We define "modulation index" m as:

$$m = \frac{-f(t)_{min}}{A_c} \quad \ldots(4\text{-}7)$$

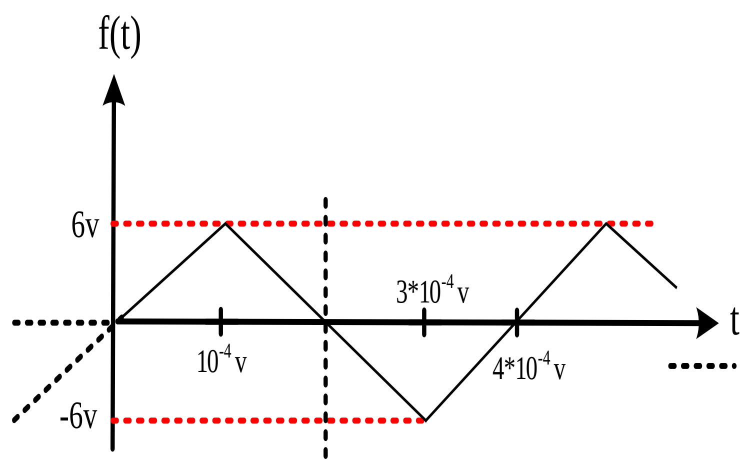
$m \leq 1$ for envelope detection, and modulation depth as:

$$D = m \times 100\% \quad \ldots(4\text{-}8)$$

Ex 4-2:

Find the modulation index and modulation depth if a carrier signal given by $8\cos 2\pi * 10^5 t$ is modulated by the signal shown below using AM/DSB-LC technique.

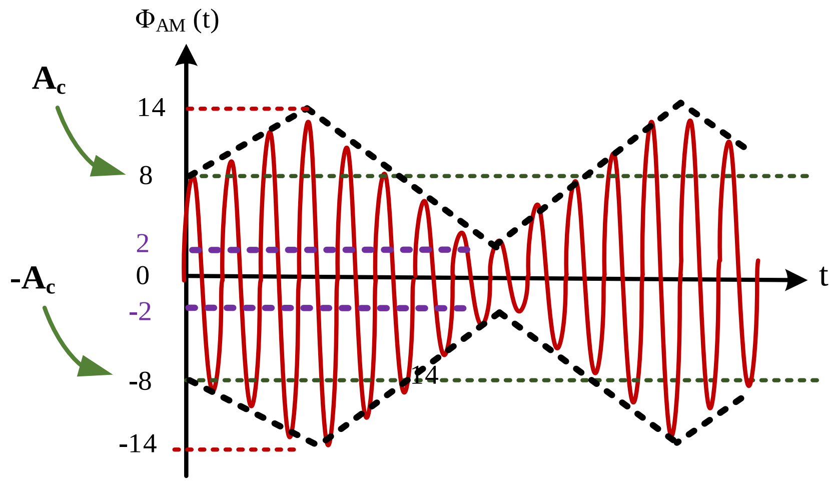
Solution:



$$m = \frac{-f(t)_{min}}{A_c} = \frac{6}{8} = 0.75 \qquad f(t)_{min} = -6\,v A_c = 8\,v$$

$D = m \times 100\%$=75%

The modulated signal waveform may be plotted as shown below



H.W:

a) Write the equation of the modulated wave in the previous example.

b) Sketch the waveform of modulated wave and write its equation if the modulation type is AM/DSB- SC.

Single Tone Modulation:

The single tone signal is given by:

$$f(t) = A_m cos\omega_m t \quad \dots(4\text{-}9)$$

It is an experimental signal commonly used in communication systems. It is a simple signal, since it has only one frequency.

The modulation index and depth for single tone modulation would be:

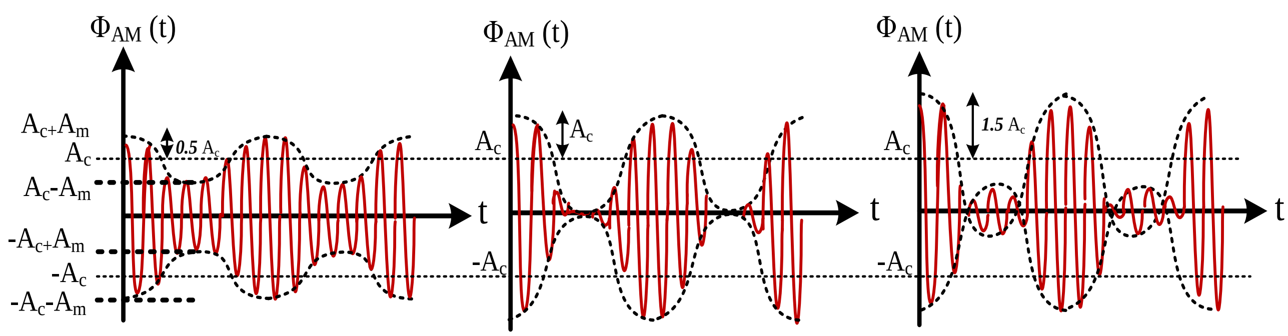
$$m = \frac{-f(t)_{min}}{A_c} = \frac{A_m}{A_c} \quad \dots (4\text{-}10)$$

$$D = \frac{A_m}{A_c} * 100\% \quad \text{... (4-11)}$$

The AM signal for tone modulation is given by:

$$\Phi_{AM} = [A_c + f(t)]cos\omega_c t$$
$$= A_c[1 + mcos\omega_m t]cos\omega_c t$$

For single tone ... (4-12)



m=0.5 $\left(A_m = \frac{A_c}{2}\right)$ Correct modulation

m=1 $(A_m = A_c)$ Critical modulation

m=1.5 $\left(A_m = \frac{3A_c}{2}\right)$ Over modulation

From the previous figure, the modulation index could be obtained using:

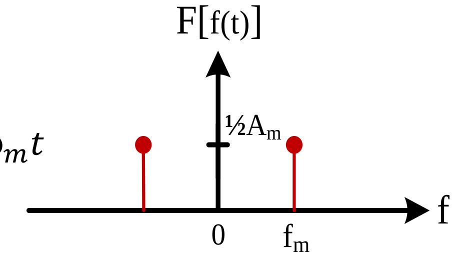
$$m = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} \quad \text{... (4-3)}$$

Where:

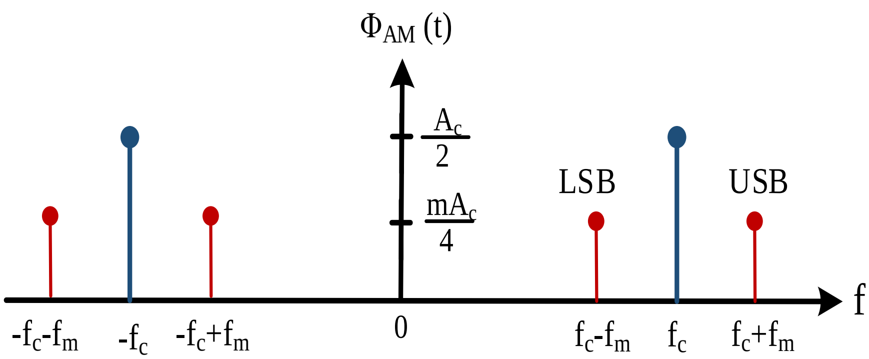
$A_{max} = A_c + A_m$ and $A_{min} = A_c - A_m$

The spectrum of single-tone AM modulation should be:

$$f(t) = A_m cos\omega_m t$$



$$\Phi_{AM_{S.T}}(t) = A_c cos\omega_c t + \frac{mA_c}{2} cos(\omega_c - \omega_m)t + \frac{mA_c}{2} cos(\omega_c + \omega_m)t$$



H.W:

A carrier signal given by $10\cos 10000\,\pi t\ \ volt$ is AM/DSB-LC modulated by single-tone signal $4\cos 100\,\pi t\ \ volt$.

1- Calculate the modulation index.
2- Sketch the spectrum of the modulated signal.
3- Calculate transmission bandwidth.

Sideband and Carrier Powers:

$$\Phi_{AM} = \boxed{A_c cos\omega_c t} + \boxed{f(t) cos\omega_c t}$$

Carrier (P_c)"Losses" Sidebands (P_s)

$$P_c = \frac{A_c^2}{2}$$ "Carrier Power" or $$P_c = \frac{A_c^2}{2R}$$ if A_c in volts and R is given

…… (4-14)

$$P_s = \frac{1}{2}\overline{f^2(t)}$$ "Sideband Power" $$\overline{f^2(t)} = \frac{1}{T}\int_0^T |f(t)|^2 dt$$

…(4-15)

$$P_t = P_c + P_s = \frac{1}{2}\left[A_c^2 + \overline{f^2(t)}\right] \qquad \text{…(4-16)}$$

$$\eta = \frac{P_s}{P_t} \times 100\%$$ "efficiency of transmission"

$$\eta = \frac{\overline{f^2(t)}}{\overline{f^2(t)} + A_c^2} \times 100\% \qquad \text{…(4-17)}$$

For Single tone modulation

$$\Phi_{AM} = A_c[1 + m\cos\omega_m t]\cos\omega_c t$$

$$P_t = \frac{A_c^2}{2} + \frac{m^2 A_c^2}{8} + \frac{m^2 A_c^2}{8} = \frac{A_c^2}{2} + \frac{m^2 A_c^2}{4}, \quad P_t = P_c\left(1 + \frac{m^2}{2}\right) \qquad \text{…(4-18)}$$

P_c (Losses) P_{USB} P_{LSB} P_c P_S

$$\eta = \frac{P_s}{P_t} = \frac{m^2}{m^2 + 2}$$ max. efficiency when m=1, …(4-19)

$$\eta_{max} = 33.33\%$$

$$\frac{P_c}{P_t} = \frac{2}{m^2 + 2} \qquad \text{…(4-20)}$$

Ex 4-3:

A transmitter transmits an AM/DSB single tone-modulating signal given by $3\cos(2\pi 10^3 t)\ volt$ with a carrier signal given by $10\cos(2\pi 10^6 t)\ volt$,Find:

1- Modulation depth.
2- USB & LSB frequencies.
3- Amplitude of sideband frequencies.
4- Efficiency of Transmission.

Solution:

A_m=3 v, A_c=10 v

$f_m=10^3$ Hz, $f_c=10^6$ Hz

1- $D = \frac{A_m}{A_c} \times 100\% = \frac{3}{10} \times 100\% = 30\%$
2- USB frequency is $f_c + f_m = 10^6 + 10^3 = 1.001\ MHz$, LSB frequency is $f_c - f_m = 10^6 - 10^3 = 0.999\ MHz$
3- Amp. of USB & LSB is $\frac{mA_c}{4} = \frac{0.3\times 10}{4} = 0.75\ volt$
4- $\eta = \frac{P_s}{P_t} = \frac{m^2}{m^2+2} = \frac{0.3^2}{0.3^2+2} = 4.3\ \%$

H.W:

Repeat the previous example assuming that the modulating signal is given by:

$$f(t) = \begin{cases} t-1, & 0 < t < 2*10^{-3} \\ 0\ , & 2*10^{-3} < t < 4*10^{-3} \end{cases}$$

Assume that the signal is band-limited to 1 kHz.

Ex. 4-4:

Broadcast transmitter transmits AM/DSB-LC signal, with total average power of 50 kW and uses a modulation index of 0.707 for a sinusoidal message signal, calculate:

1- carrier signal power (P_c)
2- Efficiency of transmission (η).
3- Maximum carrier signal amplitude if the antenna is represented as a 50Ω resistance (A_c).

Solution:

1- $\frac{P_c}{P_t} = \frac{2}{m^2+2}$

$$P_c = \frac{2}{m^2+2} P_t = \frac{2}{2+0.707^2} \times 50\ kw = 40\ kw$$

2- $\eta = \frac{P_s}{P_t} \times 100\% = \frac{m^2}{m^2+2} \times 100\% = 20\ \%$

3- $P_c = \frac{A_c^2}{2R}$

$$A_c = \sqrt{2 * R * P_c} = \sqrt{2 * 50 * 40kw}$$

$$= 2kv$$

Generation of AM/DSB- LC Signal: