Mustansiriyah University

College of Engineering

Electrical Engineering Dept.

Fundamentals of Logic Circuits Laboratory

First Year

Object: To study the axioms defining Boolean algebra and how to represent Boolean expressions in SOP and POS forms.

Theory:

Digital systems are composed of combinations logic gates described by a truth table and Boolean expression or a logic symbol diagram.

The fundamental Boolean operations of AND, OR and NOT can be summarized as follows:

$A + B = B + A$	$A \cdot B = B \cdot A$
$A + (B + C) = (A + B) + C$	$A \cdot (B \cdot C) = (A \cdot B) \cdot C$
$A \cdot (B + C) = A \cdot B + A \cdot C$	$A + B \cdot C = (A + B) \cdot (A + C)$
$A + 0 = A$	$A \cdot 1 = A$
$A + 1 = 1$	$A \cdot 0 = 0$
$A + A = A$	$A \cdot A = A$
$A + \overline{A} = 1$	$A \cdot \overline{A} = 0$
$A + \overline{A} \cdot B = A + B$	$A \cdot (A + B) = A$
$A \cdot B + A \cdot C = A \cdot (B + C)$	$(A + B) \cdot (A + C) = A + B \cdot C$

In combination logic, the output of the circuit depends only on the inputs to the circuit. Combination logic problems are normally given in the form of logical statements or a truth table. To design and implement the problem, Boolean logical expressions equations are derived for the output logic function in terms of the binary variables representing the inputs. The logic expressions are given either in the form of a sum of products (SOP) of in the form of a product of sums (POS).

CANONICAL FORMS:

For the table shown below:

Table 1.1: Input and output for CANONICAL Forms.

Input X Y Z	Output F
0 0 0	1
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	0
1 1 0	1
1 1 1	1

We can derive the logical expression for the function F:

$$F = \overline{X} \cdot \overline{Y} \cdot \overline{Z} + \overline{X} \cdot Y \cdot \overline{Z} + \overline{X} \cdot Y \cdot Z + X \cdot Y \cdot \overline{Z} + X \cdot Y \cdot Z$$

This expression is called the canonical sum of product. A product term which contains each of the n-variables factors in either complemented or uncomplemented form called minterm. So, F can be put in other form such as:

$$F = \sum 0,2,3,6,7$$

Since F=1 in rows 0, 2, 3, 6, 7

A logical equation can also be expressed as a product of sums. This done by considering the combinations for which F=0. From the truth table F=0 in rows 1, 4, 5 hence:

$$\overline{F} = \overline{X} \cdot \overline{Y} \cdot Z + X \cdot \overline{Y} \cdot \overline{Z} + X \cdot \overline{Y} \cdot Z$$

$$F = \overline{\overline{F}} = \overline{\overline{X} \cdot \overline{Y} \cdot Z + X \cdot \overline{Y} \cdot \overline{Z} + X \cdot \overline{Y} \cdot Z}$$

$$F = (X + Y + \overline{Z}) \cdot (\overline{X} + Y + Z) \cdot (\overline{X} + Y + \overline{Z})$$

The product of sums can be expressed as:

$$F = \prod 1,4,5$$

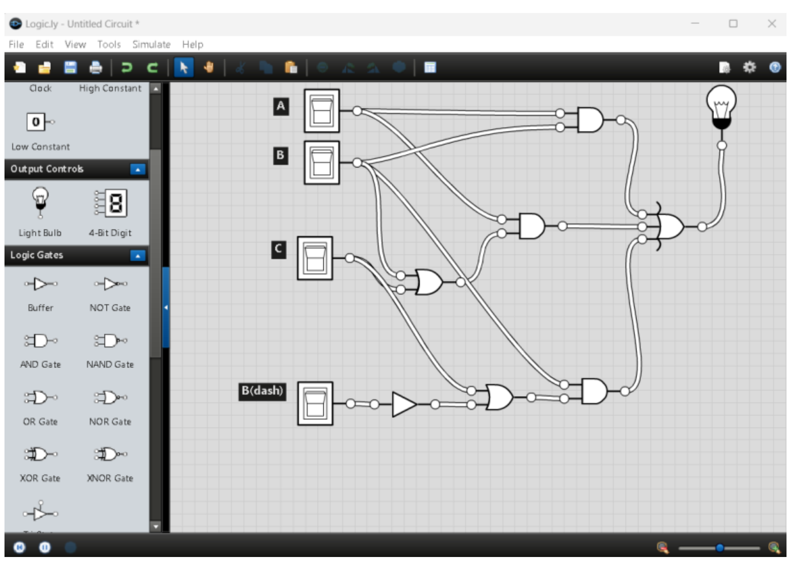
A sum which contains each of n variables complement not is called a maxterm.

Procedure:

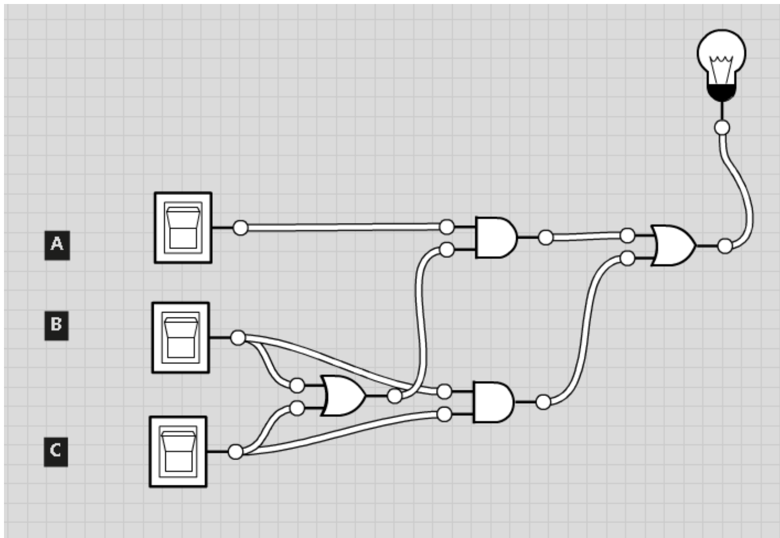
Using Boolean algebra to simplify the following expressions. Then find the truth table of each after connecting the circuits.

As presented below, use (A, B, C) as inputs and F as the output. Connect toggle switches, light bulb, and different logic gates to implement the following Boolean expressions:

a. $F_1 = A \cdot B + A \cdot (B + C) + B \cdot (\bar{B} + C)$
b. $F_2 = (A + \bar{B} + A \cdot B) \cdot (\bar{A} + \bar{B})$
c. $F_3 = (A \oplus B) \oplus (B \oplus C)$
d. $F_4 = (A \odot B) \oplus (B \odot C)$



(a)



(b)

Fig. 1.1. The logic diagram of Boolean function F_1: (a) before simplification; (b) after simplification.

Discussion:

1. Simplify the following logical expression and implement them using suitable logic gates.
   a. $F_1 = \sum 2,4,6,10,14$
   b. $F_2 = \prod 2,3,6$
2. Determine whether or not the following equalities correct:
   a. $A + B\cdot C + \bar{A}\cdot C = B\cdot \mathrm{C}$
   b. $\bar{B}(A\odot \mathrm{c}) + B\cdot\bar{C} + A(B\odot \mathrm{c}) = \overline{\bar{A}C}$
3. Convert the following expressions to SOP forms:
   a. $(A + \bar{B}\cdot C)\cdot B$
   b. $(A + C)(\bar{A}\cdot B\cdot\bar{C} + A\cdot \mathrm{C}\cdot \mathrm{D})$
4. Write a Boolean expression for the following statement :
   F is a "1" if A, B&C are all 1's or if only two of the variable is a"0".

5. Find **F** for the following Figure.