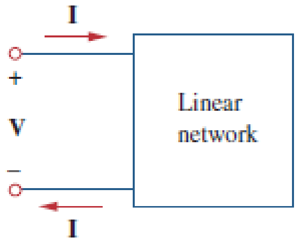
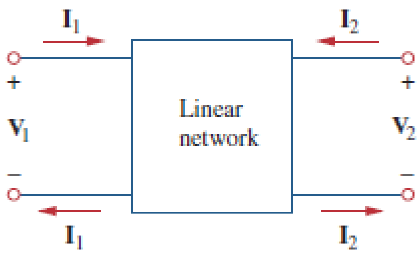
Two Port Networks

Introduction

- A pair of terminals through which a current may enter or leave a network is known as ***a port***.
- The two-port model is used to describe the performance of a circuit in terms of the voltage and current at its input and output ports.
- Two-terminal devices or elements (such as resistors, capacitors, and inductors) result in one-port networks. Most of the circuits we have dealt with so far are two-terminal or one-port circuits, represented in Fig. below.



- Four-terminal or two-port circuits involving **operation ampliers, transistors, and transformers**, as shown in Fig. below.

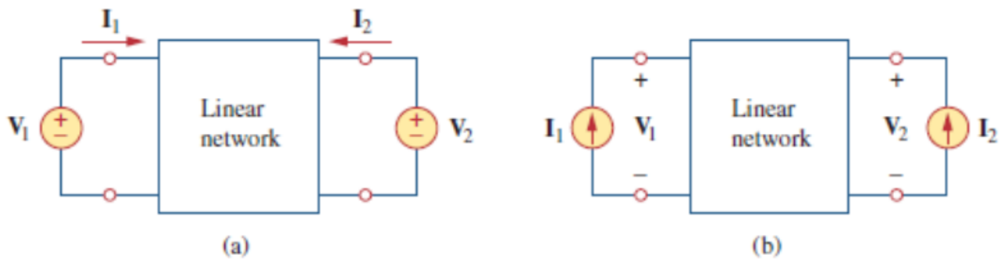


- To characterize a two-port network requires that we relate the terminal quantities V_1, V_2, I_1 and I_2. The various terms that relate these voltages and currents are called *parameters*.
- No independent sources are inside the circuit between the ports (dependent can be)
- No energy is stored inside the circuit between the ports.

1- Impedance Parameters

Impedance and admittance parameters are commonly used in the synthesis of filters. They are also useful in the design and analysis of impedance-matching networks and power distribution networks.

A two-port network may be voltage-driven as in Fig. (a) or current-driven as in Fig. (b). From either Fig. (a) or (b), the terminal voltages can be related to the terminal currents as:



$$\boxed{\begin{aligned} V_1 &= z_{11}I_1 + z_{12}I_2 \\ V_2 &= z_{21}I_1 + z_{22}I_2 \end{aligned}}$$

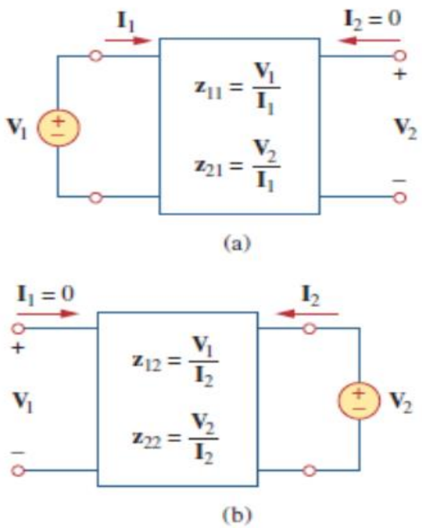
or in matrix form as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

where the terms are called the ***impedance parameters***, or simply ***z parameters***, and have units of ohms.

The values of the parameters can be evaluated by setting (input port open-circuited) or (output port open-circuited). Thus,

$$\boxed{\begin{aligned} z_{11} &= \left.\frac{V_1}{I_1}\right|_{I_2=0}, & z_{12} &= \left.\frac{V_1}{I_2}\right|_{I_1=0} \\ z_{21} &= \left.\frac{V_2}{I_1}\right|_{I_2=0}, & z_{22} &= \left.\frac{V_2}{I_2}\right|_{I_1=0} \end{aligned}}$$



Since the *z* parameters are obtained by open-circuiting the input or output port, they are also called the ***open-circuit impedance parameters***. Specifically,

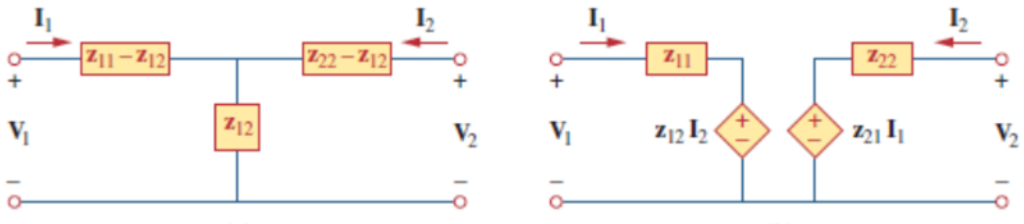
z_{11} = Open-circuit input impedance
z_{12} = Open-circuit transfer impedance from port 1 to port 2
z_{21} = Open-circuit transfer impedance from port 2 to port 1
z_{22} = Open-circuit output impedance

Sometimes z_{11} and z_{22} are called ***driving-point impedances***, while z_{21} and z_{12} are called ***transfer impedances***. A driving-point impedance is the input impedance of a two-terminal (one-port) device. Thus, is the input driving-point impedance with the output port open circuited, while is the output driving-point impedance with the input port open-circuited.

When $z_{11}=z_{22}$, the two-port network is said to be ***symmetrical.*** This implies that the network has mirror like symmetry about some center line; that is, a line can be found that divides the network into two similar halves.

When the two-port network is linear and has no dependent sources, the transfer impedances are equal ($z_{12}=z_{21}$), and the two-port is said to be ***reciprocal*.**

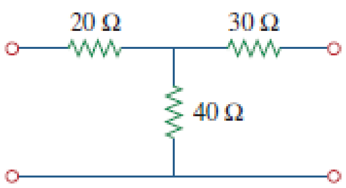
Any two-port that is made entirely of resistors, capacitors, and inductors must be reciprocal. A reciprocal network can be replaced by the T-equivalent circuit in Fig. (a). If the network is not reciprocal, a more general equivalent network is shown in Fig.(b).



(a) (b)

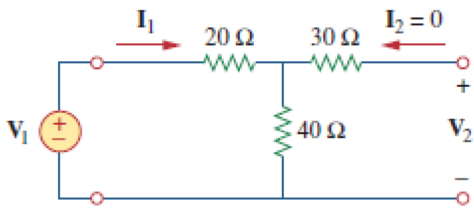
Example:

Determine the *z* parameters for the circuit in Fig. below.



Solution:

To determine $z_{11}$and z_{21}, we apply a voltage source to the input port and leave the output port open:

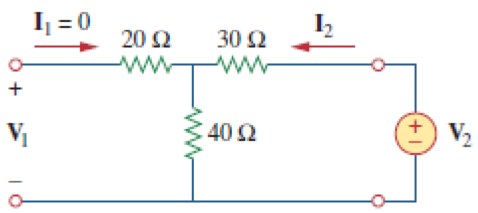


$$z_{11} = \frac{V_1}{I_1} = \frac{(20+40)I_1}{I_1} = 60\ \Omega$$

that is, z_{11} is the input impedance at port 1.

$$z_{21} = \frac{V_2}{I_1} = \frac{40I_1}{I_1} = 40\ \Omega$$

To find z_{12} and z_{22}, we apply a voltage source to the output port and leave the input port open:



$$z_{12} = \frac{V_1}{I_2} = \frac{40I_2}{I_2} = 40\ \Omega, \qquad z_{22} = \frac{V_2}{I_2} = \frac{(30+40)I_2}{I_2} = 70\ \Omega$$

Thus,

$$[z] = \begin{bmatrix} 60\ \Omega & 40\ \Omega \\ 40\ \Omega & 70\ \Omega \end{bmatrix}$$

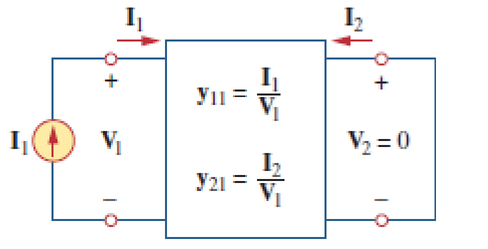
2- Admittance Parameters

They are parameters, which we obtain by expressing the terminal currents in terms of the terminal voltages. In either Fig. (a) or (b) below, the terminal currents can be expressed in terms of the terminal voltages as

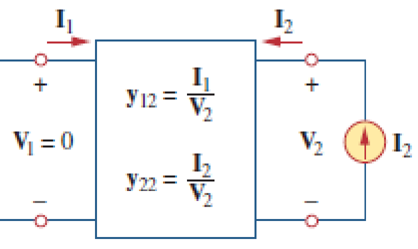
$$\boxed{\begin{aligned} I_1 &= y_{11}V_1 + y_{12}V_2 \\ I_2 &= y_{21}V_1 + y_{22}V_2 \end{aligned}}$$

or in matrix form as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$



(a)



(b)

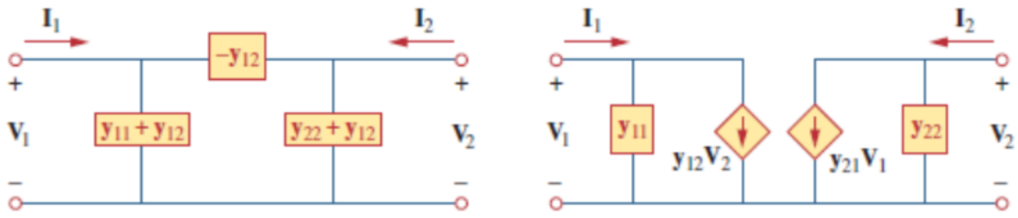
- The terms are known as the ***admittance parameters* (or, simply, *y parameters*)** and have units of siemens.
- The values of the parameters can be determined by setting (input port short-circuited) or (output port short-circuited). Thus,

$$\boxed{\begin{aligned} y_{11} &= \left.\frac{I_1}{V_1}\right|_{V_2=0}, & y_{12} &= \left.\frac{I_1}{V_2}\right|_{V_1=0} \\ y_{21} &= \left.\frac{I_2}{V_1}\right|_{V_2=0}, & y_{22} &= \left.\frac{I_2}{V_2}\right|_{V_1=0} \end{aligned}}$$

y_{11} = Short-circuit input admittance
y_{12} = Short-circuit transfer admittance from port 2 to port 1
y_{21} = Short-circuit transfer admittance from port 1 to port 2
y_{22} = Short-circuit output admittance

For a two-port network that is linear and has no dependent sources, the transfer admittances are equal ($y_{12}=y_{21}$).

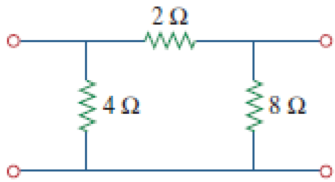
A reciprocal network ($y_{12}=y_{21}$) can be modeled by the Π equivalent circuit in Fig. (a) below. If the network is not reciprocal, a more general equivalent network is shown in Fig.(b) below.



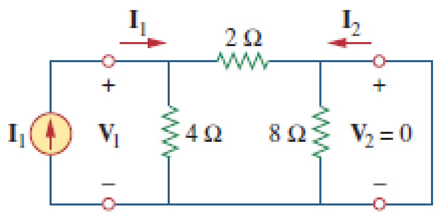
(a) (b)

Example:

Obtain the *y* parameters for the network shown in Fig. below.



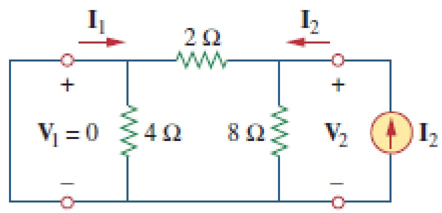
Solution:



$$V_1 = I_1(4 \parallel 2) = \frac{4}{3}I_1, \qquad y_{11} = \frac{I_1}{V_1} = \frac{I_1}{\frac{4}{3}I_1} = 0.75 \text{ S}$$

By current division,

$$-I_2 = \frac{4}{4+2}I_1 = \frac{2}{3}I_1, \qquad y_{21} = \frac{I_2}{V_1} = \frac{-\frac{2}{3}I_1}{\frac{4}{3}I_1} = -0.5 \text{ S}$$



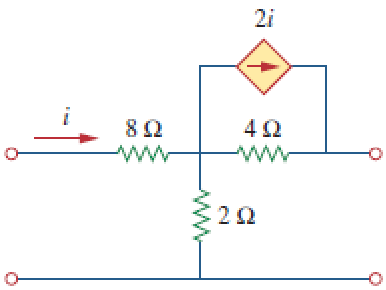
$$V_2 = I_2(8 \parallel 2) = \frac{8}{5}I_2, \qquad y_{22} = \frac{I_2}{V_2} = \frac{I_2}{\frac{8}{5}I_2} = \frac{5}{8} = 0.625 \text{ S}$$

By current division,

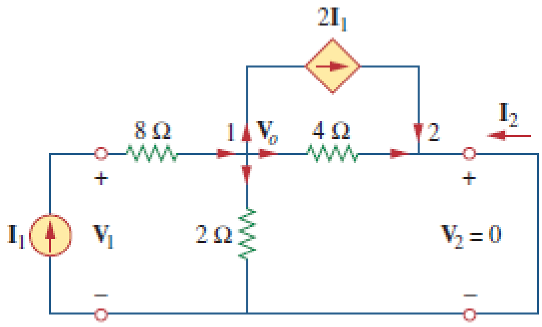
$$-I_1 = \frac{8}{8+2}I_2 = \frac{4}{5}I_2, \qquad y_{12} = \frac{I_1}{V_2} = \frac{-\frac{4}{5}I_2}{\frac{8}{5}I_2} = -0.5 \text{ S}$$

Example:

Determine the y parameters for the two-port shown in Fig. below.



Solution:



$$\frac{V_1 - V_o}{8} = 2I_1 + \frac{V_o}{2} + \frac{V_o - 0}{4}$$

But $I_1 = \frac{V_1 - V_o}{8}$; therefore,

$$0 = \frac{V_1 - V_o}{8} + \frac{3V_o}{4}$$

$$0 = V_1 - V_o + 6V_o \quad \Rightarrow \quad V_1 = -5V_o$$

Hence,

$$I_1 = \frac{-5V_o - V_o}{8} = -0.75V_o$$

and

$$y_{11} = \frac{I_1}{V_1} = \frac{-0.75V_o}{-5V_o} = 0.15 \text{ S}$$

At node 2,

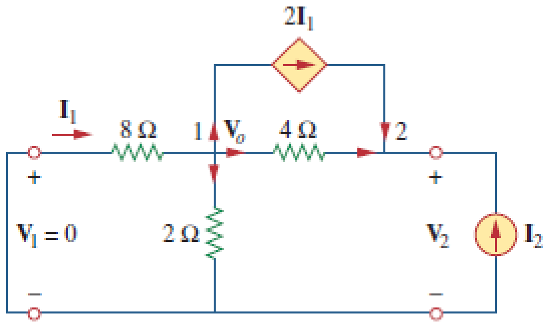
$$\frac{V_o - 0}{4} + 2I_1 + I_2 = 0$$

or

$$-I_2 = 0.25V_o - 1.5V_o = -1.25V_o$$

Hence,

$$y_{21} = \frac{I_2}{V_1} = \frac{1.25V_o}{-5V_o} = -0.25 \text{ S}$$



$$\frac{0 - V_o}{8} = 2I_1 + \frac{V_o}{2} + \frac{V_o - V_2}{4}$$

But $I_1 = \frac{0 - V_o}{8}$; therefore,

$$0 = -\frac{V_o}{8} + \frac{V_o}{2} + \frac{V_o - V_2}{4}$$

or

$$0 = -V_o + 4V_o + 2V_o - 2V_2 \quad \Rightarrow \quad V_2 =$$

Hence,

$$y_{12} = \frac{I_1}{V_2} = \frac{-V_o/8}{2.5V_o} = -0.05 \text{ S}$$

At node 2,

$$\frac{V_o - V_2}{4} + 2I_1 + I_2 = 0$$

or

$$-I_2 = 0.25V_o - \frac{1}{4}(2.5V_o) - \frac{2V_o}{8} = -0.62$$

Thus,

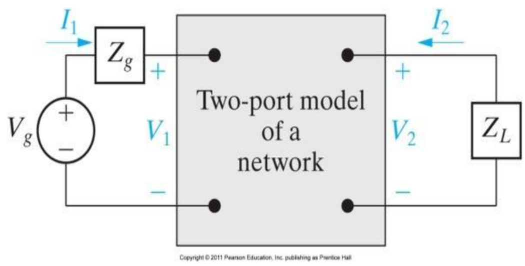
$$y_{22} = \frac{I_2}{V_2} = \frac{0.625V_o}{2.5V_o} = 0.25 \text{ S}$$

Notice that $y_{12} \neq y_{21}$ in this case, since the network is

3- Analysis of the Terminated Two-Port Circuit

Typical two-port circuits are driven in port 1 and loaded on port 2.
Where V_g & Z_g are the voltage and impedance of the source. Z_L is the load impedance.

Six characteristics of a terminated two-port circuit



- Input impedance $Z_{in} = V_1/I_1$

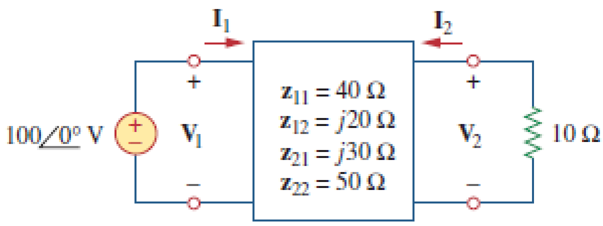
  or the input admittance $Y_{in} = I_1/V_1$
- The output current I_2
- The current gain I_2/I_1
- The voltage gain V_2/V_1
- The overall voltage gain V_2/V_g
- The Thevenin voltage and impedance (V_{Th} & Z_{Th}) with respect to port 2.
- Maximum power transfer , its depends on Thevenin voltage and impedance(V_{Th} & Z_{Th})

4- Terminated Two port Equations

z Parameters	y Parameters
$Z_{in} = z_{11} - \dfrac{z_{12}z_{21}}{z_{22} + Z_L}$	$Y_{in} = y_{11} - \dfrac{y_{12}y_{21}Z_L}{1 + y_{22}Z_L}$
$I_2 = \dfrac{-z_{21}V_g}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}$	$I_2 = \dfrac{y_{21}V_g}{1 + y_{22}Z_L + y_{11}Z_g + \Delta y Z_g Z_L}$
$V_{Th} = \dfrac{z_{21}}{z_{11} + Z_g}V_g$	$V_{Th} = \dfrac{-y_{21}V_g}{y_{22} + \Delta y Z_g}$
$Z_{Th} = z_{22} - \dfrac{z_{12}z_{21}}{z_{11} + Z_g}$	$Z_{Th} = \dfrac{1 + y_{11}Z_g}{y_{22} + \Delta y Z_g}$
$\dfrac{I_2}{I_1} = \dfrac{-z_{21}}{z_{22} + Z_L}$	$\dfrac{I_2}{I_1} = \dfrac{y_{21}}{y_{11} + \Delta y Z_L}$
$\dfrac{V_2}{V_1} = \dfrac{z_{21}Z_L}{z_{11}Z_L + \Delta z}$	$\dfrac{V_2}{V_1} = \dfrac{-y_{21}Z_L}{1 + y_{22}Z_L}$
$\dfrac{V_2}{V_g} = \dfrac{z_{21}Z_L}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}$	$\dfrac{V_2}{V_g} = \dfrac{y_{21}Z_L}{y_{12}y_{21}Z_gZ_L - (1 + y_{11}Z_g)(1 + y_{22}Z_L)}$

Example:

Find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit in Fig. below.



Solution:

Since we are looking for $\mathbf{I}_1$ and $\mathbf{I}_2$, we substitute

$$\mathbf{V}_1 = 100\underline{/0^\circ}, \qquad \mathbf{V}_2 = -10\mathbf{I}_2$$

$$\mathbf{V}_1 = 40\mathbf{I}_1 + j20\mathbf{I}_2$$

$$\mathbf{V}_2 = j30\mathbf{I}_1 + 50\mathbf{I}_2$$

$$100 = 40\mathbf{I}_1 + j20\mathbf{I}_2$$

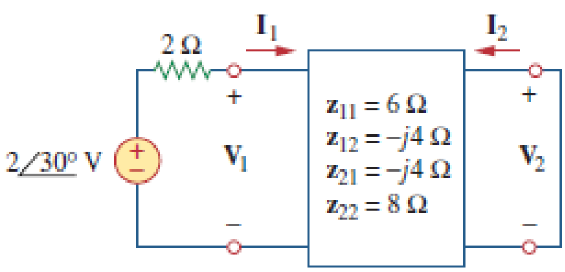
$$100 = j80\mathbf{I}_2 + j20\mathbf{I}_2 \quad \Rightarrow \quad \mathbf{I}_2 = \frac{100}{j100} = -j$$

, $\mathbf{I}_1 = j2(-j) = 2$. Thus,

$$\mathbf{I}_1 = 2\underline{/0^\circ}\text{ A}, \qquad \mathbf{I}_2 = 1\underline{/-90^\circ}\text{ A}$$

Home Work

Calculate $\mathbf{I}_1$ and $\mathbf{I}_2$ in the two-port of Fig. below



5- Hybrid Parameters

The z and y parameters of a two-port network do not always exist. So, there is a need for developing another set of parameters. This third set of parameters is based on making V_1 and I_2 the dependent variables. Thus, we obtain:

$$\boxed{\begin{aligned} V_1 &= h_{11}I_1 + h_{12}V_2 \\ I_2 &= h_{21}I_1 + h_{22}V_2 \end{aligned}}$$

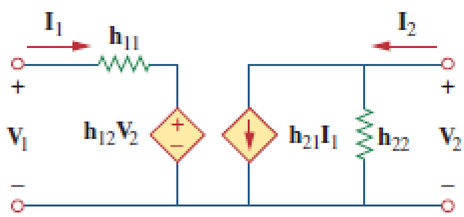
or in matrix form,

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = [h] \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

The h terms are known as the hybrid parameters (or, simply, h parameters) because they are a hybrid combination of ratios. They are very useful for describing electronic devices such as transistors it is much easier to measure experimentally the h parameters of such devices than to measure their z or y parameters.

$$\boxed{\begin{aligned} h_{11} &= \left.\frac{V_1}{I_1}\right|_{V_2=0}, \qquad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \\ h_{21} &= \left.\frac{I_2}{I_1}\right|_{V_2=0}, \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} \end{aligned}}$$

h_{11} = Short-circuit input impedance
h_{12} = Open-circuit reverse voltage gain
h_{21} = Short-circuit forward current gain
h_{22} = Open-circuit output admittance



The h-parameter equivalent network of a two-port network.

A set of parameters closely related to the h parameters are the ***g parameters* or *inverse hybrid parameters***. These are used to describe the terminal currents and voltages as:

$$\boxed{\begin{aligned} I_1 &= g_{11}V_1 + g_{12}I_2 \\ V_2 &= g_{21}V_1 + g_{22}I_2 \end{aligned}}$$

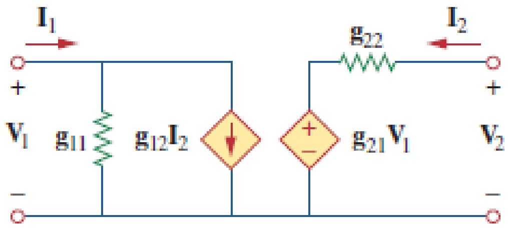
or

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = [g] \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

The values of the g parameters are determined as:

g_{11} = Open-circuit input admittance
g_{12} = Short-circuit reverse current gain
g_{21} = Open-circuit forward voltage gain
g_{22} = Short-circuit output impedance

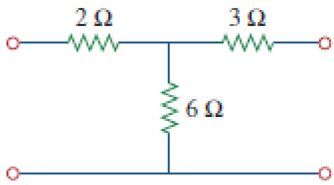
$$g_{11} = \frac{I_1}{V_1}\bigg|_{I_2=0}, \qquad g_{12} = \frac{I_1}{I_2}\bigg|_{V_1=0}$$
$$g_{21} = \frac{V_2}{V_1}\bigg|_{I_2=0}, \qquad g_{22} = \frac{V_2}{I_2}\bigg|_{V_1=0}$$



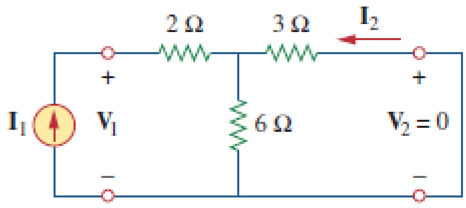
The g-parameter model of a two-port network.

Example:

Find the hybrid parameters for the two-port network of Fig. below.



Solution:

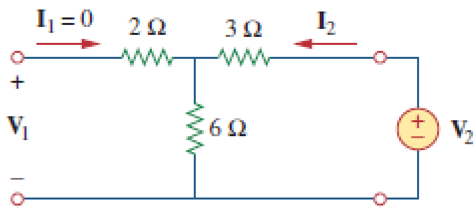


$$V_1 = I_1(2 + 3 \parallel 6) = 4I_1$$

Hence,

$$h_{11} = \frac{V_1}{I_1} = 4\ \Omega$$

$$-I_2 = \frac{6}{6+3}I_1 = \frac{2}{3}I_1$$



Hence,

$$h_{21} = \frac{I_2}{I_1} = -\frac{2}{3}$$

$$V_1 = \frac{6}{6+3}V_2 = \frac{2}{3}V_2$$

$$h_{12} = \frac{V_1}{V_2} = \frac{2}{3}$$

Also,

$$V_2 = (3 + 6)I_2 = 9I_2$$

$$h_{22} = \frac{I_2}{V_2} = \frac{1}{9}\ \text{S}$$