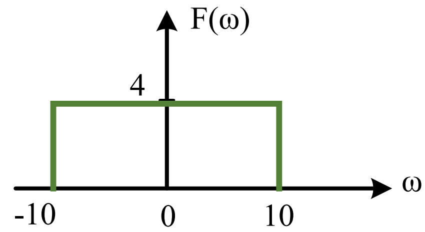
Q4: A certain function of time, $f(t)$, has a Fourier transform shown in Fig. below. Sketch the Fourier transform of

(a) $f(2t)$ (b) $[f(t)]^2$



Q5: A given periodic signal (in volts),

$f(t) = 4\sin 8\pi t + 2\cos 12\pi t$ is developed across 1Ω resistor.

(a) Determine the highest fundamental frequency possible for the signal.

(b) Sketch two-sided amplitude and phase spectrum of $f(t)$.

(c) Find the total mean (average) power in $f(t)$.

(d) Calculate the percentage of the total mean power contained in each harmonic up to the fifth harmonic using the fundamental of the part (a).

Ans: (a) 4π (c) 10 watt

(d)

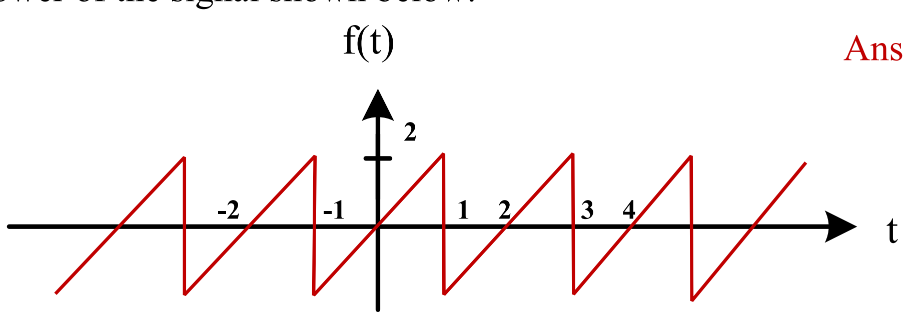
n	Power percentage	n	Power percentage
0	0%	3	20%
1	0%	4	0%
2	80%	5	0%

Q6: Determine the required numerical value of the positive real constant a if it is given that one – half the energy in $f(t) = exp\,(-at)\,u\,(t)$ lies in the spectrum range from zero to one Hz.

Ans: 2π

Q7: Find the ratio of average power in the spectral range $0 \rightarrow 2\,Hz$ to the total average power of the signal shown below.

Ans: 86.55%.



Q8: The two – sided exponential voltage $f(t) = 10e^{-|t|}$ volt is developed across a $50\ \Omega$ resistor.

(a) Calculate the total energy dissipated in the resistor.
(b) What fraction of this energy is in the frequency range of 0 – 1 rad/ sec?

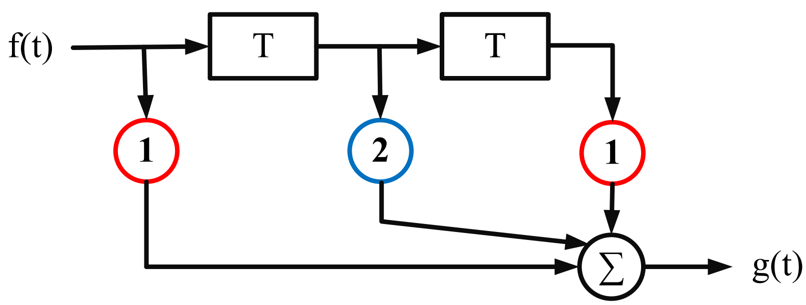
Ans: (a) 2 joule (b) 81.83%.

Q9: The spectral density of the input to a given linear time invariant system is:
$F(\omega) = [exp(-j\pi\omega)]/(1 + j\omega)$ and the corresponding output spectral density is:
$G(\omega) = [exp(-j2\pi\omega)]/(1 - \omega^2 + 2j\omega)$.

(a) Determine the transfer function of the system.
(b) Determine the impulse response of the system.

Ans: (a) $H(\omega) = \frac{1}{1+j\omega} e^{-j\pi\omega}$ (b) $h(t) = e^{-(t-\pi)} u(t-\pi)$

Q10: Determine the magnitude of the – frequency transfer function of the transversal filter, shown in Fig. below:



Ans: $|H(\omega)| = 4cos^2\left(\frac{\omega T}{2}\right)$

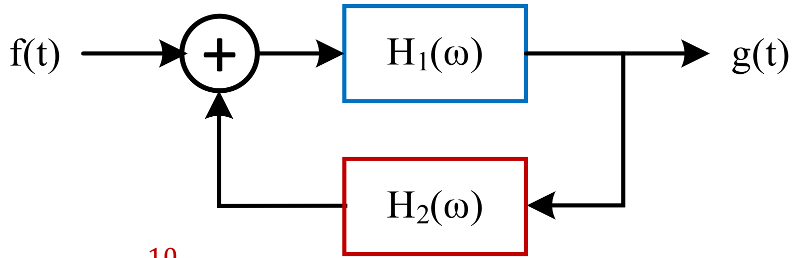
Q11: The bandwidth of a given ideal LPE is 4 rad/ sec and the low frequency gain is one. Calculate the output energy, on one ohm basis, if the input f(t):

(a) $f(t) = \delta(t)$ Ans: (a) $^4/_\pi$ joule

(b) $f(t) = e^{-4t} u(t)$ (b) $^1/_{16}$ joule

Q12: In the system shown below. If $H_1(\omega) = 10e^{-j2\omega}$ and $H_2(\omega) = e^{-j2\omega}$.

(a) Find the step response of the system.
(b) If $f(t) = \cos 10\pi t + 3\cos 20\pi t,$ find $g(t)$.



Ans: (a) $g_u(t) = \frac{10}{11}u(t-2)$

(b) $g(t) = \left[\frac{10}{11}cos10\pi(t-2) + \frac{30}{11}cos20\pi(t-2)\right]u(t-2)$

Q13: A certain signal $f(t)$ has the following PSD (assume 1Ω load):

$S_f(\omega) = \pi\left[e^{-|\omega|} + \delta(\omega-2) + +\delta(\omega+2)\right]$

(a) What is the mean power in the bandwidth $\omega \leq 1$ rad/ sec?

(b) What is the mean power in the bandwidth $0.99\ to\ 1.01$ rad/sec?

(c) What is the mean power in the bandwidth $1.99\ to\ 2.01$ rad/ sec?

(d) What is the total mean power in (t) ?

Ans: (a) 0.632 w (b) 7.36 mw.

(c) 1.0027 w (d) 2 w.

Q14: A source with a resistance of 6 k Ω resistor. Compute the mean power across the 6-kΩ resistor if the power spectral density of the source (in watt per Hz) is (a) 10^{-2}, (b) $S(\omega+10) + S(\omega-10)$

Ans: (a) 3.7 μw . (b) 11.79 μw.

Q15: A given voltage signal $f(t) = 4cos^2 20\pi t + 2cos30\pi t$ across 1Ω.

(a) Determine PSD of $f(t)$.

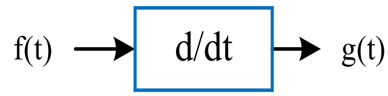
(b) Sketch $S_f(\omega)$.

(c) Calculate the mean (average) power, both in the time domain and in frequency domain, that is dissipated by $f(t)$ across the 1Ω resistor.

Ans: (c) 8 watt.

Q16: In the system shown below, if the PSD of g(t) is :$\eta/_{A^2}$ watt/Hz when $-2\pi B \le \omega \le 2\pi B$

(a) Sketch PSD of $g(t)$.
(b) Find the average power at system output



Ans: (a) $S_g(\omega) = \omega^2 S_g(\omega)$.

(b) $\frac{8\pi^2 \eta B^3}{3A^2}$ watt.

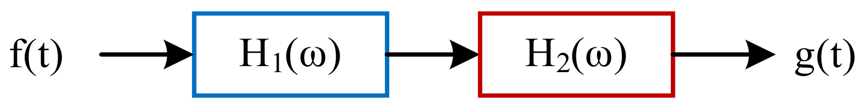
Q17: For the cascaded systems in Fig. below:

$H_1(\omega) = |3\omega| \angle -\frac{\pi}{2}$, when $0 < \omega \le 10$ rad/sec

$H_2(\omega) = |\omega + 1| \angle -\omega - \frac{\pi}{2}$, when $0 < \omega \le 20$ rad/sec

If the power spectral density of f (t) is $10\left(1 + {}^{\omega}/_{10}\right)$ watt/Hz in the range $0 < \omega \le 30$ rad/ sec.

(a) Find and sketch magnitude and phase $H_{eq}(\omega)$.
(b) Compute input and output power of the system.



Q18: A system is governed by the differential equation:

$$\frac{dg(t)}{dt} + ag(t) = b\frac{df(t)}{dt} + cf(t)$$

(a) Find and plot $|H(f)|$ and $\angle|H(f)|$ for c = 0.
(b) Find and plot $|H(f)|$ and $\angle|H(f)|$ for b = 0.

Q19: Determine the autocorrelation function of the pulse waveform $e^{-at}\,[u(t) - u(t-\tau)]$ when it is repeated periodically every $2T$ seconds.

Ans: $R_f(\tau) = \frac{1}{4aT}[e^{3a\tau} - e^{a\tau}e^{-2aT}]$ *when* $-T < \tau < 0$

$R_f(\tau) = \frac{e^{-a\tau}}{-4aT}[e^{-2aT} - e^{-2a\tau}]$ *when* $0 < \tau < T$

Chapter 3

Noise

Noise: unwanted signal, random or deterministic interfere the desired signal in a system.

Noise classification:

1- Manmade noise:

Such as:

- Electromagnetic pick-up of the radiating signals.
- Inadequate power supply filtering.
- Alias terms from poor sampling.

These sources of noise can be eliminated or minimized, by careful engineering design.

2- Naturally occurring noise.

These are not controllable. Their characteristics can be described statically.

Statistical Representation of Noise:

If $n(t)$ is a noise voltage or current and there is a $1\Omega\ resistive\ load$, then it could be described as:

$$n(t) = \overline{n(t)} + \sigma(t) \quad \ldots (3\text{-}1)$$

Where:

$$\overline{n(t)} = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} n(t)dt \quad \text{d.c component} \quad \ldots(3\text{-}2)$$

$$\sigma(t) = n(t) - \overline{n(t)} \quad \text{a.c component} \quad \ldots(3\text{-}3)$$

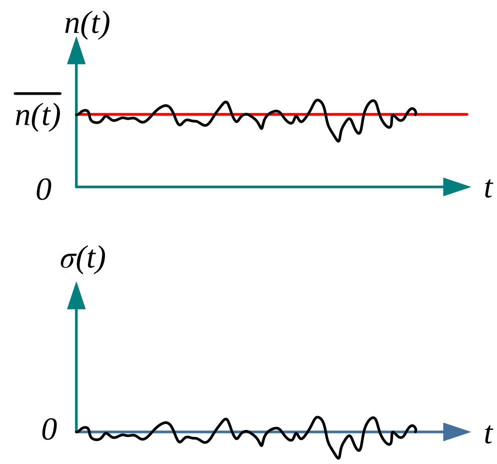
Noise Power:

a) Using Time domain:

$$\overline{n^2(t)} = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} |n(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \left|\overline{n(t)}\right|^2 dt \quad \text{d.c power}$$

$$+ \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} |\sigma(t)|^2 dt \quad \text{a.c power}$$



b) Using frequency domain:

$$\overline{n^2(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_n(\omega)d\omega$$

$S_n(\omega)$= power spectral density

… (3-5)

Note:

If $S_n(\omega)$ in watt/Hz, then $\overline{n^2(t)}$ in watt (N)

If $S_n(\omega)$ in volt^2/Hz, then $\overline{n^2(t)}$ in watt volt^2

Signal-to-Noise Ratio:

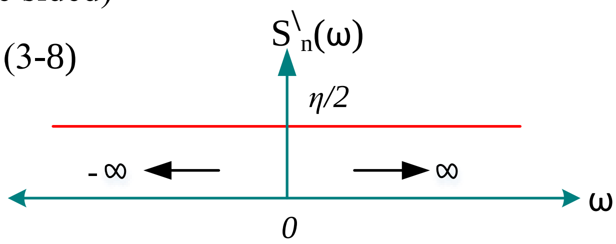
$$SNR = \frac{\overline{s^2(t)}}{\overline{n^2(t)}} \qquad \text{… (3-6)}$$

$$SNR_{dB} = 10\ log_{10}\left[\frac{\overline{s^2(t)}}{\overline{n^2(t)}}\right] \qquad \text{… (3-7)}$$

White Noise:

A noise is said to be white if it has a flat spectrum at all frequency components, like the white light.

$S_n(\omega) = \frac{\eta}{2}$ watt/Hz for all ω (double sided) … (3-8)

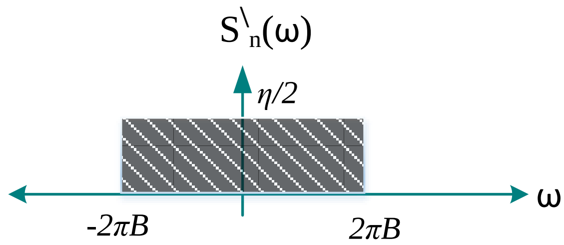


Band Limited White Noise:

It has a flat power at all frequency component extending beyond the bandwidth of a given system.

$$S'_n(\omega) = \frac{\eta}{2}\ watt/_{Hz} \qquad -2\pi B \le \omega \le 2\pi B$$

(double sided)



$$\overline{n^2(t)} = \frac{1}{2\pi}\int_{-2\pi B}^{2\pi B} \frac{\eta}{2}\, d\omega = \eta B \quad \text{watt}$$

$$= \eta R B \ \ \text{volt}^2$$

$$= \eta G B \ \ \text{amp}^2$$

… (3-9)

Selected Types of Noise:

1- Thermal Noise:

It is a type of band-limited white noise over a very broad frequency range. It is produced as a result of thermally excited random motion of free electrons in a conducting medium such as a resistor. The net effect of the motion of all electronic current which is random with a mean value of zero. The PSD of thermal noise is given by:

$$S_n(\omega) = 2kT \qquad \text{Watt/Hz} \quad \text{for } |\omega| \ll \frac{2\pi kT}{h}$$

(two sided) … (3-10) a