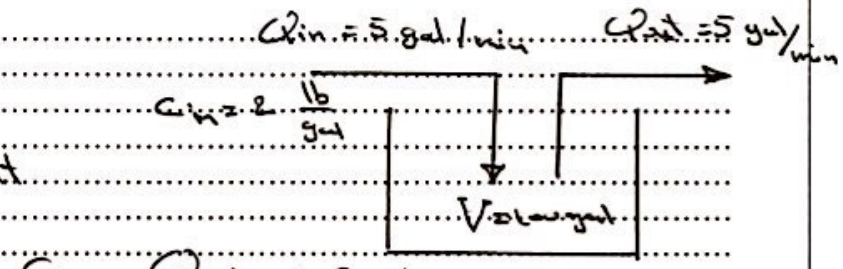
Engineering Analysis & Numerical Methods

Sol:



$Q_{in} = Q_{out}$

$\frac{dX}{dt} = Q_{in} * C_{in} - Q_{out} * C_{out}$

$C_{out} = \frac{X}{V}$

$\frac{dX}{dt} = (5 \times 2) - (5 \times \frac{X}{100})$

$\frac{dX}{dt} = 10 - 0.05X$

$\frac{dX}{10 - 0.05X} = dt \Rightarrow \frac{-1}{0.05} \ln(10 - 0.05X) = t + c$

$\ln(10 - 0.05X) = -0.05t + c_1$

$10 - 0.05X = Ce^{-0.05t} \Rightarrow 10 - 0.05X = Ae^{-0.05t}$

$X = \frac{10 - Ae^{-0.05t}}{0.05}$

$C_0 = \frac{X_0}{V_0} \Rightarrow 1 = \frac{X_0}{100} \Rightarrow X_0 = 100$ at $t = 0$

$100 = \frac{10 - Ae^{0}}{0.05} \Rightarrow 5 = 10 - A$

$A = 5 \Rightarrow X = \frac{10 - 5e^{-0.05t}}{0.05}$