2. Periodic signal is almost power signal.
3. Nonperiodic signal is almost energy signal.
4. Some of signals are neither power nor energy, (increasing or divergent signals like *tan t, 1/t,* e^{at}…etc.).
5. The power of energy signal is zero while the energy of power signal is infinity.

Ex 2-1:

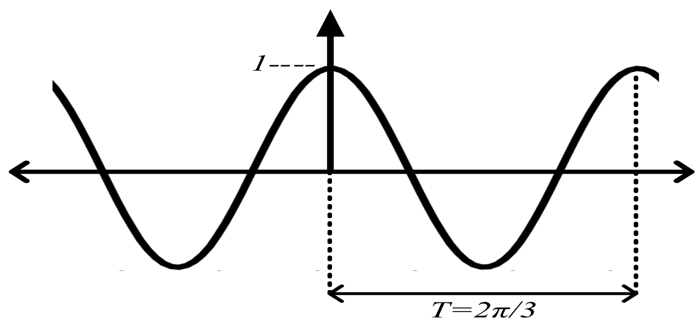
Classify the following signals:

1-$f(t) = \cos 3\,t$

2-$f(t) = e^{-2|t|}$

3-$f(t) = e^{+2t}$

Solution:



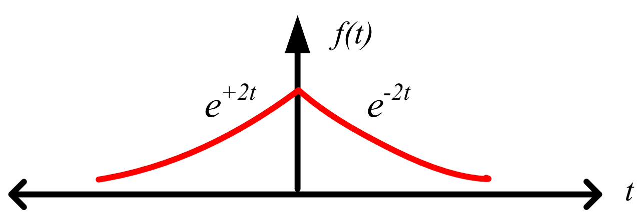
1-$f(t) = \cos 3\,t$

- Deterministic
- Periodic
- Power signal

$$P_{av} = \frac{1}{T}\int_0^{T}|f(t)|^2 dt$$

$$= \frac{3}{2\pi}\int_0^{\frac{2\pi}{3}}|\cos 3t|^2 dt \;= \frac{3}{2\pi}\int_0^{\frac{2\pi}{3}}\left(\frac{1}{2}+\frac{1}{2}\cos 6\,t\right) dt$$

=1/2 watt ∴ power signal



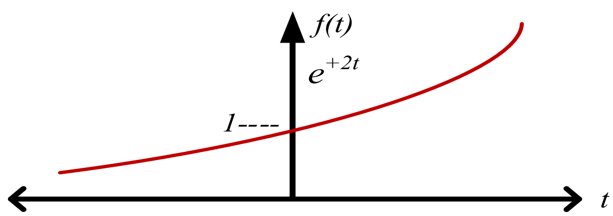
2-$f(t) = e^{-2|t|}$

- Deterministic
- Non periodic
- Energy signal

$$E = R\int_{-\infty}^{\infty}|f(t)|^2 dt = 2\int_0^{\infty} e^{-4t}\,dt = 2\left.\frac{e^{-4t}}{-4}\right|_0^{\infty} = 1/2 \text{ joule}$$

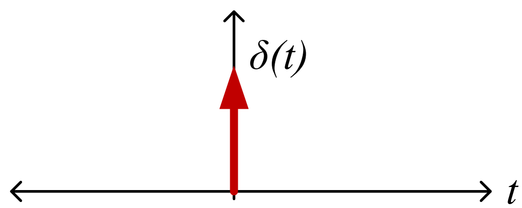
3-$f(t) = e^{+2t}$

- Deterministic
- Non periodic
- Neither power nor energy (since its divergent increasing signal)



Some important signals

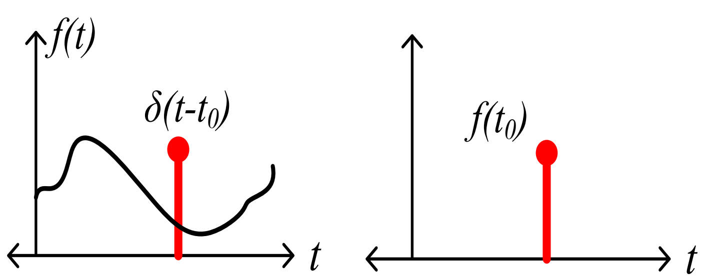
1- Impulse (Dirac)



$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & elsewhere \end{cases}$$

Properties:

- $\int_{-\infty}^{\infty} \delta(t)dt = 1$
- $\left.\begin{matrix} \int_{-\infty}^{\infty} f(t)\delta(t)dt = f(0) \\ \int_{-\infty}^{\infty} f(t)\delta(t-t_0)dt = f(t_0) \end{matrix}\right\}$ **sampling property**



Ex 2-2:

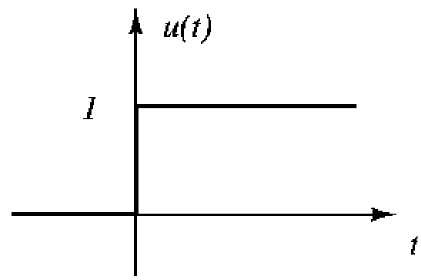
Solve $\int_{-\infty}^{\infty} t^2\, e^{-sint} cos2t\, \delta(t-\pi)dt$

Solution: $= \pi^2 e^{-sin\pi} \cos 2\pi = \pi^2 . e^0 . 1 = \pi^2$

H.W:

- Show that (a) $\delta(at) = \frac{1}{|a|}\delta(t)$, (b) $\delta(t) = \delta(-t)$,
- Solve $\int_{-\infty}^{\infty} \delta(t - \pi t)\cos\left(\frac{1}{T}\right)\, dt$… Ans: $\left(\frac{1}{\pi}\right)$

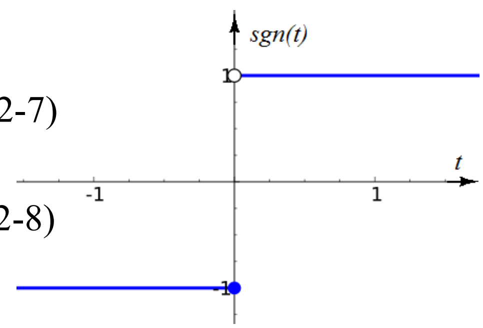
2- Unit step:



$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases} \quad \text{… (2-5)}$$

$$\delta(t) = \frac{d}{dt}u(t)\,; u(t) = \int_{-\infty}^{t} \delta(t)'\, dt' \quad \text{… (2-6)}$$

3- Signum:

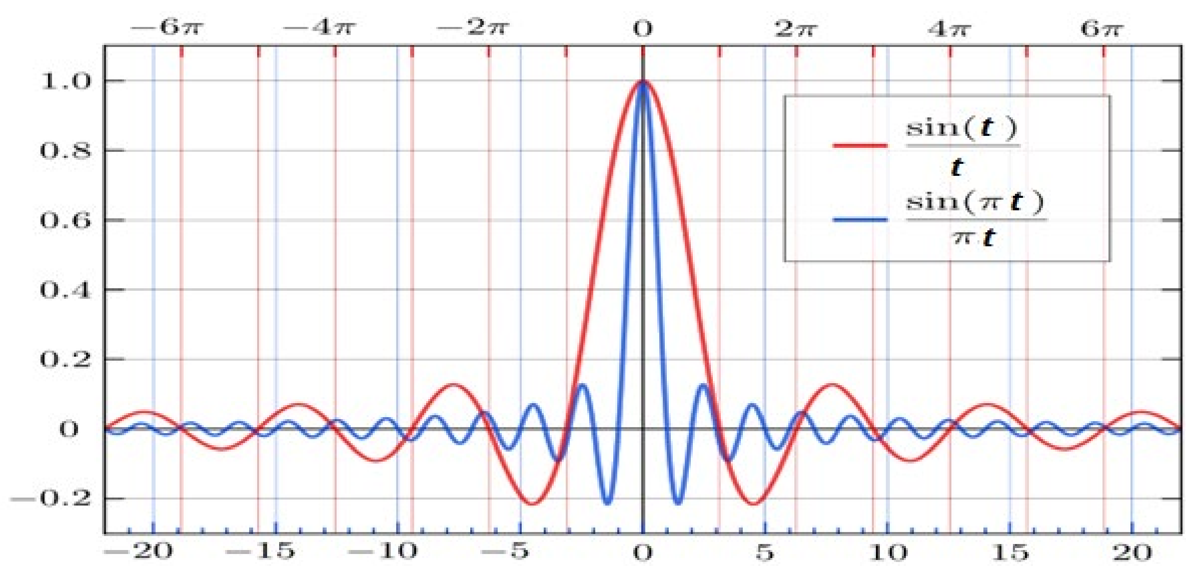


$$sgn(t) = \begin{cases} -1, & t < 0 \\ 0, & t = 0 \\ +1, & t > 0 \end{cases} \quad \text{… (2-7)}$$

$$sgn\,(t) = 2u(t) - 1 \quad \text{…(2-8)}$$

4- Sinc:

$$sinc(t) = \frac{sin\pi t}{\pi t} \quad \text{…(2-9)}$$



5- <u>Sa</u>:

$$Sa(t) = \frac{sint}{t} \qquad \dots(2\text{-}10)$$

H.W:

Show that, $sinc(0) = Sa(0) = 1$

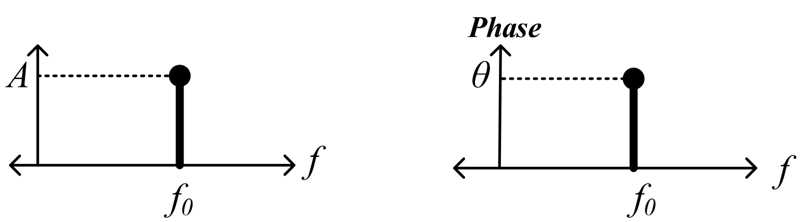
Signal spectrum:

The spectrum is the frequency representation of a signal:

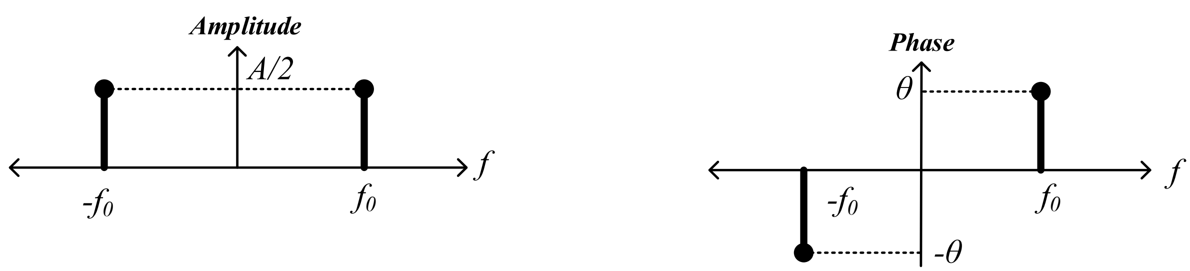
1- Spectrum of the sinusoidal signals:

$$f(t) = A\cos(2\pi f_0 t + \theta)$$

a) Single sideband spectrum:



b) Double sideband spectrum:



Rules:

1. The amplitude is always positive.

   $-A\cos 2\pi f_0 t \Rightarrow A\cos(2\pi f_0 t - 180^0)$

2. The phase ids always measured from the real axis.
   $A\sin 2\pi f_0 t \Rightarrow A\cos(2\pi f_0 t - 90^0)$
3. If the angular frequency is used (ω), the amplitude is multiplied by 2π.

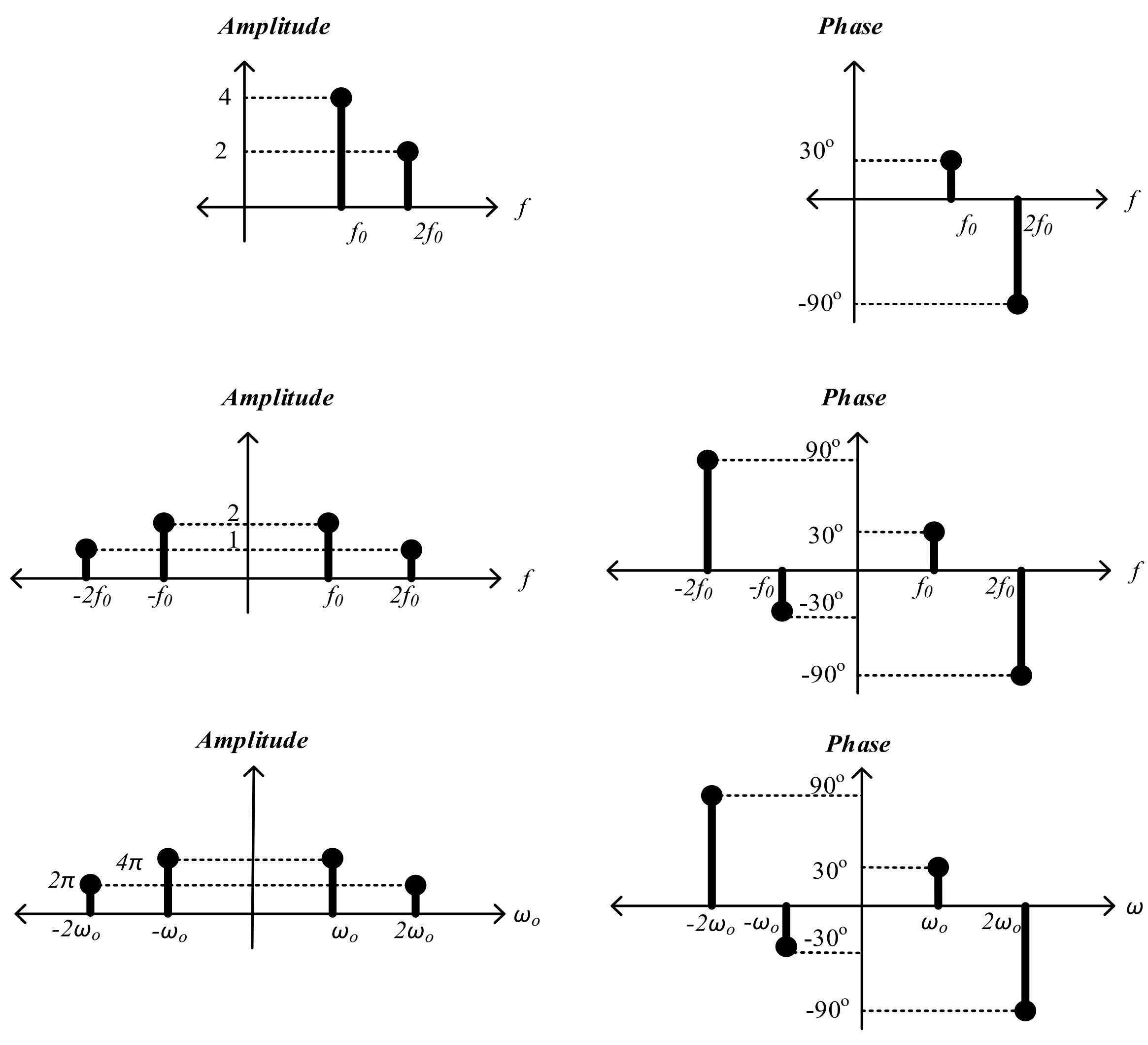
Ex 2-3:

Plot the amplitude and phase spectrum (single and double sided) for the following signal:

$$f(t) = 4\cos(2\pi f_o t + 30^o) + 2\sin(4\pi f_o t)$$

Solution:

$$f(t) = 4\cos(2\pi f_o t + 30^o) + 2\cos(4\pi f_o t - 90)$$



If the x-axis is ω *(not f)*, the amplitude is multiplied by 2π.

H.W:

Plot the amplitude and phase spectrum (single and double sided) for the following signal:

1. $f(t) = 5 + 3\sin(2\pi * 10^3 t + 30^o) - 4\cos(4\pi * 10^3 t - 20^o)$
2. $f(t) = -2 + 10\sin(2\pi * 10^5 t - 20^o) - 14\cos(6\pi * 10^5 t + 30^o) + 7\sin(4\pi * 10^5 t - 50^o)$
3. $f(t) = 1 + 4\cos(10000t + 30^o)$

2- Spectrum of periodic signals

The spectrum of any periodic signal double sided) can be obtained by plotting $|C_n|$versus nf_o and θ_n versus nf_o, where:

$$|C_n| = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_o t}dt = \frac{a_n}{2} - j\frac{b_n}{2} \quad \dots (2\text{-}11)$$

$$|\theta_n| = tan^{-1}\frac{b_n}{a_n} \quad \dots (2\text{-}12)$$

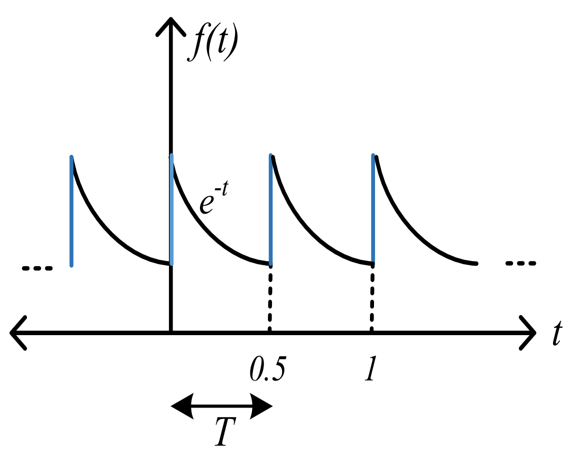
Where T is the period of the periodic signal

$$|C_n| = \sqrt{\left(\frac{a_n}{2}\right)^2 + \left(\frac{b_n}{2}\right)^2} \quad \dots (2\text{-}13)$$

Ex 2-4:

Plot the double-sided amplitude and phase spectrum of the signal shown below:

Solution:



$$C_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_o t}dt$$

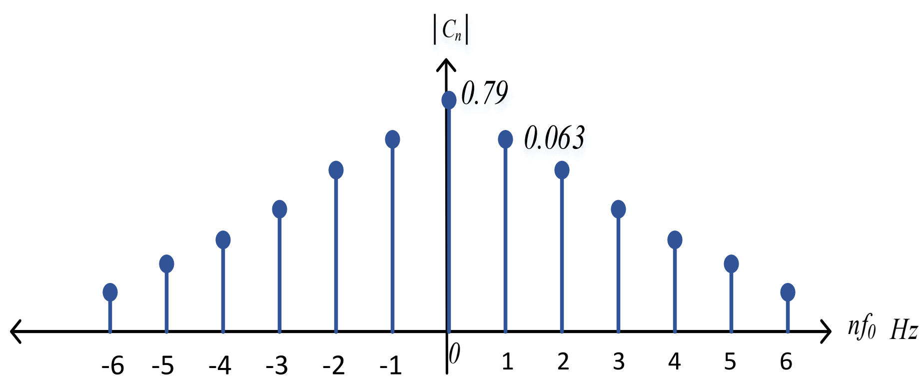
$$= \frac{1}{0.5}\int_0^{0.5} e^{-t}e^{-jn\omega_o t}dt$$

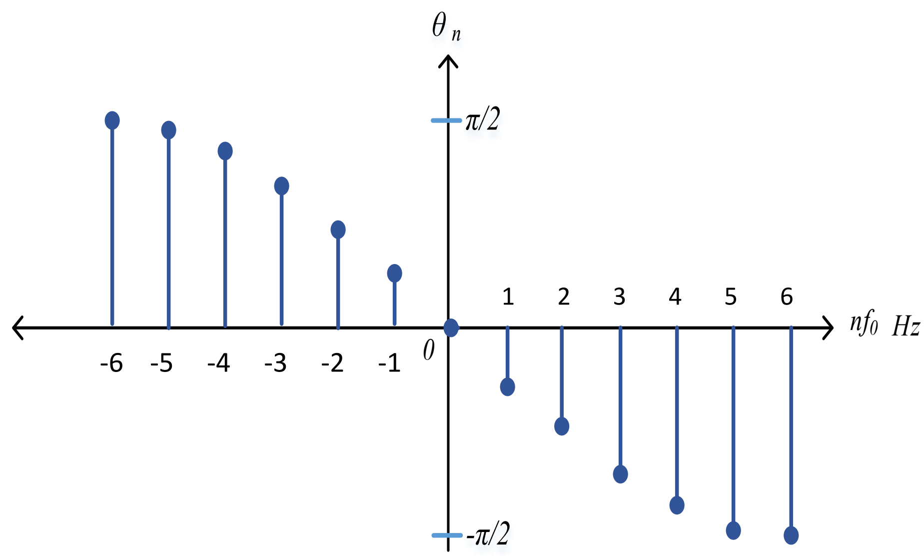
$= 2\int_0^{0.5} e^{-(1+j4\pi n)t}dt$ T=0.5, w_o= 4π, f_o= 2 Hz

$= \dfrac{0.79}{1+j4\pi n}$

$|C_n| = \dfrac{0.79}{\sqrt{1+16\pi^2 n^2}}$ $\theta_n = -tan^{-1}(4\pi n)$

n	$\lvert C_n \rvert$	n	θ_n
0	0.79	0	0
±1	0.063	±1	.
±2	.	±2	.
±3	.	±3	.
.	.	.	.





We conclude that the spectrum of periodic signals is discrete.