received x(t) is "0" or "1" according to the decision rules:

if y(Tb) > Vth then decide the received x(t) is a "1".

if y(Tb) < Vth then decide the received x(t) is a "0".

The problem is to find the best choice of Vth such that the overall net error probability (average error prob) of "1" & "0" is minimum. This optimum Vth depends on type of the binary signals and the PDF of n(t).

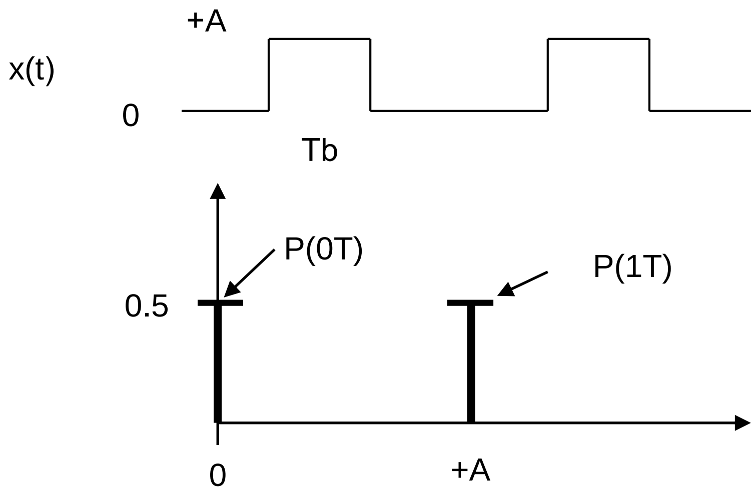
In this course, we will assume that the noise is AWGN with PDF:

$$f(n) = \frac{1}{\sqrt{2\pi}\ \sigma}\ e^{-\frac{n^2}{2\sigma^2}}$$

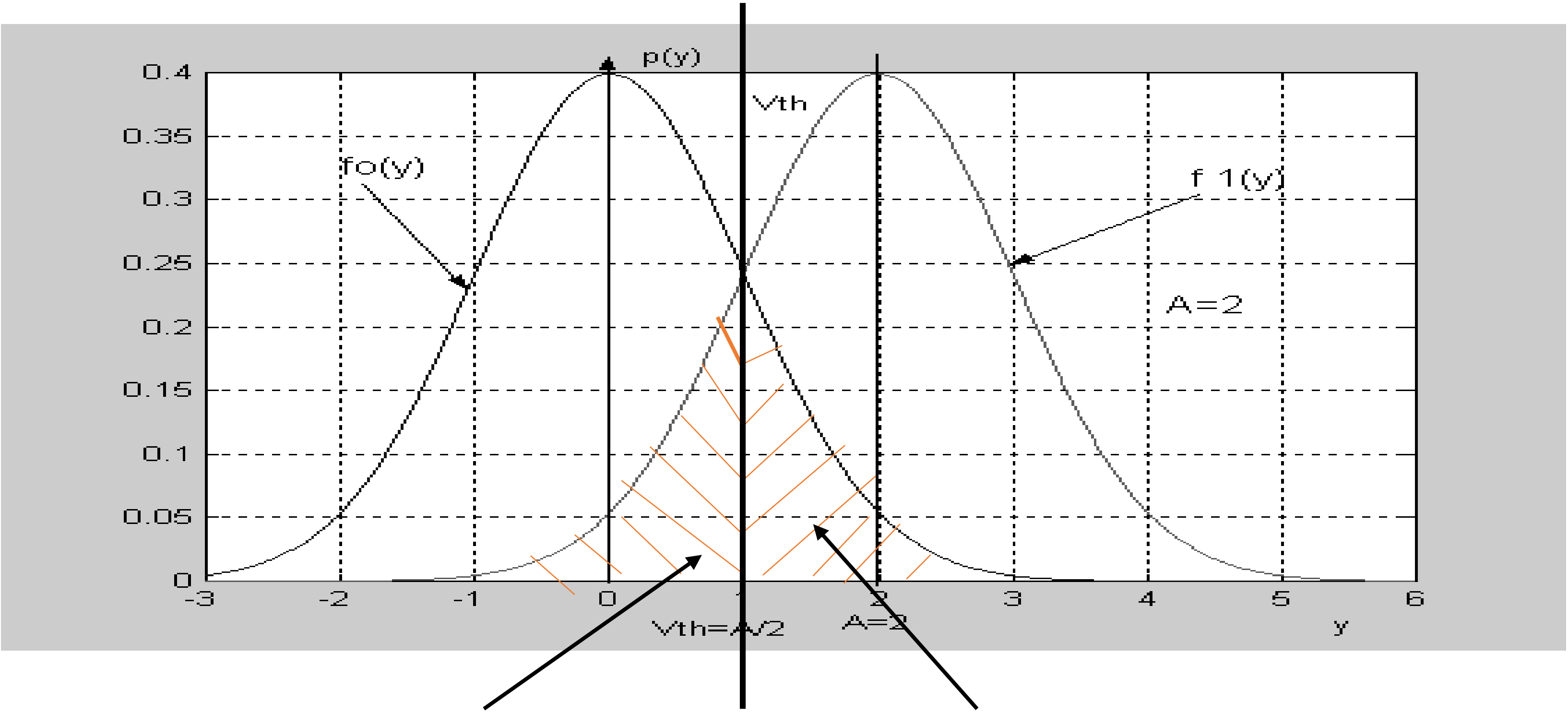
and the binary signals “0” and “1” are equiprobable. First, we will take the simplest case of unipolar signals

Unipolar binary signals:

First we discuss the simplest case if x(t)=0 volt for logic"0" and x(t)=+A volt for logic"1" over the bit duration Tb.



Note that for noiseless case (y(t)=x(t)) then the PDF of y(t) is discrete similar to the PDF of x(t). When n(t)≠0, the PDF of y(t) will be the same as the PDF of n(t) (f(n)) but d.c. shifted by 0 volt or +A volts corresponding to x(t). Let f1(y) be the PDF of y when "1" is transmitted and fo(y) be the PDF of y when "0" is transmitted.



area=p(0R/1T) area=p(1R/0T)

If we assume equiprobable then the best(optimum) threshold is the mid-way between the “0” volt and +A volt which is +A/2 as shown.

$$p(0_R/1_T)=\int_{-\infty}^{Vth} f1(y)dy \quad and \quad p(1_R/0_T)=\int_{Vth}^{\infty} fo(y)dy$$

Note that since Vth is in the middle, then the areas representing the error prob. p(0R/1T) or p(1R/0T) are equal.

To find pe= p(0R/1T) = p(1R/0T), then, we find the area under the curve:

$$pe = \int_{Vth}^{\infty} fo(y)dy = \int_{A/2}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{y^2}{2\sigma^2}} dy$$

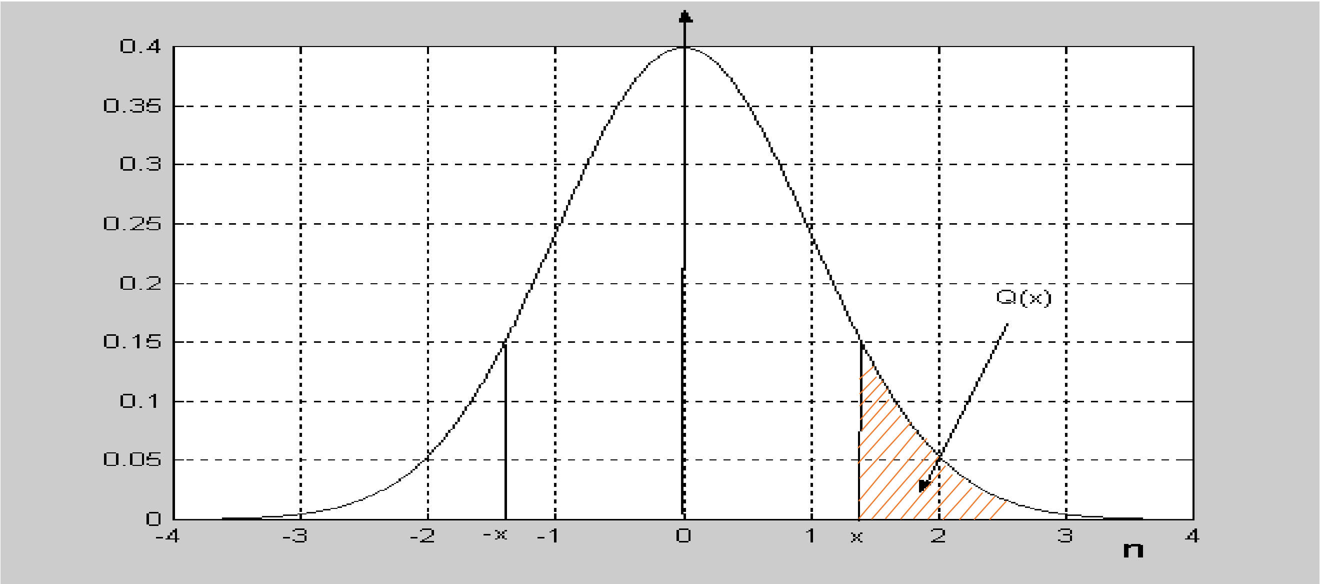
Note that this definite integral can not be evaluated analytically. The following substitution will help to evaluate this integral numerically:

Let $z=y/\sigma$, then $dy=\sigma dz$.

For y=A/2, then z=A/(2σ), and for y→∞, then z→∞. Changing the variable and the limits of this integral reduces it into:

$$pe = \int_{\frac{A}{2\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz = Q(\frac{A}{2\sigma})$$

where Q(x) is a function called Marcum function which gives the area under the standard Gaussian curve (σ=1, μ=0) from x=(A/2σ) to ∞.



Red area = Q(x)

$$Q(x) = \int_{x}^{\infty} p(z)dz = \int_{-\infty}^{-x} p(z)dz$$

This Q(x) is numerically evaluated and tabulated for x<3, or can be approximated for large x (say x>2) as:

$$Q(x) \approx \frac{1}{\sqrt{2\pi}\, x}(1 - \frac{0.7}{x^2})e^{-0.5x^2}$$

Example: Equiprobable binary signals are represented by 0 & 2 volts. For AWGN with $\sigma^2 = 0.16$ volts^2, find the optimum threshold setting and the net error prob.

Solution: For equiprobable case with unipolar signals affected by AWGN, then Vth=A/2(mid-way) =2/2=1 volt,

and σ^2=0.16, then σ=0.4 hence pe=Q(A/2σ)=Q(2/2*0.4)=Q(2.5)

$$Pe=Q(2.5) \approx \frac{1}{\sqrt{2\pi}\ 2.5}\left(1-\frac{0.7}{6.25}\right)e^{-0.5*6.25} \approx 6.2*10^{-3}$$

Note: In general, for equiprobable signals A1 volts and A2 volts and for AWGN, then:

$Vth|_{op}=(A_1+A_2)/2$ (mid-way between A1 and A2)

and $p(0_R/1_T)=p(1_R/0_T)=pe=Q[d/(2\sigma)]$, where $d=A_2-A_1$

Where d is the absolute distance between A1 and A2