r^2$$

$$\pi R^2 \frac{dy}{dt} = -\pi r^2 \sqrt{2g}\ \left(y^{1/2} + (y - H/2)^{1/2}\right)$$

Case 2 :- $(y \le H/2)$



$$\because \frac{dV}{dt} = Q_{in} - Q_{out}$$

$$\pi R^2 \frac{dy}{dt} = -\sqrt{2gy}\ \pi r^2$$

$$\frac{dy}{\sqrt{y}} = -\left(\frac{r}{R}\right)^2 \sqrt{2g}\ dt$$

$$2y^{1/2} = -\left(\frac{r}{R}\right)^2 \sqrt{2g}\ t + c$$

B.C.S at $t = ?$ , $y = H/2$

$$\frac{4}{3h}\left[(H/2)^{3/2} - 0\right] = -\left(\frac{r}{R}\right)^2 \sqrt{2g}\ t + 0.862 H^{1/2}$$

$$-0.3906\, h^{1/2} = -\left(\frac{r}{R}\right)^2 \sqrt{2g}\ t$$

at $t = \dfrac{0.3906\sqrt{h}}{\left(\frac{r}{R}\right)^2 \sqrt{2g}}$ , $y = H/2$

$$2(H/2)^{1/2} = -\left(\frac{r}{R}\right)^2 \sqrt{2g} \cdot \frac{0.3906\sqrt{h}}{\left(\frac{r}{R}\right)^2 \sqrt{2g}} + c$$

$$\Longrightarrow c = 1.805\, h^{1/2}$$