For equiprobable $p(0_T)=p(1_T)=0.5$, and for Gaussian case:

$$pe = Q\left(\sqrt{\frac{\int_0^{Tb}[s_o(t)-s_1(t)]^2dt}{2\eta_o}}\right) = Q\left(\sqrt{\frac{E_o+E_1-2E_{01}}{2\eta_o}}\right) \quad \text{-------------(1)}$$

<u>Error prob for digital carrier systems using matched filter (coherent detection):</u>

<u>1- ASK (OOK) (ON-OFF) keying:</u>

Here the two signals are $s_o(t)=0$, $s_1(t)=A\cos\omega t$ over bit duration Tb. We use the general equation:

$$pe= Q\left(\sqrt{\frac{E_o+E_1-2E_{01}}{2\eta_o}}\right)$$

where for OOK, $E_o = E_{01} = 0$ (since So(t)=0) , and:

$E_1 = (A^2 Tb)/2$ ($s_1(t)$ is a sinusoid whose normalized power=$A^2/2$), putting E_o, E_{01} and E_1 then:

$$pe = Q(\sqrt{\frac{A^2 Tb}{4\eta_o}})$$

and express in terms of average signal power, then:

$S = [0.5E_o + 0.5E_1]/Tb = A^2/4$, then: $pe = Q(\sqrt{\frac{S}{\eta_o Rb}})$ and if $\lambda = \frac{S}{\eta_o Rb} = \frac{E_b}{\eta_o} = energy/bit$

Then: $pe = Q(\sqrt{\lambda})$

<u>2-BPSK signals (Binary Phase Shift Keying):</u>

Here the two signals are $s_o(t) = -A\cos\omega t$, $s_1(t) = A\cos\omega t$ over bit duration Tb. Again, using the same general equation, then:

$E_o = E_1 = [A^2 Tb]/2$ since both $s_o(t)$ and $s_1(t)$ are two sinusoids, also:

$E_{01} = \int_0^{Tb} -A^2 \cos^2 \omega t \, dt = -[A^2 Tb]/2$.

Also $S=[0.5E_o+0.5E_1]/Tb=A^2/2$=average signal power, then one can show that after putting E_o, E_1 and E_{01} in the general equation:

$$pe = Q(\sqrt{\frac{2S}{\eta_o Rb}}) = Q(\sqrt{2\lambda})$$

which is better than OOK

3-FSK (frequency shift keying):

Here the two signals are $s_o(t)=A\cos\omega_1 t$, $s_1(t)=A\cos\omega_2 t$ over bit duration Tb. Note that pe depends also on

$$\Delta\omega = \omega_d = \omega_2 - \omega_1$$

After a similar longer derivation, one can show that:

$$pe = Q(\sqrt{\lambda(1 - \frac{\sin \omega_d Tb}{\omega_d Tb})})$$

which is a general formula used for matched filter detection of FSK signals.(where $\lambda = \frac{S}{\eta_o Rb} = \frac{E_b}{\eta_o} = energy/bit$ as before)

Special cases in FSK:

1-if $\omega_d Tb=\pi, 2\pi, 3\pi,..$, or f_d=0.5Rb, Rb, 1.5Rb,... this gives $E_{o1}=0$ (orthogonal FSK), and $pe=Q(\sqrt{\lambda})$ (similar to OOK)

2-if $\omega_d Tb$=4.49rad, or f_d=0.715Rb, then: $\frac{\sin\omega_d Tb}{\omega_d Tb}=-0.217$

And this gives $pe=Q(\sqrt{1.217\lambda})$ which is the best (optimum)performance of FSK.

(Note: Use the general equation $pe=Q(\sqrt{\lambda(1-\frac{\sin\omega_d Tb}{\omega_d Tb})})$ for FSK in solving problems, do not mix with above special cases.)

Example: Matched_filter detection is used to detect BPSK signals at a rate of 600bps. If transmitted power is 5KW over an HF channel having estimated path losses of 150dB, find the error prob if the noise at detector input has one sided spectral density of 10^{-15} Watt/Hz.

- Solution:

P_T =trans power=5000 W , P_T = 10 $Log_{10}(5000)$ = 36.989 dB

then, P_R=S= P_T - P_{Loss} =36.989 – 150 = -113.01 dB = $10^{(-113.01/10)}$

S=5*10^{-12} W=average signal power.

$$\lambda = \frac{S}{\eta_o Rb} = \frac{5*10^{-12}}{10^{-15}*600} = 8.333 \text{ then } \quad pe = Q(\sqrt{2\lambda}) = Q(\sqrt{2*8.3333}) = Q(4.08) \cong 2*10^{-5}$$

Example: Repeat previous example for FSK signals with f_1=700Hz, f_2=2000Hz.

Solution:

Here ω_d=2π(2000-700)=2600π, and ω_d Tb=2600π/600=4.3333π, then and for the same λ=8.333:

$$pe = Q(\sqrt{\lambda(1-\frac{\sin\omega_d Tb}{\omega_d Tb})}) = Q(\sqrt{8.333(1-\frac{\sin 4.333\pi}{4.333\pi})}) = Q(2.84) \cong 2.3*10^{-3}$$

Which is worst than BPSK for the same λ.(remember that both BPSK and FSK have the same average power of A^2 /2, but the performance of BPSK is better)

Q1: A signal g (t), containing three frequencies is given by:

$g(t) = 10\cos 150\pi t + 5\, cos300\ \pi t,$

This signal is required to be transmitted using pulse techniques. Determine the Nyquist rate of sampling.

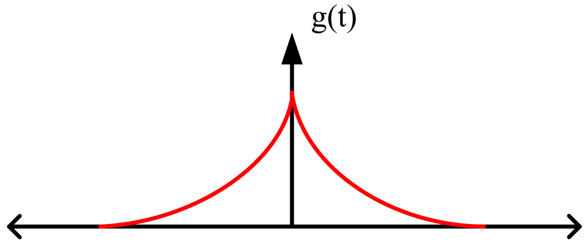
Ans: 300Hz.

Q2: The signal g (t), defined by the relation:

$g(t) = 10\cos 100\pi t \cos 200\pi t$ is required to be transmitted using pulse-modulation techniques. Determine the Nyquist rate of sampling.

Ans: 300 Hz.

Q3: The signal shown below, is not bandlimited, but it can be approximated to be band – limited signal. Assuming a suitable criterion, for approximation, determine the corresponding minimum sampling rate, with $g(t) = e^{-2|t|}$.



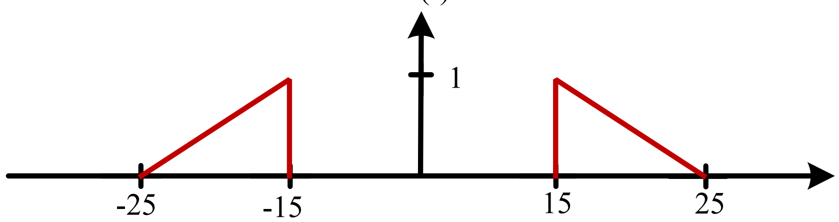
Q4: Determine the minimum transmission bandwidth required to transmit the following two signals:

$f_1(t) = 10\cos 150\ \pi t + 15\cos 200\ \pi t$ and $f_2(t) = 10\cos 100\ \pi t\cos 200\ \pi t$

in a PAM/TDM system. Compare the result if FDM is used with AM& SSB techniques.

Ans: 300Hz.

Q5: Sketch the spectrum of $g(t)$ if it is sampled at the following sampling frequencies: 25 KHz, 50 KHz and 60 KHz. Indicate if and how the signal can be recovered at each sampling freq.



Q6: Binary channel with bit rate $R_b = 36000$ bits/sec is available for PCM voice transmission. Find appropriate values of the sampling rate $\mathbf{f_s}$, the quantizing level **L**, and binary digits **n** assuming $\mathbf{f_{max}}$= 3.2 KHz.

[redacted]

n=5 , L=32 fs=7200 Hz

Q7: An analog signal is quantized and transmitted by using PCM system. If each sample at the receiving end of the system must be within ± 0.5 percent of the peak -to-peak full-scale value. How many binary digits must each sample contain?

Ans: 7

Q8: 24-speech signal are modulated using pulse code modulator (PCM) of 256 quantizing levels then multiplexed in time. Determine minimum possible transmission bandwidth.

Q9: Determine the output SNR in a DM system for a 1 kHz sinusoid, sampled at 32 kHz, without slope overload, and followed by a 4 kHz post reconstruction filter.

Ans: 24.9 dB.

Q10: It is required to transmit a sinusoidal signal of 1- volt amplitude and 800 Hz by delta modulation. Determine the minimum step size so that the slope – overload distortion is avoided, if the sampling rate is 4000 samples per second.

Ans: 0.1256 volt.

Q11: A signal $f_1(t)$ is band – limited to 3.6 kHz, and three other signals $f_2(t), f_3(t)$ *and* $f_4(t)$, are band – limited to 1.2 kHz each. These signals are to be transmitted by means of time – division multiplexing.

(a) Set up scheme for accomplishing this multiplexing requirement, with each signal sampled at its Nyquist rate.

(b) What must be the speed of the commutator (in samples per second)?

(c) If the commutator output is quantized with L = 1024 and the result is binary-coded, what is the output bit rate?

(d) Determine the minimum transmission bandwidth of the channel.

Ans: (b) 14400 samples/sec. (c) 144 kb/sec. (d) 7.2 kHz.

Q12: Consider a binary sequence with a long sequence of 1s followed by a single "0" and then a long sequence of "1s" as shown below. Draw waveforms for this sequence, using the following formats:

(a)Unipolar RZ signaling.

(b) Bipolar NRZ signaling.

(c) Spilt – phase (Manchester) signaling.

Binary sequence: 11111011111