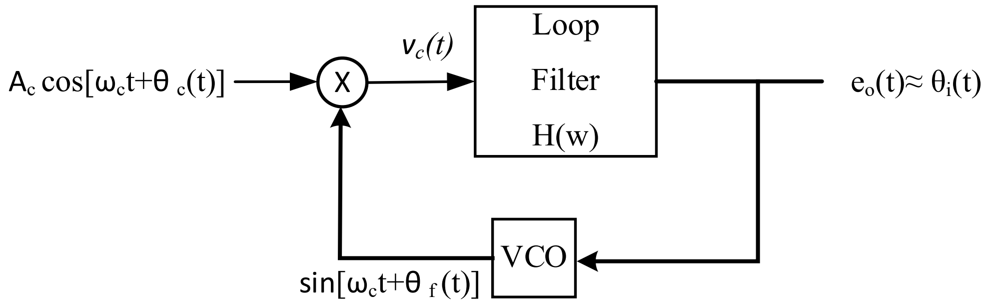
2- Indirect Method (Phase- Locked Loop (PLL))



$$e_o(t) = v_c(t)\otimes h(t)$$

$$\theta_i(t) = k_f \int_{-\infty}^{t} f(\propto) d \propto$$

$$\theta_f(t) = k_f \int_{-\infty}^{t} f(\propto) d \propto - \theta_e$$

$$\theta_e(t) = \theta_i'(t) = k_f f(t) \quad \dots \text{ (5-32)}$$

PLL is more effective than direct method in the presence of strong noise Low SNR

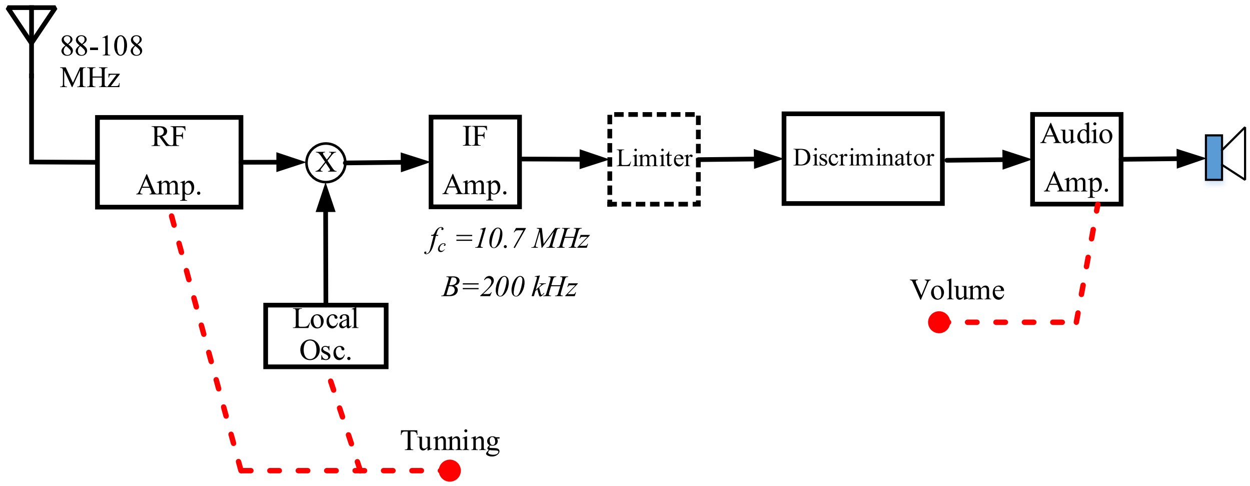
Commercial FM Broadcast Tx:

- Carrier frequency range: 88-108 MHz
- Spacing: 200 kHz
- Peak frequency deviation: 75 kHz (WBFM)
- Armstrong Modulator

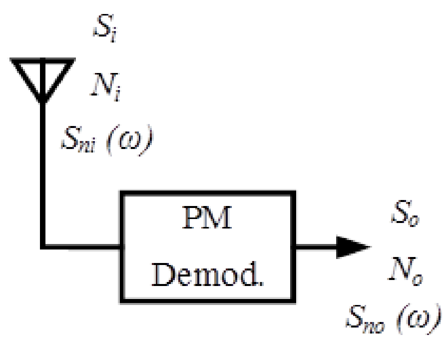
Commercial FM Broadcast Tx:

F_{IF}=10.7 MHz

Discrimination Demodulators.



Noise in Angle Modulated signals



PM

$S_i = \frac{A_c^2}{2}$ … (5-33)

$S_o = k_p^2 \overline{f^2(t)}$ …(5-34)

If the channel noise is white and double-sided PSD is $\eta/2$ watt/Hz. Then PSD at the demodulator output is:

$S_{no}(\omega) = \frac{\eta}{A_c^2} \quad \Longrightarrow \quad N_o = \frac{2\eta B}{A_c^2}$ …(5-35)

where B is the baseband filter BW (see ref.4)

$\frac{S_o}{N_o} = k_p^2 \overline{f^2(t)} \left(\frac{A_c^2/2}{\eta\, B} \right) = k_p^2 \overline{f^2(t)} \frac{S_i}{N_i}$, since $\Delta\omega = k_p f_p'$ then,

$$\frac{S_o}{N_o} = (\Delta\omega)^2 \frac{\overline{f^2(t)}}{f_p'^2} = \frac{S_i}{N_i} \quad \text{…(5-36)}$$

For tone modulation:

$$f(t) = A_m cos\omega_m t \quad \Longrightarrow f'(t) = -A_m \omega_m sin\omega_m t$$

$$\overline{f^2(t)} = \frac{A_m^2}{2} \Longrightarrow f_p'^2 = A_m^2 \omega_m^2$$

$$\frac{S_o}{N_o} = \frac{1}{2}\left(\frac{\Delta\omega}{\omega_m}\right)^2 \left(\frac{S_i}{N_i}\right) = \frac{1}{2}\beta^2 \left(\frac{S_i}{N_i}\right) \qquad \ldots(5\text{-}37)$$

FM

$$S_i = \frac{A_c^2}{2} \quad , \quad S_o = k_f^2 \overline{f^2(t)} \qquad \ldots(5\text{-}38)$$

‘

$$S'_{no}(\omega) = \frac{\mu\omega^2}{A_c^2} \Longrightarrow N_o = \frac{8\pi^2 \mu B^3}{3A_c^2} \qquad \ldots(5\text{-}39)$$

(see ref. 4)

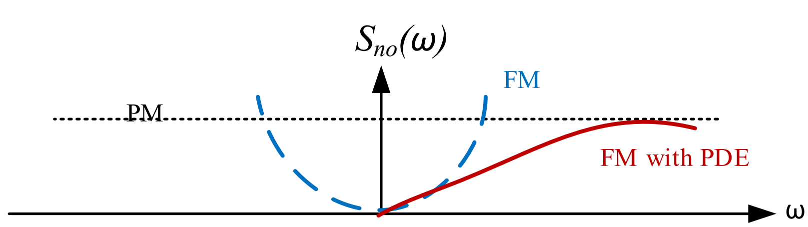
$$\frac{S_o}{N_o} = 3\beta^2 \frac{\overline{f^2(t)}}{f_p^2}\left(\frac{S_i}{N_i}\right) \qquad \ldots(5\text{-}40)$$

where $\beta = \frac{k_f f_p}{2\pi B}$

For tone modulation:

$$f(t) = A_m cos\omega_m t \quad \Longrightarrow \quad f_p = A_m$$

$$\frac{S_o}{N_o} = \frac{3}{2}\beta^2 \left(\frac{S_i}{N_i}\right) \qquad \ldots(5\text{-}41)$$



Ex 5-5:

An FM signal with 75 kHz deviation, has an input signal-to-noise ratio of 15 dB, with a modulating frequency of 10 kHz.

a) Find SNR_o at demodulator o/p.
b) Find SNR_o at demodulator o/p if AM is used with m=0.5
c) Compare the performance in case a) and b).

Solution:

a) FM

SNR_i=15 dB=31.6

Δf=75 kHz, f_m=10 kHz

$$\beta = \frac{\Delta f}{f_m} = \frac{75}{10} = 7.5$$

$$\frac{S_o}{N_o} = \frac{3}{2}\beta^2\left(\frac{S_i}{N_i}\right) = \frac{3}{2} * 7.5^2 * 31.6 = 2000 = 33.01\ dB$$

b) AM

$$SNR_o = \left(\frac{2m^2}{2+m^2}\right)\left(\frac{S_i}{N_i}\right) = \left(\frac{2 * 0.5^2}{2+0.5^2}\right)(31.6) = 3.51 = 5.45\ dB$$

$$\frac{(SNR_o)_{FM}}{(SNR_o)_{AM}} = \frac{1.5\beta^2(2+m^2)}{2m^2} = \frac{2000}{3.51} = 569.8$$

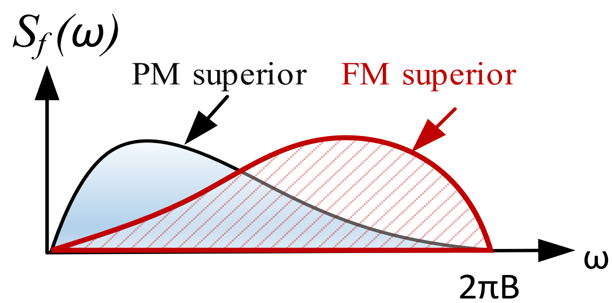
H.W

When $f(t) = 8cos\omega_o t + cos5\omega_o t$, show that for a given transmission bandwidth, the output SNR in PM is four times that of FM.

Comparing the expressions of output SNR for both FM and PM demodulators, we conclude the following.

- In wideband PM and FM, the output SNR increases as the square of transmission bandwidth, quadruples the output SNR (increases by 6 dB).
- For the modulation FM, yields three times as much SNR as does PM, but does not means that FM is superior to PM always (see previous homework). If $f(t)$ has a large peak amplitude and its derivative$f(t)'$ has a relatively small peak amplitude, PM tends to be superior to FM, for opposite conditions, FM tend to be superior to PM. If the PSD of $f(t)$ is predominantly concentrated at higher frequencies, the signal is a low frequency signal f_p' will tend to be larger, and FM will perform better than PM.

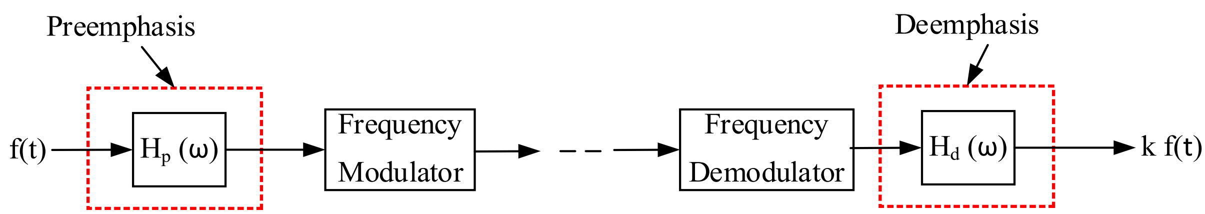
Most signals in real life, including voice and music have PSDs are at low frequencies; in such case PM is superior than FM.



It may seems logical for radio broadcast station to use PM rather than FM, but in fact the broadcast stations do not really FM, but are FM modified by preemphasis.

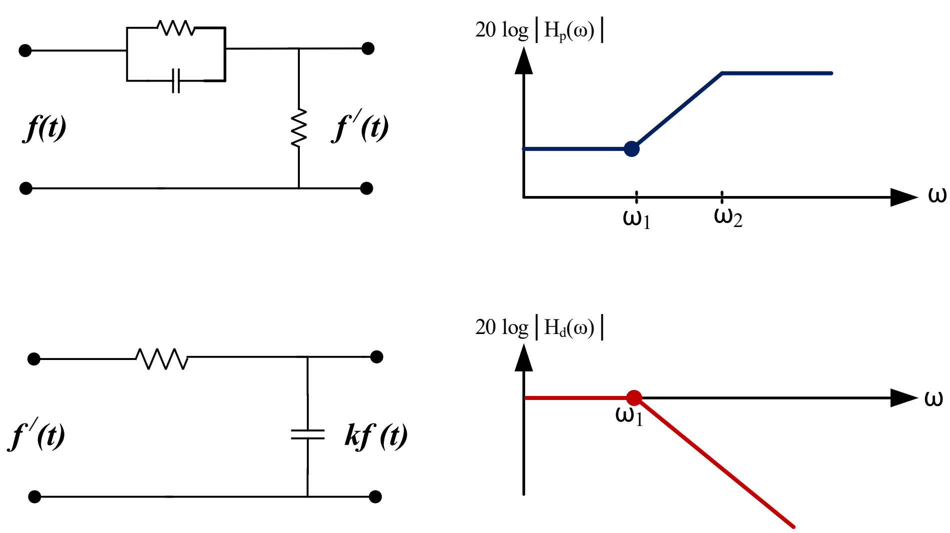
FM with Preemphasis and Deemphasis:

$H_p(\omega)$ is the preemphasis filter, which redistribute the PSD of the baseband signal. At Rx, the incoming signal is demodulated and deemphasized throw a complementary filter $H_d(\omega)$.



For radio broadcast, the baseband signal *f(t)* has a bandwidth of 15 kHz, even though the PSD of *f(t)* is concentrated within 2 kHz and is small beyond 2 kHz, the output noise on the other hand is parabolic. Hence, the noise is strongest in the frequency range where the signal is the weakest. If we boost the high frequency components of the signal at the transmitter (Preemphasis), we get back *f(t)* undistorted. However, the noise will be considerable weakened. This is because unlike *f(t)* the noise enters after the transmitter and is not boosted. It undergoes only deemphasis or attenuation of high frequency components, at the Rx. Because the noise PSD is parabolic, attenuation of high frequency component cuts down the noise significantly.

The Filters $H_p(\omega)$ and $H_d(\omega)$are shown below:



The frequency f_1 is 2.1 kHz and f_2 is typically 30 kHz or more, the preemphasis transfer function is:

$H_p(\omega) = k\frac{j\omega+\omega_1}{j\omega+\omega_2}$ …. (5-42)

Where k, is set at a value of ω_2/ω_1 , thus,

$H_p(\omega) = \left(\frac{\omega_2}{\omega_1}\right)\frac{j\omega+\omega_1}{j\omega+\omega_2}$ …(5-43)

For $\omega \ll \omega_1$; $H_p(\omega) = 1$

For frequencies $\omega_1 \ll \omega \ll \omega_2$; $H_p(\omega) = \frac{j\omega}{\omega_1}$

Thus, the preemphasizer acts as a differentiator at intermediate frequencies (2.1 to 15 kHz). This means that FM with PDE is FM over the modulating signal frequency range 0 to 2.1 kHz and is nearly PM over the range 2.1 to 15 kHz. The deemphasis filter H_d (ω) is given by:

$H_d(\omega) = \frac{\omega_1}{j\omega+\omega_1}$ … (5-44)

Note that for $\omega \ll \omega_2$, $H_p(\omega) \cong \frac{j\omega+\omega 1}{\omega_1}$. Hence,

$H_p(\omega)H_d(\omega) \approx 1$ over the baseband 0 to 15 kHz.

Ex 5-6:

Compute the improvement in SNR resulting from using FM with PDE rather than traditional FM.

Solution:

We observe that parabolic PSD of the output noise $S_{no}(\omega) = \frac{\eta\omega^2}{A_c^2}$ passes through a deemphasis filter

$$H_d(\omega) = \frac{\omega_1}{j\omega+\omega_1}$$

Thus N_o' , the noise power at deemphasis filter output is

$$N_o' = 2\int_0^B S_{no}(\omega)|H_d(\omega)|^2 df$$

$$= 2\int_0^B \frac{\eta\omega^2}{A_c^2} \times \frac{\omega_1^2}{\omega^2+\omega_1^2} df$$