$$S_n(\omega) = 2kTR$$ volt^2/Hz (two sided) … (3-10) b

$$S_n(\omega) = 2kTG$$ amp^2/Hz (two sided) … (3-10) c

where:

T= Temperature of the conducting medium in Kelvin ($k^o = C^o+273$)

k =Boltzmann's constant =1.38 x 10^{-23} joule/ k^o

h = Blank's constant =6.625 x 10^{-34} joule.sec

- For frequencies above $\frac{kT}{h}$, it is no longer white.
- At 290 $k^{o,}$ $\frac{kT}{h}$=600 GHz.
- An ideal L&C has no thermal noise.

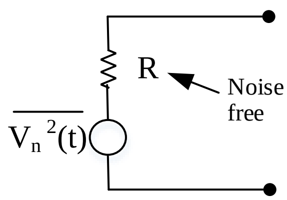
Thermal Noise Power:

The thermal noise power in a resistor R can be found either using voltage model or current model.

Voltage model:

$$\overline{V_n^2(t)} = \frac{1}{2\pi}\int_{-2\pi B}^{2\pi B} 2kTRd\omega \quad = 4kTRB \text{ volt}^2$$

(or $4kTB$ watt) … (3-11)

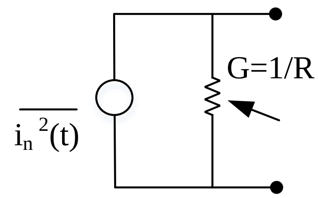


r.m.s noise voltage=$\sqrt{\overline{V_n^2(t)}}$ volt

Current model:

$$\overline{i_n^2(t)} = \frac{1}{2\pi}\int_{-2\pi B}^{2\pi B} 2kTG d\omega \quad = 4kTGB \text{ amp}^2$$

(Or $4kTB$ watt) … (3-12)



r.m.s noise current= $\sqrt{\overline{i_n^2(t)}}$ amp

Ex 3-1:

Calculate the rms noise voltage arising from thermal noise in two resistors 100 Ω and 150 Ω at T=300 ºk and a bandwidth of 1 MHz:

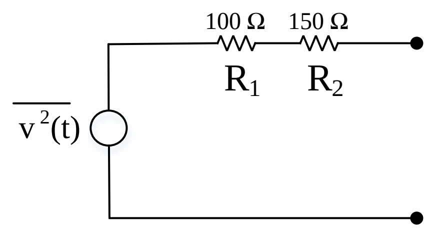
a) Connected in series
b) Connected in parallel

Solution:

a) $\overline{v_n^2(t)} = 4kTRB$ v^2

$= 4kTB(R_1 + R_2) = 4 \times 1.38 \times 10^{-23} \times 300 \times 10^6 \times (100 + 150)$

$= 4.14 \times 10^{-12}$ volt^2



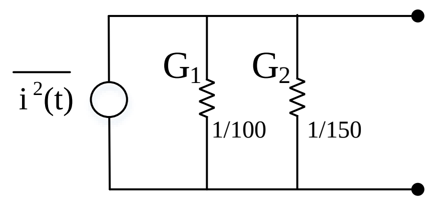
$V_n rms = \sqrt{\overline{v_n^2(t)}}$=2.3 µv

b) $\overline{i_n^2(t)} = 4kTGB$ amp^2

$= 4kTB(G_1 + G_2)$

$= 4 \times 1.38 \times 10^{-23} \times 300 \times 10^6 \times (\frac{1}{100} + \frac{1}{150})$

$= 2.76 \times 10^{-16}$ amp^2



$i_n rms = \sqrt{\overline{i_n^2(t)}}$=0.0166 µA

The equivalent parallel resistance $R_{eq} = \frac{1}{G_1+G_2} = 60\ \Omega$

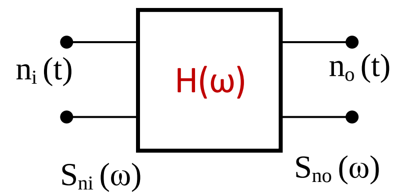
$\therefore V_{rms} = i_n rms \times R_{eq}$

$=(0.166*10^{-6})*60$

$=0.997\ \mu V$

Transmission of Thermal Noise through Linear System

- In General



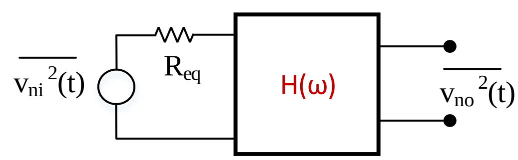
$$S_{no}(\omega) = S_{ni}(\omega)|H(\omega)|^2 \quad \text{… (3-13)}$$

Noise Power at System Input:

$$\overline{n_i^2(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S'_n(\omega)d\omega \quad \text{… (3-14)}$$

$$\overline{n_o^2(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S'_{no}(\omega)d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} S'_{ni}(\omega)|H(\omega)|^2 d\omega \quad \text{… (3-15)}$$

- **For thermal Noise:**



$$S_{ni}(\omega) = 2kTR_{eq} \quad \text{… (3-16)}$$

$$\therefore \overline{V_{ni}^2(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} 2kTR_{eq} d\omega \quad \text{… (3- 17)}$$

$$\therefore \overline{V_{no}^2(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} 2kTR_{eq}|H(\omega)|^2 d\omega \quad \text{… (3-18)}$$

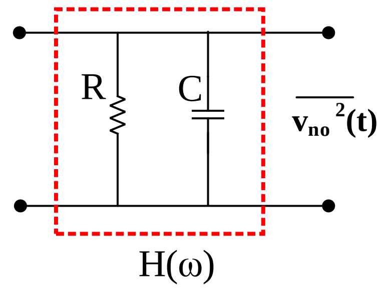
When R_{eq} is the input resistance of the system

$$R_{eq} = Re[Z_{in}] \quad \text{… (3-19)}$$

Ex 3-2:

Find the rms noise voltage across 1 μf capacitor over the entire frequency band, when the capacitor is shunted by 1000 Ω resistor maintained at 300 ^{o}k.

Solution:



V_{rms} across C $= \sqrt{\overline{V_{no}^2(t)}}$

$$\overline{V_{no}^2(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} 2kTR_{eq}|H(\omega)|^2 d\omega$$

$$Z_{in} = R // \frac{1}{j\omega c} = \frac{\frac{R}{j\omega c}}{R+\frac{1}{j\omega c}} = \frac{R}{1+j\omega Rc}$$

$$R_{eq} = Re[Z_{in}] = \frac{R}{1+\omega^2R^2C^2}$$

$$\overline{V_{no}^2(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} 2kT\frac{R}{1+\omega^2R^2C^2}d\omega$$

$$= \frac{kT}{\pi}\int_{-\infty}^{\infty}\frac{R}{1+\omega^2R^2C^2}d\omega = \frac{2kT}{\pi C}[tan^{-1}(\omega RC)]_0^{\infty}$$

$$= \frac{kT}{C} = \frac{1.38\times10^{-23}\times300}{10^{-6}} = 4.14\times10^{-15}\ volt^2$$

$$V_{rms} = 0.0645\mu v$$

H.W:

A resistor of resistance R=1000 Ω is maintained at 17 ºC and it shunted by 100 µH inductor. Determine the rms noise voltage across the inductor over a frequency bandwidth of:

i) 15.9 kHz Ans: $182\ x10^{-9}$ volt
ii) 159 kHz Ans: $9.22\ x10^{-8}$ volt
iii) 1590 kHz Ans: $2.34\ x10^{-6}$ volt

2- Sky noise (Additive noise):

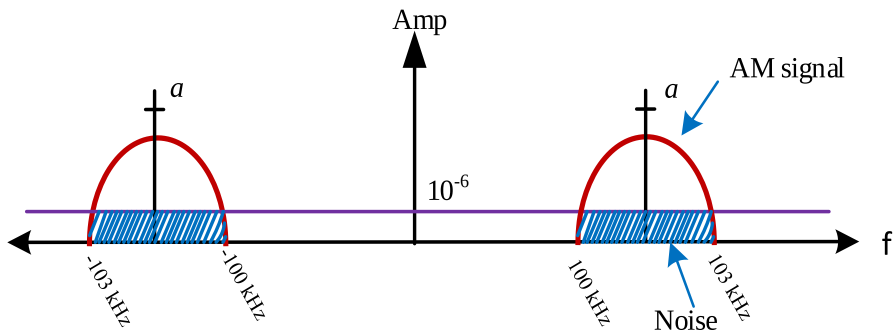
It is referred to cosmic noise and galactic noise. It is added to modulated signals that use the free space as a transmission channel such as AM, FM, …signals. The well-known of probability distribution of this type of noise is Gaussian (AGWN

Additive White Gaussian Noise). Its power is usually computed from the power spectral density function $S_n(\omega)$.

Ex 3-3:

An AM signal of 50 watt power is transmitted in a frequency range 100-103 kHz in a transmission channel. If the additive noise power spectral density (two-sided) in a transmission channel is 1µ watt/Hz. Find the signal-to-noise ratio in the transmission channel.

Solution:



$S = 50\ watt$

$$N = \frac{2}{2\pi}\int_{200\pi*10^3}^{206\pi*10^3} S_n(\omega)d\omega = \frac{1}{\pi}\int_{200\pi*10^3}^{206\pi*10^3} 10^{-6}d\omega = 6\times10^{-3}watt$$

Or simply $N = \eta B = 10^{-6}\,\frac{watt}{Hz} * 2 * 3000\ Hz = \ 6\times10^{-3}watt$

$SNR = \frac{S}{N} = \frac{50}{6\times10^{-3}} = 8333.33$

$SNR_{dB} = 10\log\ 8333.33 = 39.208\ dB$

H.W 3-2:

Compute SNR for the previous example if PSD of noise is not white and given by 10^{-6} f watt/Hz. (what is the conclusion you obtained for this colored noise assumption as compared with white case?).

Problem Sheet for Noise

Q1: Two resistors R_1, R_2 are both at a temperature T. Determine the required relationship between R1 and R2 if the rms thermal noise voltage across the series combination is twice that across the parallel combination.

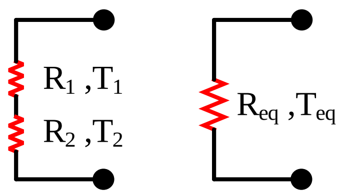
Ans: $R_1 = R_2$.

Q2: Two resistors connected in series and at differing temperature are shown below.

(a) Derive relations for the noise – equivalent resistor and temperature R_{eq}, T_{eq}.

(b) Calculate R_{eq}, T_{eq} for the specific case where:

$$R_1 = 1k\Omega \,, R_2 = 2k\Omega \,, T_1 = 300^o k \;\;, T_2 = 390^o k$$

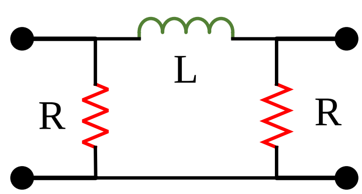


Ans: (a) $\frac{R_1T_1+R_2T_2}{R_1+R_2}$, (b) $T_{eq} = 360^o k$

Q3: Two: identical resistors at a temperature T are connected to an ideal inductor as shown below:

(a) Derive an expression for the thermal noise voltage spectral density developed across each resistor.

(b) What is the mean-square noise voltage across each resistor within the noise bandwidth B.



Q4: A system has input resistance of 4 kΩ and transfer function:

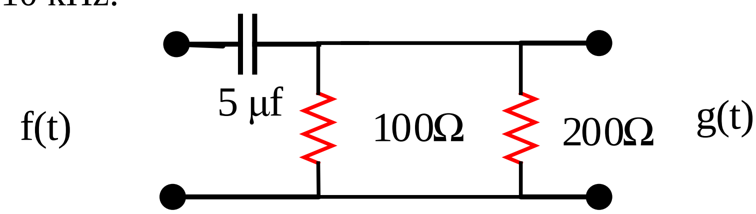
$H(\omega) = 8/(5 + j\omega)$ find:

(a) The impulse response of the system.

(b) The thermal noise power at system input and output.

Q5: Calculate signal-to-thermal noise ratio at the input and output of the network shown below, if the PSD of the input signal is:

$S_f(\omega) = \pi^2\delta(\omega - 10) + \pi^2\delta(\omega + 10)$ watt/Hz, and $T = 290^o k$, assume bandwidth= 10 kHz.



Q6: An AM signal of 50 $watt$ power is transmitted in a frequency range $200 - 210$ kHz in a transmission channel with power spectral density shown in figure. below. Calculate signal-to-noise ratio in the transmission channel.