Note:

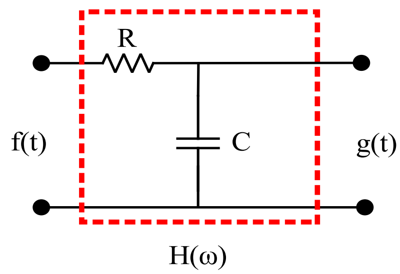
In some systems, H(ω) can be found using:

$$H(\omega) = \frac{Z_{out}}{Z_{in}} \quad \ldots (2\text{-}28)$$

Ex 2-13:

Find H(ω) for the system shown below:

Solution:



$$Z_{in} = R + \frac{1}{j\omega c}$$

$$Z_{out} = \frac{1}{j\omega c}$$

$$H(\omega) = \frac{Z_{out}}{Z_{in}} = \frac{\frac{1}{j\omega c}}{R+\frac{1}{j\omega c}} = \frac{1}{1+j\omega Rc}$$

Spectral Density and Correlation:

Energy Spectral Density (ESD):

It shows the distribution of energy at each frequency component of ***nonperiodic*** signal.

$$\psi_f(\omega) = |F(\omega)|^2 \quad \text{joule/Hz} \quad \ldots (2\text{-}29)$$

To find the total energy from the spectrum, we use:

$$E = \frac{1}{2\pi}\int_{-\infty}^{\infty} \psi_f(\omega)\, d\omega \quad \text{joule} \quad \ldots (2\text{-}30)$$

Power Spectral Density (PSD):

It shows the distribution of power at each frequency component of ***periodic*** signal.

$$S_f(\omega) = 2\pi \sum_{n=-\infty}^{\infty} |C_n|^2 \delta(\omega - n\omega_o)$$ Watt/Hz … (2-31)

To find the total power from the spectrum, we use:

$$P = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_f(\omega)\, d\omega$$ Watt … (2-32)

Note:

Power spectral density exists for ***deterministic*** and ***random*** signals, such as noise.

Ex 2-14:

A given voltage signal $f(t) = 4\cos 20\,\pi t + 2\cos 30\,\pi t$ across 2 Ω resistor.

a) Determine PSD of f(t).
b) Sketch $S_f(\omega)$.
c) Calculate the average power [(i) using *time domain*, (ii) using *spectral density*]

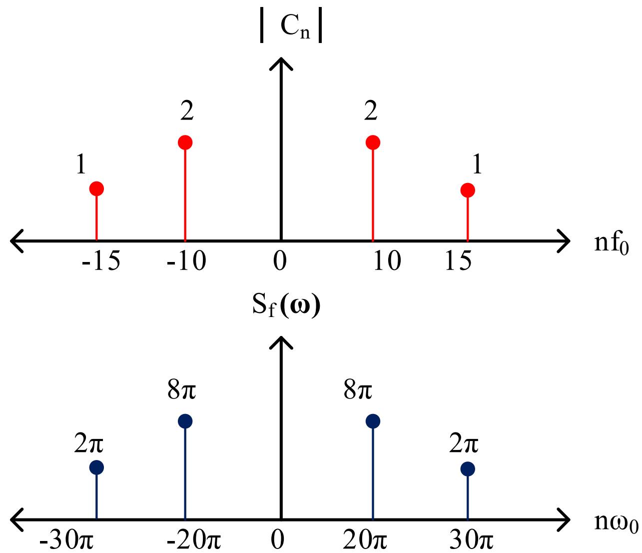
Solution:

(a) $S_f(\omega) = 2\pi \sum_{n=-\infty}^{\infty} |C_n|^2 \delta(\omega - n\omega_o)$

$$= 2\pi(1)^2\delta(\omega + 30\pi) + 2\pi(2)^2\delta(\omega + 20\pi) + 2\pi(2)^2\delta(\omega - 20\pi)$$

$$+2\pi(1)^2\delta(\omega - 30\pi)$$

(b)



(c)

$$(i): \quad P_{av} = \frac{1}{T}\int_0^T |f(t)|^2 dt = \frac{4^2}{2} + \frac{2^2}{2} = 8 + 2 = 10\ volt^2/R$$

$$P_{av} = \frac{10}{2} = 5 \text{ watt}$$

$$(ii): \quad P_{av} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_f(\omega)d\omega = \frac{2}{2\pi}\int_0^{\infty} S_f(\omega)d\omega$$

$$= \frac{1}{\pi}\int_0^{\infty} [8\pi\delta(\omega - 20\pi) + 2\pi\delta(\omega - 30\pi)]\, d\omega$$

$$= \frac{1}{\pi}(8\pi + 2\pi) = 10\ volt^2/R$$

$$P_{av} = \frac{10v^2}{2\Omega} = 5 \text{ watt} \quad \text{(the same result)}$$

Correlation:

It is the inverse Fourier Transform of the power spectral density. It is a measure of similarity between two signals or a signal and its replica shifted by τ seconds.

$$R_f(\tau) = F^{-1}\{S_f(\omega)\} \qquad \text{Watt} \quad \dots (2\text{-}33)$$

Cross Correlation:

$R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t)y(t+\tau)dt$ *Nonperiodic signals* …(2-34)a

$R_{xy}(\tau) = \frac{1}{T}\int_{0}^{T} x(t)y(t+\tau)dt$ *Periodic signals* …(2-34)b

Auto Correlation:

$R_{x}(\tau) = \int_{-\infty}^{\infty} x(t)x(t+\tau)dt$ *Nonperiodic signals* …(2-35)a

$R_{x}(\tau) = \frac{1}{T}\int_{0}^{T} x(t)x(t+\tau)dt$ *Periodic signals* …(2-35)b

Properties of Correlation:

(1) When $\tau = o$

$$R_f(0) = E \; for\ energy\ signals$$

$$R_f(0) = P_{av} \; for\ power\ signals$$

(2) $R_f(\tau) \le R_f(0)$

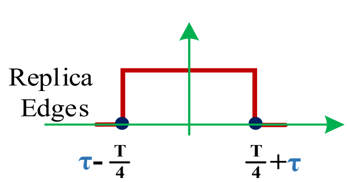
(3) If $z(t) = x(t) + y(t)$ *then,*

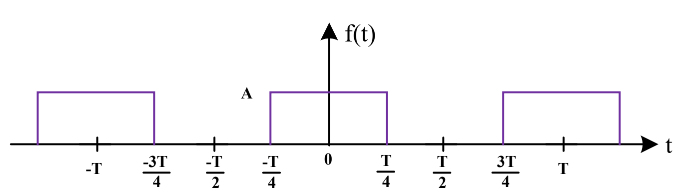
$$R_z(\tau) = R_x(\tau) + R_{xy}(\tau) + R_{yx}(\tau) + R_y(\tau)$$

Ex 2-15:

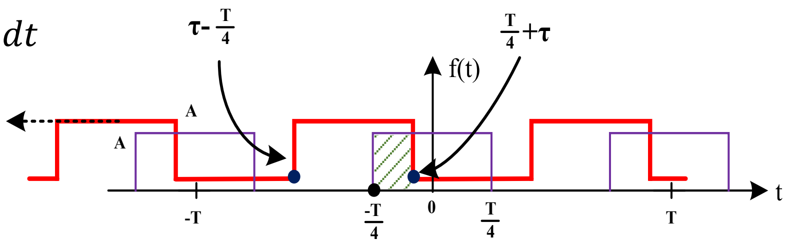
Determine and sketch the autocorrelation function of periodic square wave shown below:

Solution:





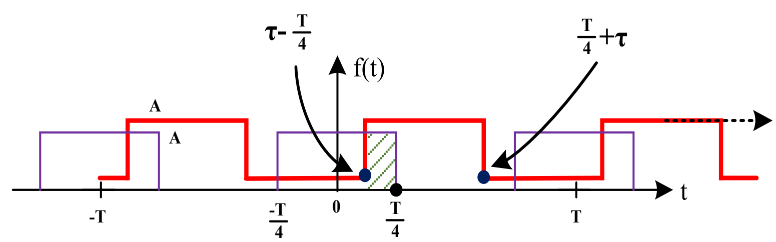
$$R_f(\tau) = \frac{1}{T}\int_0^T f(t)f(t+\tau)dt$$



(1) When $\frac{-T}{2} < \tau < 0$

$$R_f(\tau) = \frac{1}{T}\int_{-\frac{T}{4}}^{\frac{T}{4}+\tau} A^2 dt$$

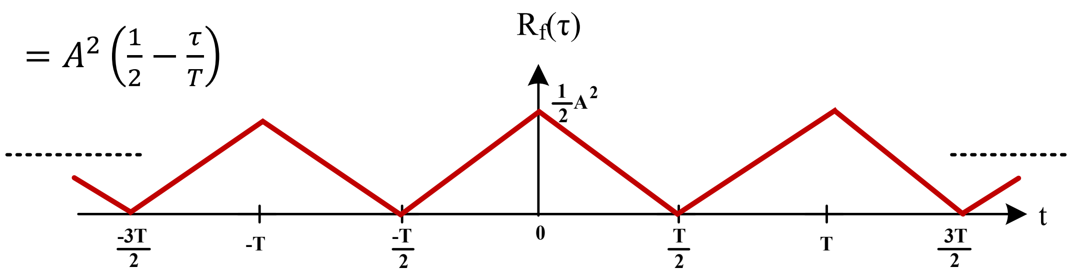
$$= A^2\left(\frac{1}{2} + \frac{\tau}{T}\right)$$



(2) When $0 < \tau < \frac{T}{2}$

$$R_f(\tau) = \frac{1}{T}\int_{\tau-\frac{T}{4}}^{\frac{T}{4}} A^2 dt$$

$$= A^2\left(\frac{1}{2} - \frac{\tau}{T}\right)$$



The autocorrelation is useful for the detection of signals, in which masked by additive noise, see the following figures.

H.W:

A sinusoidal waveform, $3\sqrt{2}cos\omega_1 t$, is added to a second $4\sqrt{2}cos\omega_2 t$, determine the *rms* value of the sum, if (a) $\omega_1 = \omega_2$, (b) $\omega_1 \neq \omega_2$

Ans: a=7, b=5

H.W:

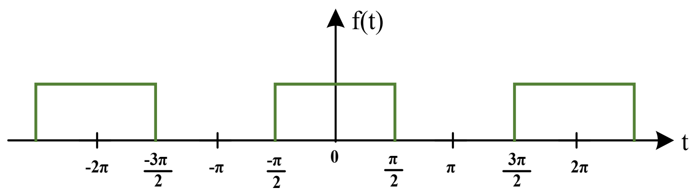
For the system shown below, find:

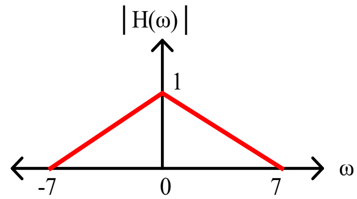
a) $g(t)$

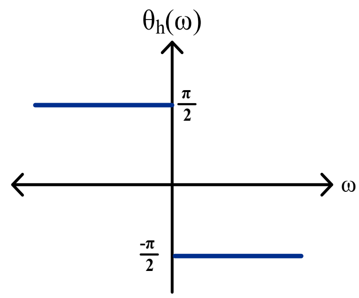
b) Average power at the system i/p & o/p.

c) PSD of $f(t)$ and $g(t)$.

d) Average power at system i/p using $P_{av} = R_f(0)$


f(t)
t
-2π
-3π/2
-π
-π/2
0
π/2
π
3π/2
2π


|H(ω)|
1
-7
0
7
ω


θh(ω)
π/2
ω
-π/2

Problem Sheet of Signal Analysis

Q1: Sketch the single and double sided amplitude and phase spectrum of the following signals:

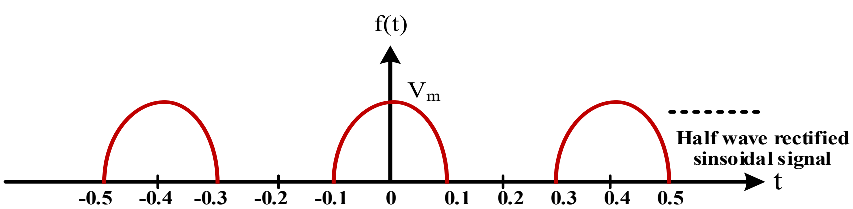
(a) $f(t) = -7\sin(3\pi t) - 5\cos(6\pi t + 90^o)$
(b) $f(t) = -4\sin(10^6\pi t) + 8\cos(10^7\pi t + 170^o)$
(c) $f(t) = \sum_{n=0}^{3}(-0.5)^n cos[n(\omega_o t + 10^o)]$

Q2: If $f(t)$ is a periodic signal in the period $-\tau/2 < t < \tau/2$ and` is given by:

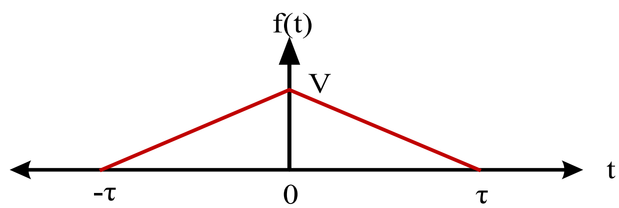
$f(t) = 2t$; find the double-sided spectrum and the ratio of the power in first three harmonics to the total average power of the signal.

Q3: Sketch the two – sided amplitude and phase spectrum of the signals shown below.

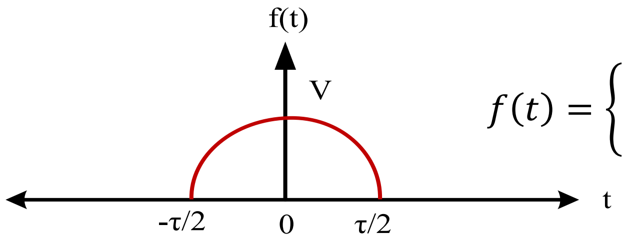
ⓐ



ⓑ

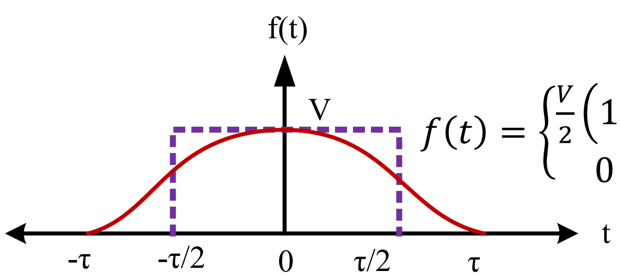


ⓒ



$$f(t) = \begin{cases} V cos\pi \frac{t}{\tau} & |t| < \tau/2 \\ 0 & elese\ where \end{cases}$$

ⓓ



$$f(t) = \begin{cases} \frac{V}{2}\left(1 - cos\pi \frac{\pi t}{\tau}\right) & |t| < \tau \\ 0 & elese\ where \end{cases}$$