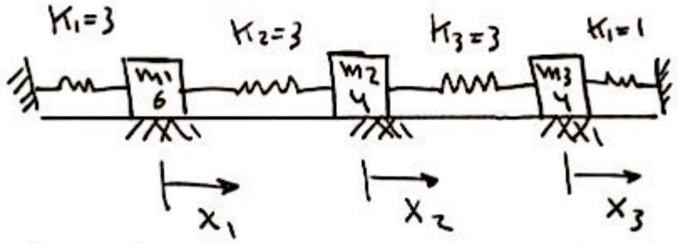
Example :- The three masses shown in fig. are initially displaced so that.

$(X_1)_0 = 2 \;,\; (X_2)_0 = 1 \;,\; (X_3)_0 = 1$

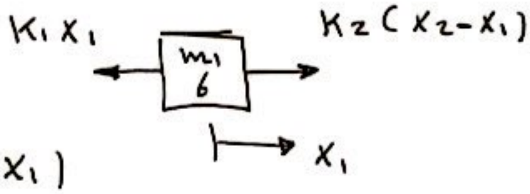


From these positions they begin to move with

$(\dot{X}_1)_0 = 1 \;,\; (\dot{X}_2)_0 = 2 \;,\; (\dot{X}_3)_0 = 0$

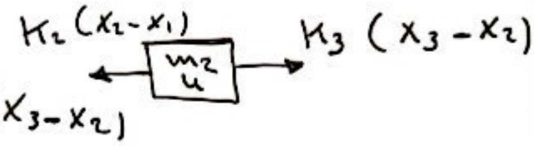
Assuming that there is no friction in the system, determine the subsequent of motion of each mass.

$ma = \Sigma F$



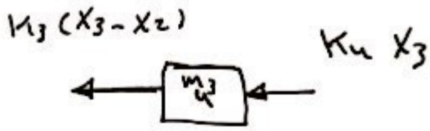
mass 1:-

$$6 \frac{d^2x_1}{dt^2} = -3X_1 + 3(X_2 - X_1)$$



mass 2 :-

$$4 \frac{d^2x_2}{dt^2} = -3(X_2 - X_1) + 3(X_3 - X_2)$$



mass 3:-

$$4 \frac{d^2X_3}{dt^2} = -3(X_3 - X_2) - X_3$$

$$(4D^2-1)(D^2-1)(4D^2-9)x_1=0$$
$$(4D^2-1)(D^2-1)(4D^2-9)x_2=0$$
$$(4D^2-1)(D^2-1)(4D^2-9)x_3=0$$

$$m_{1,2}=\mp\frac{1}{2}i \quad , m_{3,4}=\mp i \quad , m_{5,6}=\mp\frac{3}{2}i$$

$$x_1=c_1\cos\frac{t}{2}+c_2\sin\frac{t}{2}+c_3\cos t+c_4\sin t$$
$$+c_5\cos\frac{3}{2}t+c_6\sin\frac{3}{2}t.$$
$$x_2=c_7\cos\frac{t}{2}+c_8\sin\frac{t}{2}+c_9\cos t+c_{10}\sin t+$$
$$c_{11}\cos\frac{3}{2}t+c_{12}\sin\frac{3}{2}t.$$
$$x_3=c_{13}\cos\frac{t}{2}+c_{14}\sin\frac{t}{2}+c_{15}\cos t+c_{16}\sin t$$
$$c_{17}\cos\frac{3}{2}t+c_{18}\sin\frac{3}{2}t.$$

$$\begin{Bmatrix}x_1\\x_2\\x_3\end{Bmatrix}=c_1\begin{Bmatrix}2\\3\\3\end{Bmatrix}\cos\frac{t}{2}+c_2\begin{Bmatrix}2\\3\\3\end{Bmatrix}\sin\frac{t}{2}+c_3\begin{Bmatrix}1\\0\\-1\end{Bmatrix}\cos t$$
$$+c_4\begin{Bmatrix}1\\0\\-1\end{Bmatrix}\sin t+c_5\begin{Bmatrix}2\\-5\\3\end{Bmatrix}\cos\frac{3}{2}t+c_6\begin{Bmatrix}2\\-5\\3\end{Bmatrix}\sin\frac{3}{2}t$$

at $t=0 \quad (x_1)_0=2 \quad , (x_2)_0=-1 \quad , (x_3)_0=1$

$$\begin{Bmatrix}2\\-1\\1\end{Bmatrix}=\begin{bmatrix}2&1&2\\3&0&-5\\3&-1&3\end{bmatrix}\begin{Bmatrix}c_1\\c_3\\c_5\end{Bmatrix}$$

$$c_1=\frac{1}{4} \quad , c_3=\frac{4}{5} \quad , c_5=\frac{7}{20}$$

$$6\ddot{X}_1 + 6X_1 - 3X_2 = 0$$
$$4\ddot{X}_2 + 6X_2 - 3X_1 - 3X_3 = 0$$
$$4\ddot{X}_3 + 4X_3 - 3X_2 = 0$$

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{Bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \\ \ddot{X}_3 \end{Bmatrix} + \begin{bmatrix} 6 & -3 & 0 \\ -3 & 6 & -3 \\ 0 & -3 & 4 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$[M]\{\ddot{X}\} + [K]\{X\} = \{0\}$$

$$\begin{bmatrix} (6D^2+6) & -3 & 0 \\ -3 & (4D^2+6) & -3 \\ 0 & -3 & (4D^2+4) \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\Delta = \begin{vmatrix} 6D^2+6 & -3 & 0 \\ -3 & 4D^2+6 & -3 \\ 0 & -3 & 4D^2+4 \end{vmatrix}$$

$$\Delta X_1 = \begin{vmatrix} 0 & -3 & 0 \\ 0 & 4D^2+6 & -3 \\ 0 & -3 & 4D^2+4 \end{vmatrix} = 0$$

$$\Delta X_2 = 0 \quad , \quad \Delta X_3 = 0$$