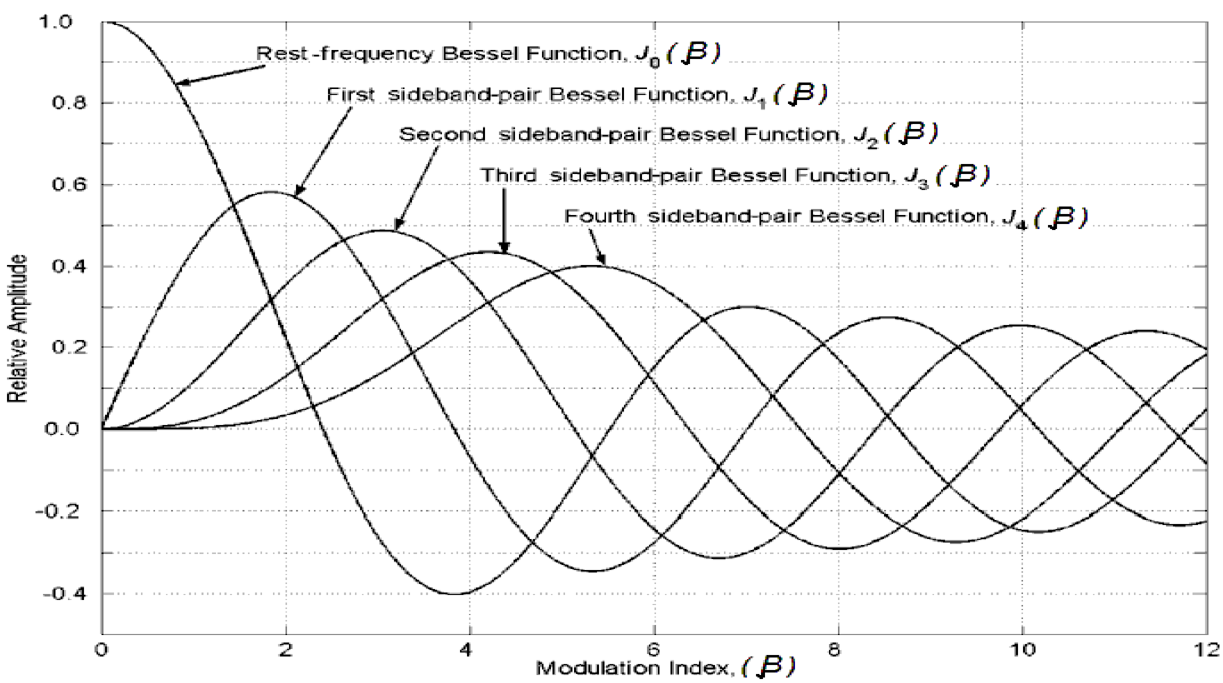
Table of Bessel function

x	Bessel-function order, n																
β	J_0	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}	J_{11}	J_{12}	J_{13}	J_{14}	J_{15}	J_{16}
0.00	1.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—	—	—	—	—	—	—
2.0	0.22	0.58	0.35	0.13	0.03	—	—	—	—	—	—	—	—	—	—	—	—
2.41	0	0.52	0.43	0.20	0.06	0.02	—	—	—	—	—	—	—	—	—	—	—
2.5	−.05	0.50	0.45	0.22	0.07	0.02	0.01	—	—	—	—	—	—	—	—	—	—
3.0	−.26	0.34	0.49	0.31	0.13	0.04	0.01	—	—	—	—	—	—	—	—	—	—
4.0	−.40	−.07	0.36	0.43	0.28	0.13	0.05	0.02	—	—	—	—	—	—	—	—	—
5.0	−.18	−.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	—	—	—	—	—	—	—	—
5.53	0	−.34	−.13	0.25	0.40	0.32	0.19	0.09	0.03	0.01	—	—	—	—	—	—	—
6.0	0.15	−.28	−.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	—	—	—	—	—	—	—
7.0	0.30	0.00	−.30	−.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	—	—	—	—	—	—
8.0	0.17	0.23	−.11	−.29	−.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03	—	—	—	—	—
8.65	0	0.27	0.06	−.24	−.23	0.03	0.26	0.34	0.28	0.18	0.10	0.05	0.02	—	—	—	—
9.0	−.09	0.25	0.14	−.18	−.27	−.06	0.20	0.33	0.31	0.21	0.12	0.06	0.03	0.01	—	—	—
10.0	−.25	0.04	0.25	0.06	−.22	−.23	−.01	0.22	0.32	0.29	0.21	0.12	0.06	0.03	0.01	—	—
12.0	0.05	−.22	−.08	0.20	0.18	−.07	−.24	−.17	0.05	0.23	0.30	0.27	0.20	0.12	0.07	0.03	0.01

Bessel function first kind



Ex. 5-3

A given FM transmitter is modulated with a single sinusoid. The output for no modulation is 100 watt, into a 50 Ω resistive load. If the power provided for the first sideband is made zero, find:

a) Carrier power.
b) All sidebands power.
c) Average power in second order sidebands.

Solution:

$P_1 = 0 \Rightarrow J_1(\beta)^2 = 0 \Rightarrow J_1(\beta) = 0, \therefore \beta \cong 3.8$

a) $J_1(\beta) = 0$ at $\beta \cong 3.8$

$P_c = \frac{1}{2} A_c^2 J_0^2(\beta) = P_t J_0^2(3.8) = 16\, watt$

b) $P_s = P_t - P_c = 100 - 16 = 84\, watt$

c) $P_2 = 2\left[\frac{1}{2} A_c^2 J_2^2(\beta)\right] = 2 \times 100 \times J_2^2(3.8) = 34\, watt$

H.W

A carrier signal given by $10 \cos 2\pi \times 10^8 t\ volt$ is FM modulated by single tone message $4 \cos 2\pi \times 10^3 t\ volt$ if the modulation constant is $1000\ \frac{Hz}{Volt}$,

a) Compute max. frequency deviation and deviation ratio.
b) Write the equation of modulated wave.
c) Sketch the spectrum.
d) Calculate sidebands carrier and total power.

Generation of Wideband FM signals

1- Indirect method (Armstrong Method)

In this method, a NBFM signal is first generated with small β (modulation index) then increased using frequency multiplier (nonlinear device and BPF).

$$e_i(t) = A_c \cos(\omega_c t + \beta sin\omega_m t)$$

$$e_0(t) = a e_i^2(t)$$

$$= a A_c^2 cos^2(\omega_c t + \beta sin\omega_m t)$$

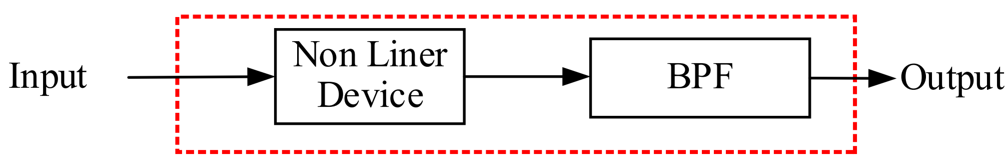
$= (\frac{1}{2}) a A^2 [1 + \cos\,(2\omega_c t + 2\beta sin\omega_m t]$, where $\omega_c and\ \beta$ are doubled.

Using a nonlinear device with I/O c/cs:

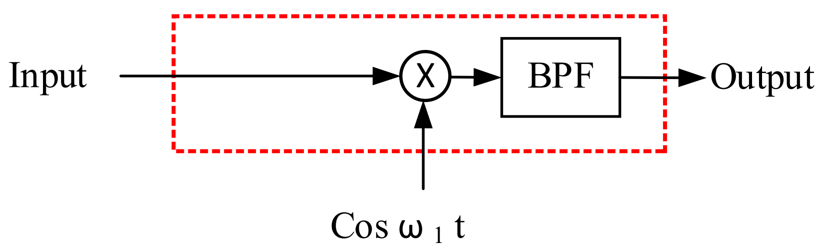
$$e_0(t) = a_0 + a_1 e_i(t) + a_2 e^2(t) + \cdots + a_n e^n(t)$$

Then we will have $n\beta$, i.e. WBFM,

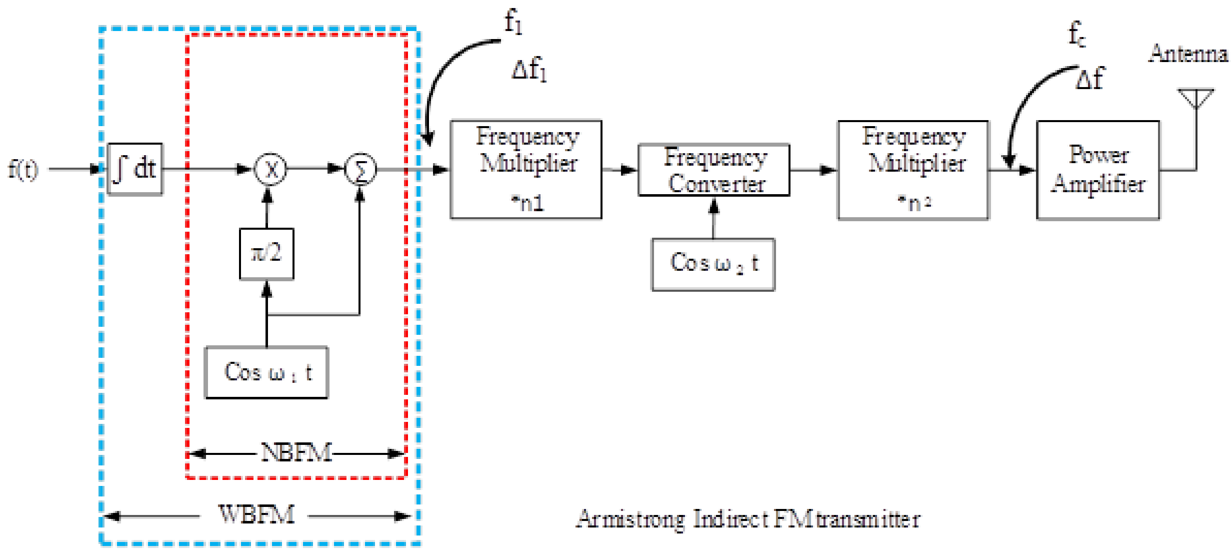
But we also have $n\omega_c$. Therefore we use frequency convertor to control the value of ω_c.



Frequency Multiplier



Frequency Converter



Armistrong Indirect FM transmitter

The frequency deviation is given by: $\Delta f = n_1 n_2 \Delta f_1$ …(5-27)

And the carrier frequency is: $f_c = (n_1 f_1 \pm f_2) n_2$ …(5-28)

Practically the value of Δf_1 is 25 Hz, in order to maintain $\beta \ll 1$ (as required in NBFM), the balanced spectrum range from 50 Hz to 15 kHz (audio frequencies).

$$\beta = \frac{\Delta f}{f_m}, at\ f_m = 15\ kHz \Rightarrow \beta = \frac{25}{15k} = 0.00167$$

$$at\ f_m = 50\ Hz \Rightarrow \beta = \frac{25}{50} = 0.5 \text{ , Worse possible case}$$

The two values of carrier frequency due to positive and negative signs are compared with the FM band [88 MHz to 108 MHz], so we choose the value which lies in that range.

The values n_1 & n_2 are chosen such that they can be generated using doublers and triplers I cascaded more easier practically

(ex. 64 =2*2*2*2*2*2, 48= 2*2*2*2*3, ……)

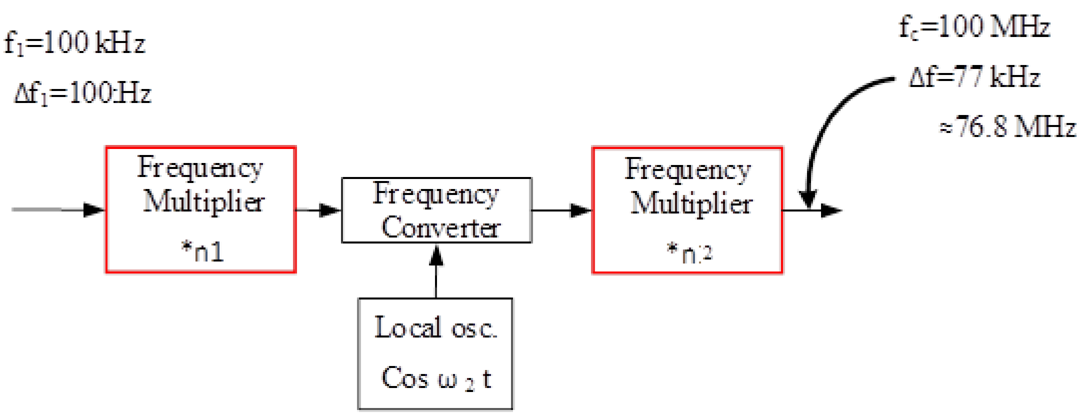
Ex 5-4

Consider the signal $2\cos(2\pi \times 10^5\, t + 0.1 \sin 2\pi \times 10^3\, t)$ is used to generate WBFM with Δf no more than 77 kHz, and f_c of 100 MHz.

a) Design a circuit of Armstrong transmitter.

b) Determine an estimate of BW of both signals.

Solution



$$\beta = 0.1$$

f_m= 10^3= 1kHz

$\Delta f_1 = \beta f_m = 0.1 * 10^3 = 100$ Hz

$$n = \frac{77kHz}{100\ Hz} = 770$$

$$n \cong 2^8 * 3 = 768$$

Say $n_1 = 2^5 = 32$

$n_2 = 2^3 * 3 = 24$

$\Delta f = n_1 n_2 \Delta f_1$=768 *100Hz

= 76.8 kHz

2	770
	~~385~~
2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

$$f_c = (n_1 f_1 \pm f_2)n_2$$

Since $n_1 n_2 f_1$=768 *100 k=76.8 MHz which is less than f_c (100 MHz).

The sign in above equation is +, i.e:

$$f_c = (n_1 f_1 + f_2)n_2$$

$100*10^6=(32*100*10^3+f_2)*24$

f_2=33.2 MHz.

a) $BW_{NBFM}=2f_m$=2 kHz

$BW_{WBFM}=2\Delta f$=153.6 kHz

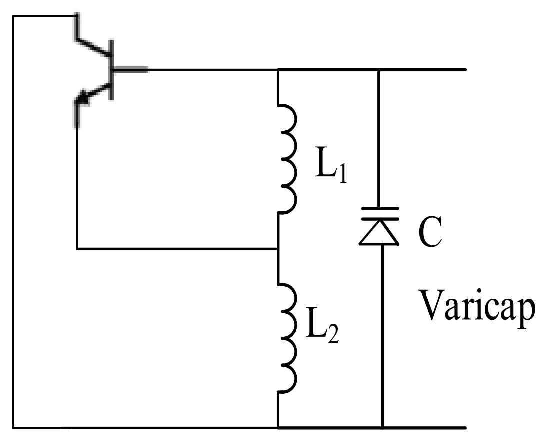
2- Direct Method

Using simple LC oscillator by varying either C or L, depending on the message signal, the frequency generated is ω_c. e.g. Hartly oscillator.

$$\omega_o = \frac{1}{\sqrt{LC}}, \quad L=L_1+L_2$$

$$c = c_o + kf(t)$$

$$\omega_o = \frac{1}{\sqrt{LC_o\left[1+\frac{kf(t)}{C_o}\right]}}$$



$$\omega_o = \frac{1}{\sqrt{LC}\left[1+\frac{kf(t)}{c_o}\right]^{1/2}} \cong \frac{1}{\sqrt{LC_o}}\left[1+\frac{kf(t)}{2C_0}\right], \qquad \text{when } \frac{kf(t)}{C_0} \ll 1$$

Using binomial $(1+x)^n \cong 1 + nx \;\; for\; x \ll 1$

$$\omega_o = \omega_c\left[1+\frac{kf(t)}{2C_0}\right] \qquad ;\omega_c = \frac{1}{\sqrt{LC_o}}$$

$$\boxed{\omega_o = \omega_c + k_f f(t)} \quad \text{…. (5-29)}$$

$$; k_f = \frac{\omega_c k}{2c_o} \qquad \text{.... (5-30)}$$

Demodulation of FM Signals

1- Direct method (Discrimination)

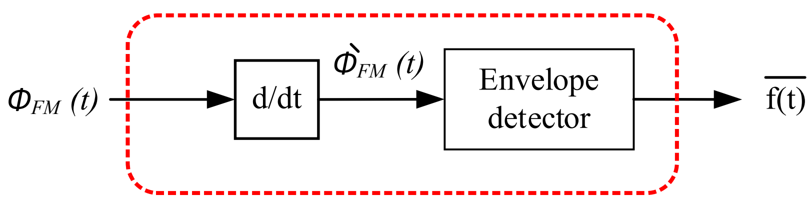
Using differentiator and envelope detector

$$\varphi_{FM}(t) = A_c cos\left[\omega_c t + k_f \int_{-\infty}^{t} f(\alpha)d\alpha\right]$$

$$\frac{d\varphi_{FM}(t)}{dt} = A_c[\omega_c + k_f f(t)]sin\left[\omega_c t + k_f \int_{-\infty}^{t} f(\alpha)d\alpha\right] \text{...(5-31)}$$

↑ Const. ↑ Const ↑ Const

If $k_f f(t) \ll \omega_c$, then envelope of $\varphi_{FM}'(t)$ is $A_c[\omega_c + k_f f(t)]$



Discriminator

In fact, the discriminator changes the FM signal into AM with only the slight difference that the new carrier frequency has some frequency variation.