The output SNR:

$$\frac{S_o}{N_q} = \frac{3L^2\overline{f^2(t)}}{f_p^2} \quad \text{Volt}^2 \qquad \text{.... (6-8)}$$

Note:

$$N_o = N_q\ ; \qquad \frac{S_o}{N_o} = \frac{S_o}{N_q}$$

- For tone modulation: $\overline{f^2(t)} = \frac{A^2}{2}$; $\quad f_p = A$

$$\frac{S_o}{N_q} = \frac{3L^2}{2} \qquad \text{... (6-9)}$$

$$\left(\frac{S_o}{N_q}\right)_{dB} = 1.76 + 20\ log\ L = 1.76 + 6.02\ n \qquad \text{... (6-10)}$$

Bandwidth Requirement of PCM

The information rate of PCM channel is nf_s bits/sec, if message bandwidth is f_{max} and the sampling rate is $f_s(\geq 2f_{max})$ then nf_s binary pulses must be transmitted per second.

Assuming the PCM signal is a low-pass signal of bandwidth BW_{PCM} , the required minimum sampling rate is $2BW_{PCM}$. Thus:

$$2BW_{PCM} = nf_s$$

$$BW_{PCM} = \frac{n}{2}f_s \geq nf_{max} \quad \text{Hz} \qquad \text{... (6-11)}$$

$$BW_{PCM_{minimum}} = nf_{max} \quad \text{Hz} \qquad \text{... (6-12)}$$

Ex 6-5:

In a binary PCM system, the output signal-to-quantization ratio is to be hold to a minimum of 40 dB. If the message is a single tone with f_m=4 kHz. Determine:

1- The number of required levels, and the corresponding output signal-to-quantizing noise ratio.

2- Minimum required system bandwidth.

Solution:

1) $L = 2^n$

$$\frac{S_o}{N_q} = 10000 = 40\ dB$$

$$\frac{S_o}{N_q} = \frac{3L^2}{2} \quad \text{(S.T)}$$

$$\therefore L = \sqrt{\frac{2}{3} * 10000} = [81.6] = 82$$

$$n = \log_2 82 = [6.36] = 7$$

$$\therefore L = 2^7 = 128$$

2) Minimum system bandwidth $= nf_{max}$=7*4 kHz=28 kHz

H.W:

Consider a single tone signal of frequency 3300 Hz. A PCM is generated with a sampling rate of 8000 sample/sec. the required output signal-to-quantizing noise ratio is 30 dB.

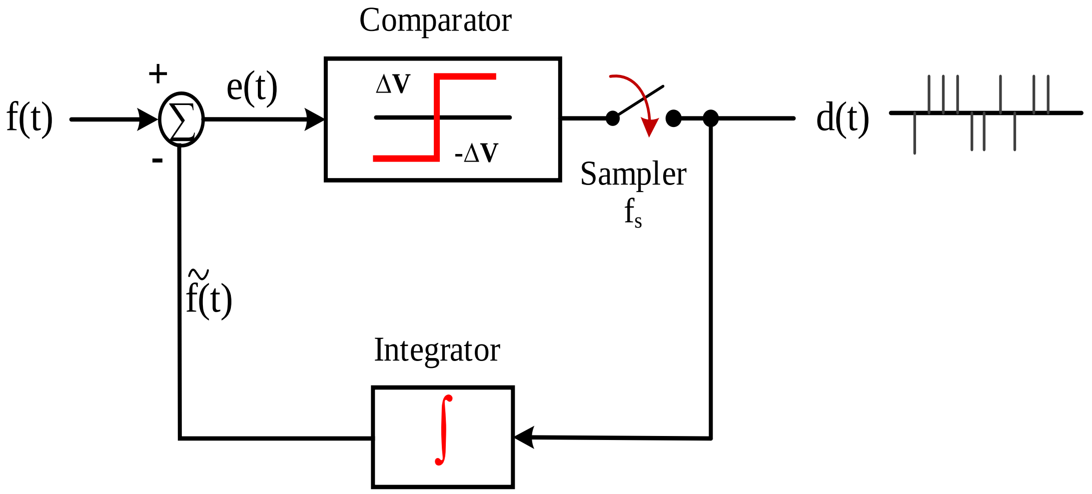
1) What the minimum number of uniform quantizing levels needed?. And what the minimum number of bits per sample needed?

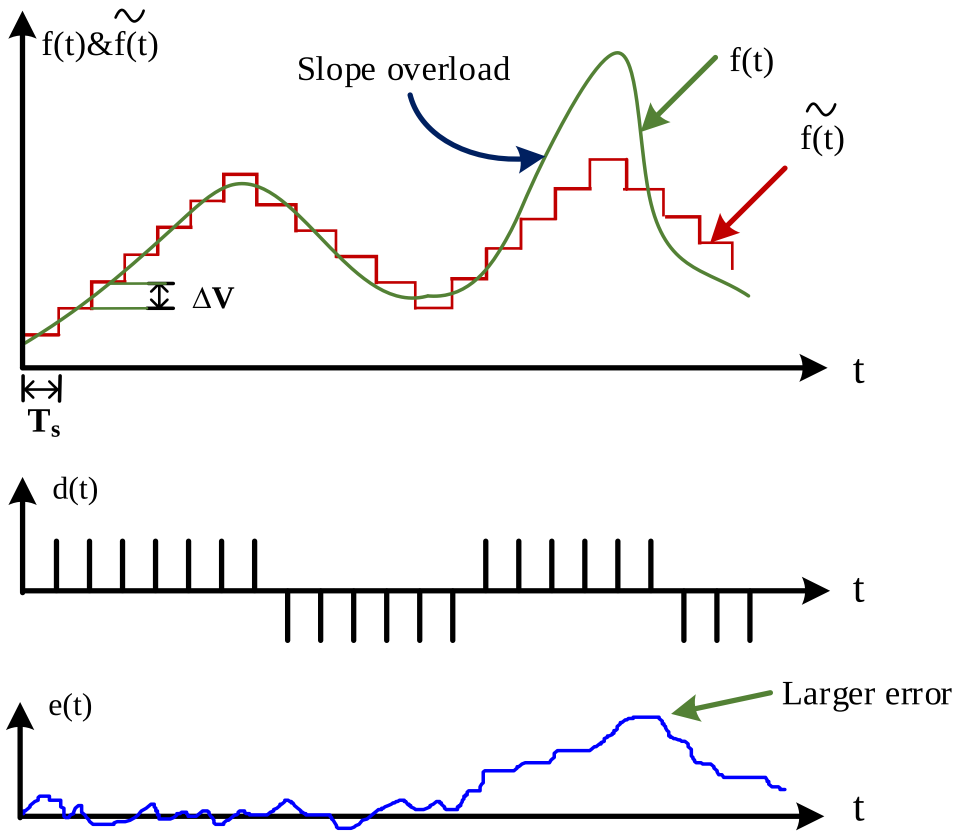
2) Calculate minimum system bandwidth required.

Ans: (a) 26.5 (b) 20 kHz

2- Delta Modulation:

It is a sampling way to convert analog signal into digital with reduced bandwidth





Its produces information about the difference between successive samples.

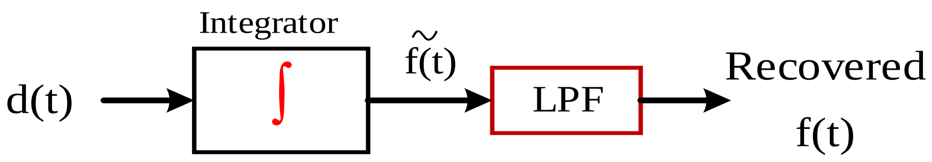
$e(t) = f(t) - \tilde{f}(t)$, where $\tilde{f}(t)$ is a stair case approximation of $f(t)$

The sampler with rate ($f_s \gg Nyquist\ rate$) produces pulse train $d(t)$ where:

$$d(t) = \Delta sgn[e(t)] = \begin{cases} \Delta V & e(t) > 0 \\ -\Delta V & e(t) < 0 \end{cases}$$

$d(t)$ represents the derivative of $f(t)$

The demodulator will integrate $d(t)$ to produce $f_s(kT_s)$ smoothed by LPF with BW of f_{max}



Slope overload problem:

Due to finite step size ΔV of integrator and if the slope of $f(t)$ is Larger than $\tilde{f}_s(t)$ will not track $f(t)$ in its value [$\left(\tilde{f}_s(t)\right)$ and $f(t)$ will diverge from each other]. This will produce distortion at Rx side when $d(t)$ is used to construct $\tilde{f}_s(t)$.

To avoid slope overload, the step size must be kept such that:

$$\left|\frac{df(t)}{dt}\right|_{max} < \Delta V . f_s \qquad \text{... (6-13)}$$

For single tone case $f(t) = A_m cos\omega_m t$

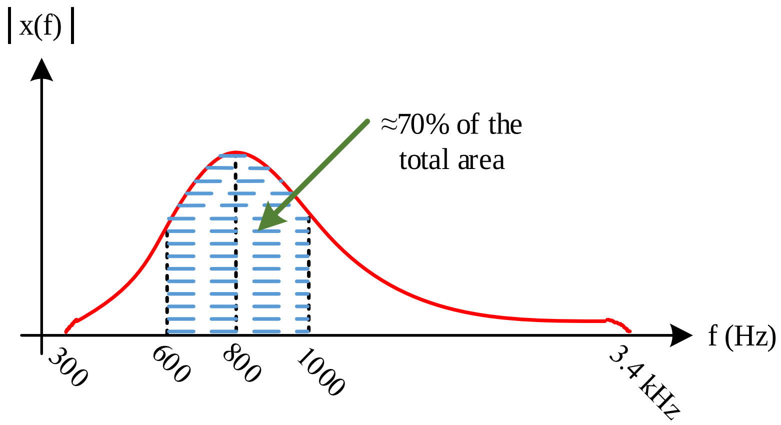
$\left|\frac{df(t)}{dt}\right|_{max} = A_m\omega_m$, therefore

$$\Delta V_{min} = \frac{A_m\omega_m}{f_s} \qquad \text{... (6-14)}$$

- For speech signal, the typical frequency analysis show that about 70% of total energy lies between 600 and 1000 Hz indicating that peak energy is located that almost at frequency of 800 Hz called response frequency f_r=800 Hz, then we could assume ΔV_{min} for speech to be:

$$\Delta V_{min} = \frac{2\pi(800)A_m}{f_s} \quad \text{... (6-15)}$$

where f_p in the maximum amplitude of the speech signal.



Quantizing Error:

Assuming quantizing error is equally likely in the interval (-ΔV,ΔV)

$$N_q = \frac{B}{f_s}.\frac{(\Delta V)^2}{3} \quad \text{... (6-16)}$$

Where B is the preconstruction filter bandwidth

Output Signal to Noise Ratio:

$$\frac{S_o}{N_q} = \frac{3f_s\overline{f^2(t)}}{(\Delta V)^2 B} \quad \text{... (6-17)}$$

For single tone message $f(t) = A_m cos\omega_m t$

$$\frac{S_o}{N_q} = \frac{3{f_s}^2}{8\pi^2 {f_m}^2 B} \quad \text{... (6-18)}$$

Ex 6-6:

A DM has sampling frequency of 64 kHz is used to encode speech signal of ±1 volt:

1- Find minimum step size to avoid step overloading.

2- Find SNR_q assuming speech has uniform probability density function (PDF) over the interval [-1, 1] volt.

Solution:

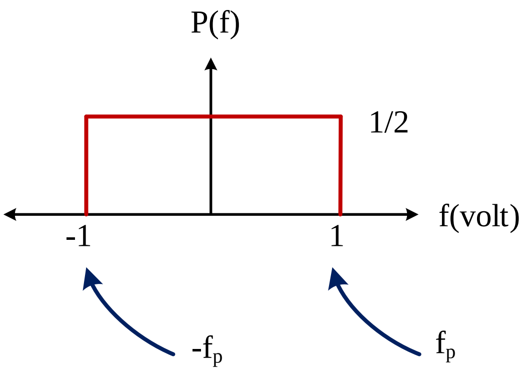
1. For speech signal $\Delta V_{min} = \frac{2\pi(800)f_p}{f_s}$

$$\Delta V_{min} = \frac{2\pi(800)(1)}{64000} \cong 78\ mV$$

2. $\frac{S_o}{N_q} = \frac{3f_s\overline{f^2(t)}}{(\Delta V)^2 B}$

$$\overline{f^2(t)} = \int_{-1}^{1} f^2 p(f) df$$

$$= 2\int_0^1 \frac{1}{2} f^2 df = \frac{1}{3}$$



$$\frac{S_o}{N_q} = \frac{(3)(64000)(1/3)}{(0.078)^2(3400)} \cong 35\ dB$$

Note:

Compare this result of 35 dB with PCM at 64000 bps $(f_s = 8\ kHz,\ n = 8\ bits/sample)$ then $SNR_q \cong 48\ dB$. i.e PCM is better than DM for the same bit rate.

H.W:

A DM system is designed to operate at 3 times the Nyquist rate for the signal with a 3 kHz bandwidth. The quantization step size is 250 mV. Determine:

a) Maximum amplitude of a 1 kHz input sinusoid for which the delta modulator does not show slop over load.

b) The post filter output signal-to-quantizing noise ratio for the signal in part a.