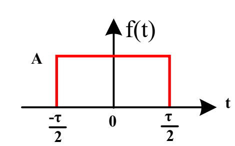
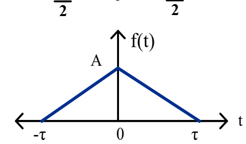
Selected F.T Pairs:

$f(t)$	$\Leftrightarrow$ $F(\omega)$	
$e^{-at}u(t)$	$\frac{1}{a+j\omega}$	
$te^{-at}u(t)$	$\frac{1}{(a+j\omega)^2}$	
$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2+\omega^2}$	
$sgn(t)$	$\frac{2}{j\omega}$	
$\frac{j}{\pi t}$	$sgn(\omega)$	
$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	
$\delta(t)$	1	
1	$2\pi\delta(\omega)$	
$e^{\pm j\omega_o t}$	$2\pi\delta(\omega\mp\omega_o)$	
$cos\omega_o t$	$\pi[\delta(\omega-\omega_o)+\delta(\omega+\omega_o)]$	
$sin\omega_o t$	$-j\pi[\delta(\omega-\omega_o)-\delta(\omega+\omega_o)]$	
$A\prod\left({}^{t}/_{\tau}\right)$	$A\tau Sa\left(\frac{\omega\tau}{2}\right)$	Rectangular pulse
$A\Lambda\left({}^{t}/_{\tau}\right)$	$A\tau\left[Sa\left(\frac{\omega\tau}{2}\right)\right]^2$	Triangular pulse





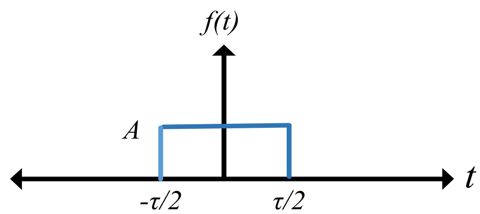
Rayleigh's Energy Theorem.

It corresponds to Parseval's power theorem but concern with energy of non-periodic signals.

$$E = \int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(\omega)|^2 \, d\omega \qquad (2\text{-}18)$$

Ex 2-9:

Find the ratio of Energy up to the first zero crossing point to the total energy for the rectangular pulse of amplitude A and width τ.



Solution:

$E_t = \int_{-\infty}^{\infty} |f(t)|^2 dt = \int_{-\tau/2}^{\tau/2} A^2 dt = A^2\tau$ joule

The first zero crossing point for $A\Pi(\tau/T)$ occurs at $\omega = \frac{2\pi}{\tau}$ rad/sec

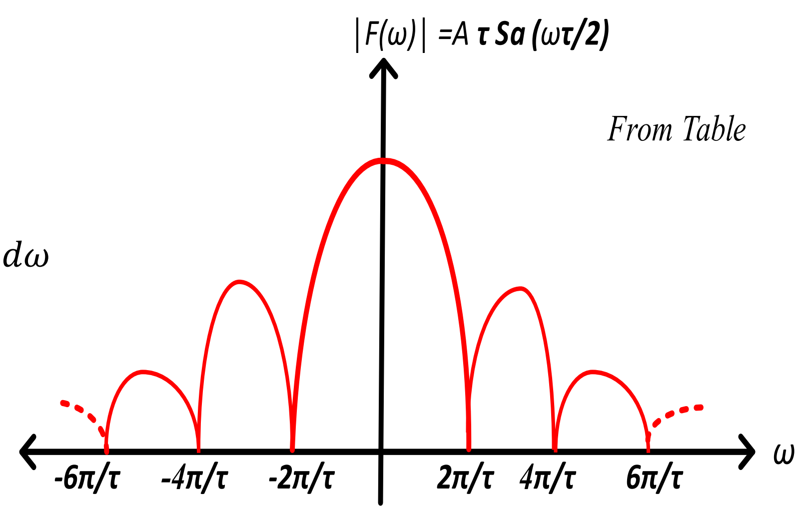
$$E_{\frac{2\pi}{\tau}} = \frac{1}{2\pi} \int_{-\frac{2\pi}{\tau}}^{\frac{2\pi}{\tau}} |F(\omega)|^2 \, d\omega$$

$$= \frac{1}{2\pi} \int_{-\frac{2\pi}{\tau}}^{\frac{2\pi}{\tau}} (A\tau)^2 \left|Sa\left(\frac{\omega\tau}{2}\right)\right|^2 d\omega$$

$= 0.92\ A^2\tau$

$$\frac{E_{\frac{2\pi}{\tau}}}{E_t} = \frac{0.92A^2\tau}{A^2\tau}$$

$= 92\ \%$



H.W:

Calculate the ratio of energy of the frequency range (10→20) Hz up to total energy for the signal $f(t) = \begin{cases} |t|, & -0.5 < 0 < 0.5 \\ 0, & else\ where \end{cases}$

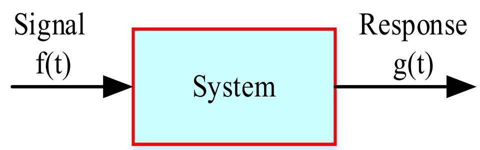
System:

A group of objects that can interacts harmoniously and combined in a manner intended to achieve a desired objective.

Liner system:

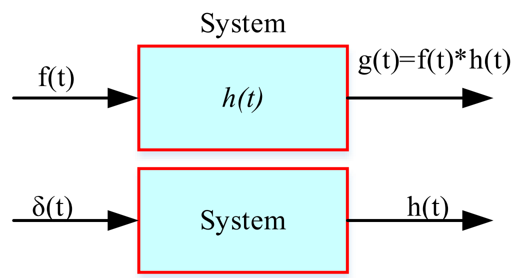
The system that satisfies the following:

If $g_1(t)$ is the output when $f_1(t)$ is the input and $g_2(t)$ is the output when $f_2(t)$ is the input, then, when the input is $f_1(t)+f_2(t)$, the output would be $a_1\, g_1(t)+a_2\, g_2(t)$.



System Impulse Response:

When an input signal is applied to a system, then the output would be;



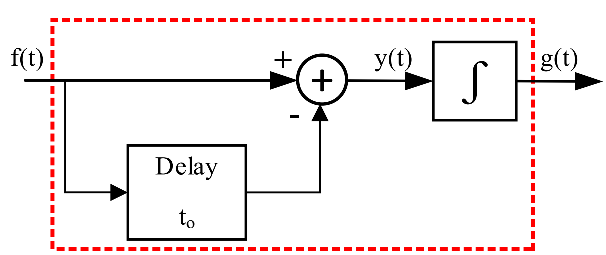
$$g(t) = f(t) * h(t) = \int_{-\infty}^{\infty} f(\tau)h(t-\tau)\, d\tau \qquad (2\text{-}19)$$

where $h(t)$ is called the impulse response of the system, h(t) is found by applying δ(t) at the system input, where the output would be h(t),

$g(t) = h(t)$ when the input is δ(t).

Ex 2-10:

Determine the impulse response of the system shown below.



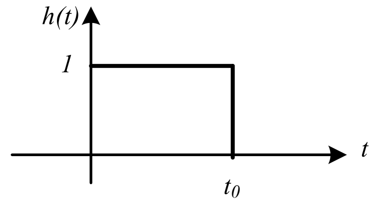
Solution:

Let $f(t) = \delta(t)$

$$y(t) = \delta(t) - \delta(t - t_o)$$

$$g(t) = \int_{-\infty}^{t} y(t')dt' = \int_{-\infty}^{t} [\delta(t') - \delta(t' - t_o)]dt'$$

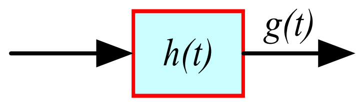
$$h(t) = u(t) - u(t - t_o)$$

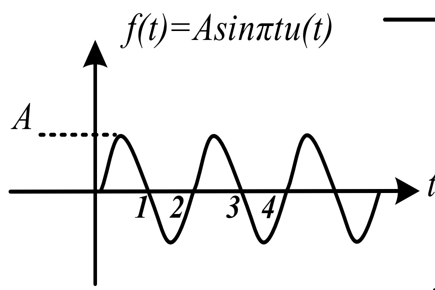


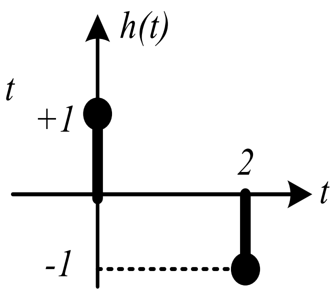
Ex 2-11:

Find the output of the system shown below:

Solution:







$$f(t) = A\, sin\pi t\; u(t)$$

$$h(t) = \delta(t) - \delta(t - 2)$$

$$g(t) = f(t) * h(t)$$

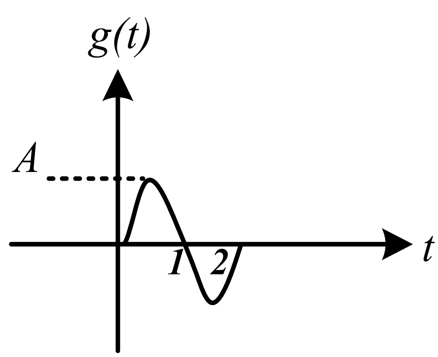
$$= \int_{-\infty}^{\infty} f(\tau)h(t - \tau)d\tau$$

$$= \int_{-\infty}^{\infty} [A\, sin\pi\tau\; u(\tau)][\delta(t - \tau) - \delta(t - 2 - \tau)]\, d\tau$$

$$= \int_{-\infty}^{\infty} A\, sin\pi\tau\; u(\tau)\delta(-(\tau - t))d\tau - \int_{-\infty}^{\infty} A\, sin\pi\tau\; u(\tau)\delta(-(\tau - (t - 2))d\tau$$

$$= A\, sin\pi\, t\; u(t) - A\, sin\pi(t - 2)\; u(t - 2)$$

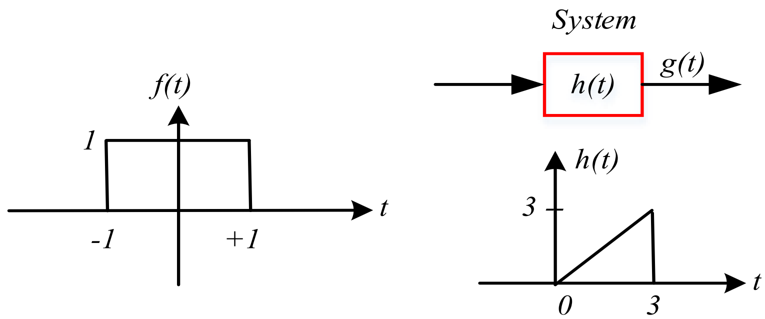
$$\therefore g(t) = \begin{cases} 0, & t < 0 \\ Asin\pi t, & 0 \le t \le 2 \\ 0, & t > 2 \end{cases}$$



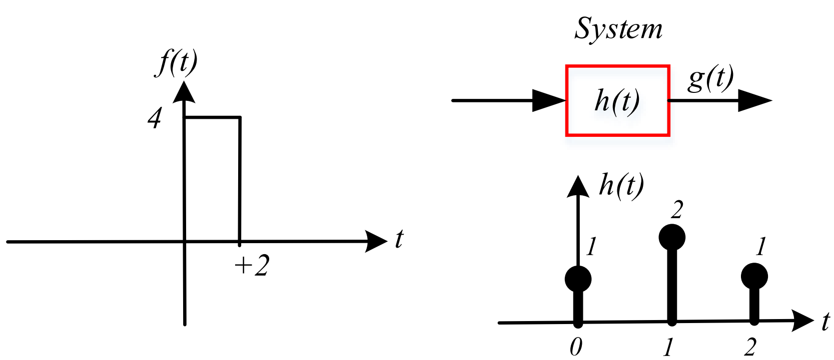
H.W:

Find the output of the following system (plot the waveforms)

1)



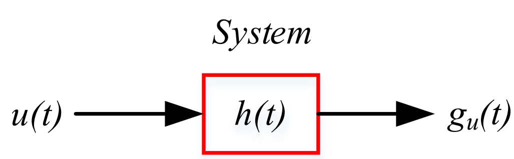
2)



Step response

The output of the system when the input is unit step u(t).

$h(t) = \frac{d}{dt}[g_u(t)]\ldots$ (2-20)

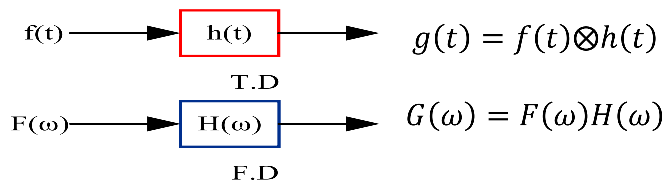


System Transfer Function:

It is the F.T of the system impulse response

$$H(\omega) = F[h(t)] \quad \ldots \quad (2\text{-}21)$$

Using the time convolution property of Fourier transform, we have:



$$G(\omega) = F(\omega).H(\omega) \quad \ldots \quad (2\text{-}22)$$

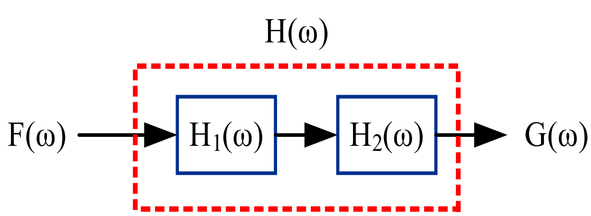
The amplitude and phase spectrum for g(t) would be:

$$|G(\omega)| = |F(\omega)|.|H(\omega)| \quad \ldots \quad (2\text{-}23)$$

$$\theta_g(\omega) = \theta_f(\omega) + \theta_h(\omega) \quad \ldots \quad (2\text{-}24)$$

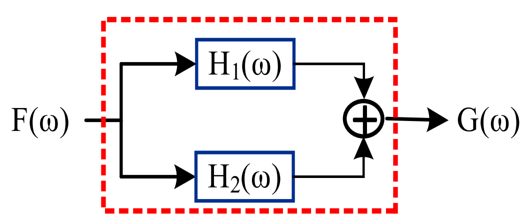
Equivalent Frequency Transfer Function:

Cascaded Series:



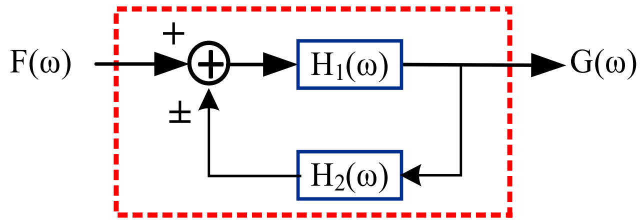
$$H_{eq}(\omega) = H_1(\omega)H_2(\omega) \quad \ldots(2\text{-}25)$$

Parallel:



$$H_{eq}(\omega) = H_1(\omega) + H_2(\omega) \quad \ldots(2\text{-}26)$$

Feedback:

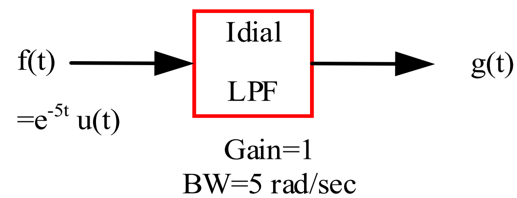


$$H_{eq}(\omega) = \frac{H_1(\omega)}{1 \pm H_1(\omega)H_2(\omega)} \quad ...(2\text{-}27)$$

Ex: 2-12:

Calculate the energy of input and output signal for the system shown below:

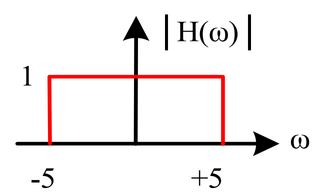
Solution:



For the input signal

$$E_f = \int_{-\infty}^{\infty} |f(t)|^2 dt$$

$$= \int_0^{\infty} e^{-10t}\, dt = 0.1\ joule$$



For the output signal

$$E_g = \frac{1}{2\pi}\int_{-\infty}^{\infty} |G(\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2 |H(\omega)|^2 d\omega$$

$$e^{-at}u(t) \Leftrightarrow \frac{1}{a+j\omega} \Rightarrow F[e^{-5t}u(t)] = \frac{1}{5+j\omega}$$

$$E_g = \frac{1}{2\pi}\int_{-5}^{5} \frac{1}{25+\omega^2} d\omega = 0.05\ joule$$

H.W:

A signal $f(t) = 2e^{-at}u(t)$ is applied to the input of system have frequency transfer function $|H(\omega)| = \frac{b}{\sqrt{\omega^2+a^2}}$ determine the required relations between the constants *a* and *b* such as exactly 50% of the input energy on a 1-ohm basis, is transferred to the output.