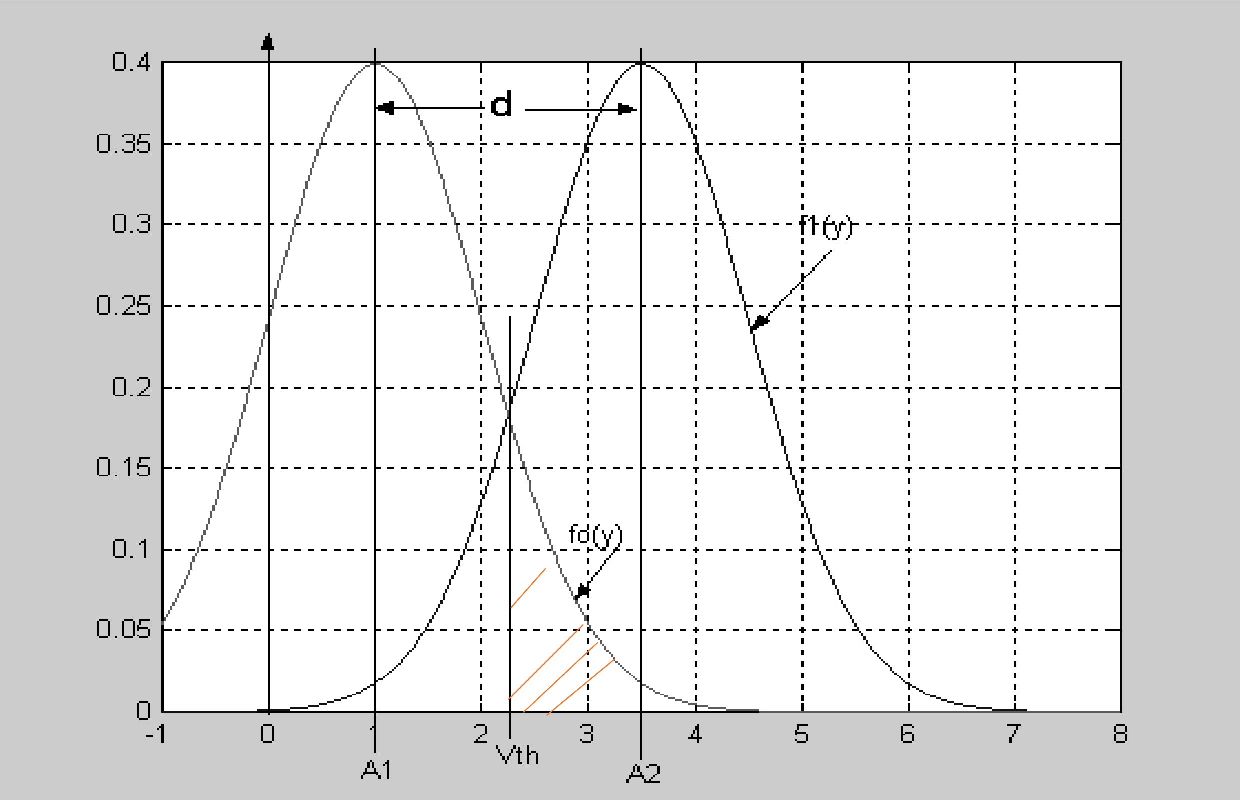


red area=pe

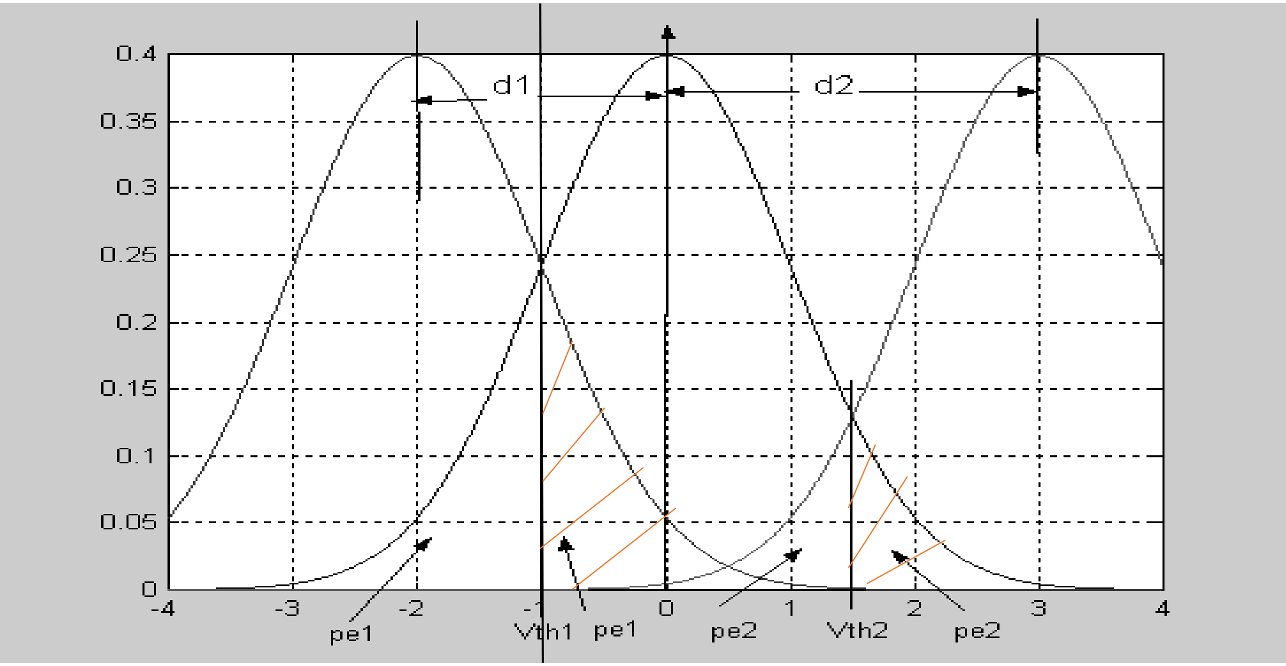
Example: Bipolar equiprobable binary signals ± A are affected by AWGN with variance σ^2, find the threshold and the error prob.

solution: Here, we have A1=-A and A2=+A then vth=(+A+(-A))/2=0 volt and pe=Q[d/(2σ)], where d= +A-(-A)=2A,hence:pe=Q[2A/(2σ)]=Q[A/ σ],

Example: The three equiprobable signals +3A, 0, -2A are affected by AWGN with σ=0.4A. Find the values of the thresholds and the overall net error prob.

Solution:

Here, we have two thresholds, one Vth1 that Separates -2A from 0 volts And Vth2 that separates +3A from 0 volts.



Since equprobable, then Vth1=(-2A+0)/2=-A volts and Vth2=(+3A+0)/2=1.5A .

To find the error prob, we find d1=2A (absolute distance between -2A and 0 volts) and d2= 3A (absolute distance between +3A and 0 volts) also we will assume -2A volt for "0" logic state , 0 volt for "1" logic state and +3A volts for "2" logic state, then:

$pe1=p(0_R/1_T)=p(1_R/0_T)$= errors between -2A and 0 volts, then:

$pe1=Q[d1/(2\sigma)]=Q[2A/(2*0.4*A)]=Q(2.5)=6.2*10^{-3}$

$pe2=p(2_R/1_T)=p(1_R/2_T)$= errors between +3A and 0 volts, then:

$pe2=Q[d2/(2\sigma)]=Q[3A/(2*0.4*A)]=Q(3.75)=8.9*10^{-5}$

net error prob=p(0T) p(1R/0T)+ p(1T) p(0R/1T)+p(1T) p(2R/1T)+ p(2T) p(1R/2T)

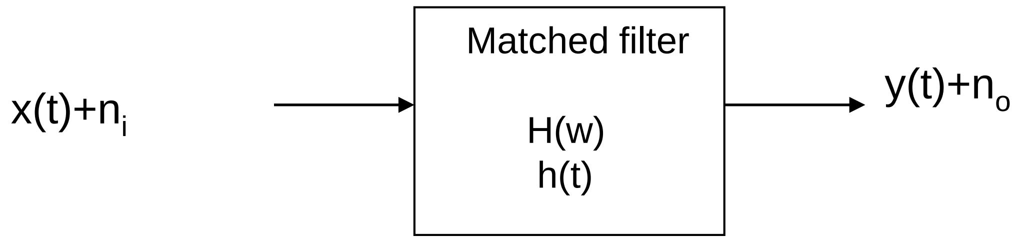
$=(1/3)*pe1+(1/3)*pe1+(1/3)*pe2+(1/3)*pe2$

$=(2/3)*pe1+(2/3)*pe2\approx 4.19*10^{-3}$

Where p(0T)=p(1T)=p(2T)=1/3 (equiprobable)

Matched Filter

<u>Detection problem:</u> Detection of the signal x(t) embedded in AWGN noise n(t) will produce the output y(Tb) which was the signal used at the decision block (comparator block). The device that carries out the function of detection is called a *Matched Filter* . This matched filter is simply a linear system with impulse response h(t) (or frequency response H(ω)) that acts as a filter. The job of this filter is to maximize the ratio $\frac{|y(Tb)|^2}{n_o^2(t)}$.



This filter is called "*Matched* " since for a certain signal x(t), there exists a filter with impulse response h(t) matched to it (maximizes the above ratio).

After long derivation, the following two equations are obtained:

1- h(t)=x(Tb-t).

Which gives the impulse response h(t) of a matched filter matched to the signal x(t). Hence the impulse response of a matched filter matched to the signal x(t) is the negative time of x(t) shifted by Tb.

2-

$$\left[\frac{|y(Tb)|^2}{n_o^2(t)}\right]\max = \frac{E}{(\eta_o/2)}$$

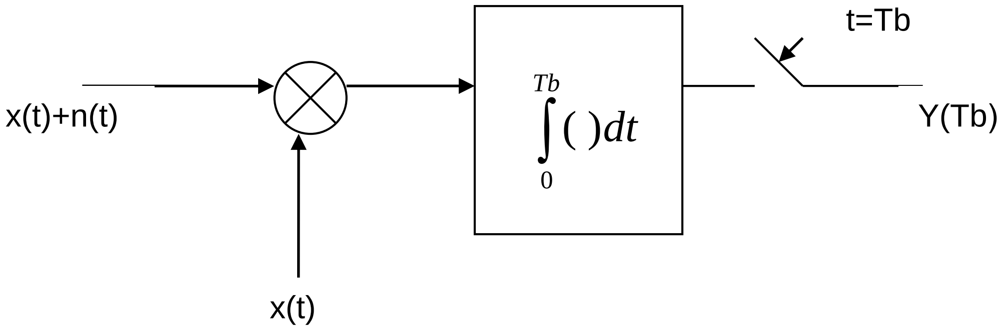
Which gives Max SNR at matched filter output. where

η_o=one-sided AWGN spectral density (Watt/Hz) and

$and\ E = \int_0^{Tb}[x(t)]^2$ dt= energy of the signal over bit duration Tb.

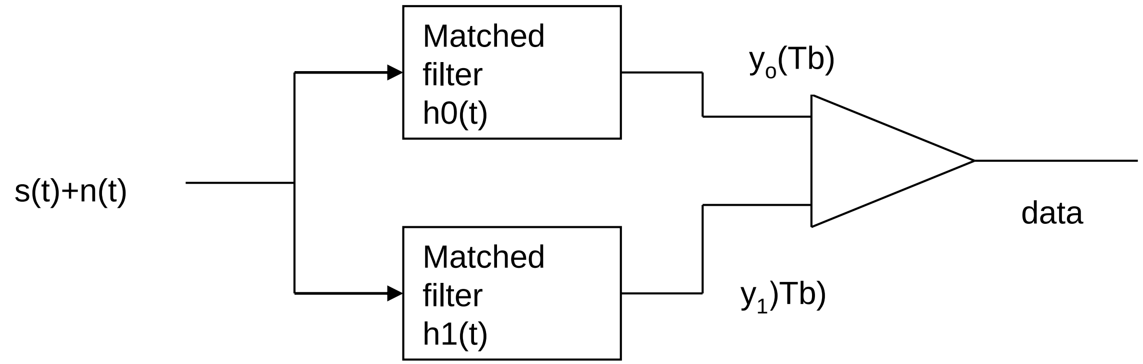
Practical Implementation of the Matched filter:

The practical implementation of a matched filter can be deduced as the product of the input x(t)+n(t) and x(t) then integrate the result over t=0 to t=Tb. The figure below shows the practical implementation of the matched filter:



Error probability of binary signal detection using matched filter:

Assume that the matched filter is used to detect the binary signal waveforms $s_o(t)$ and $s_1(t)$ embedded in AWGN. Here, we need two matched filters, one matched to $s_o(t)$ with impulse response $h_o(t)=s_o(Tb-t)$ and the other matched to $s_1(t)$ with impulse response $h_1(t)=s_1(Tb-t)$



Where: $E_o = \int_0^{Tb} s_o^2(t)dt$ = energy of $s_o(t)$ =o/p of $h_o(t)$ if s(t)= $s_o(t)$ and

$E_1 = \int_0^{Tb} s_1^2(t)dt$ = energy of $s_1(t)$ =o/p of $h_1(t)$ if s(t)= $s_1(t)$ also:

$E_{o1} = \int_0^{Tb} s_o(t)s_1(t)dt$ =cross energy between $s_o(t)$ and $s_1(t)$

=o/p of $h_o(t)$ if s(t)= $s_1(t)$

= o/p of $h_1(t)$ if s(t)= $s_o(t)$