Newton-Raphson Method

The pipe network can also be analyzed using the Newton–Raphson method, where unlike the Hardy Cross method, the entire network is analyzed altogether. The Newton–Raphson method is a powerful numerical method for solving systems of nonlinear equations.

Suppose that there are three nonlinear equations to be solved

$$F1(Q1, Q2, Q3) = 0$$
$$F2(Q1, Q2, Q3) = 0$$
$$F3(Q1, Q2, Q3) = 0$$

Adopt a starting solution (Q1, Q2, Q3). Also consider that (Q1+ΔQ1, Q2+ΔQ2, Q3+ΔQ3) is the solution of the set equations that is

$$F1\,(Q1+\Delta Q1,\ Q2+\Delta Q2,\ Q3+\Delta Q3) = 0$$
$$F2\,(Q1+\Delta Q1,\ Q2+\Delta Q2,\ Q3+\Delta Q3) = 0$$
$$F3\,(Q1+\Delta Q1,\ Q2+\Delta Q2,\ Q3+\Delta Q3) = 0$$

Expanding the above equations as Taylor's series

$$F1 + \left(\frac{\partial F1}{\partial Q1}\right)\Delta Q1 + \left(\frac{\partial F1}{\partial Q2}\right)\Delta Q2 + \left(\frac{\partial F1}{\partial Q3}\right)\Delta Q3 = 0$$
$$F2 + \left(\frac{\partial F2}{\partial Q1}\right)\Delta Q1 + \left(\frac{\partial F2}{\partial Q2}\right)\Delta Q2 + \left(\frac{\partial F2}{\partial Q3}\right)\Delta Q3 = 0$$
$$F3 + \left(\frac{\partial F3}{\partial Q1}\right)\Delta Q1 + \left(\frac{\partial F3}{\partial Q2}\right)\Delta Q2 + \left(\frac{\partial F3}{\partial Q3}\right)\Delta Q3 = 0$$

Arranging the above set of equations in matrix form to find $\Delta Q1$, $\Delta Q2$ and $\Delta Q3$

$$\begin{bmatrix} \frac{\partial F1}{\partial Q1} & \frac{\partial F1}{\partial Q2} & \frac{\partial F1}{\partial Q3} \\ \frac{\partial F2}{\partial Q1} & \frac{\partial F2}{\partial Q2} & \frac{\partial F2}{\partial Q3} \\ \frac{\partial F3}{\partial Q1} & \frac{\partial F3}{\partial Q2} & \frac{\partial F3}{\partial Q3} \end{bmatrix} \begin{bmatrix} \Delta Q1 \\ \Delta Q2 \\ \Delta Q3 \end{bmatrix} = - \begin{bmatrix} F1 \\ F2 \\ F3 \end{bmatrix}$$

Solving the above equations

$$\begin{bmatrix} \Delta Q1 \\ \Delta Q2 \\ \Delta Q3 \end{bmatrix} = - \begin{bmatrix} \frac{\partial F1}{\partial Q1} & \frac{\partial F1}{\partial Q2} & \frac{\partial F1}{\partial Q3} \\ \frac{\partial F2}{\partial Q1} & \frac{\partial F2}{\partial Q2} & \frac{\partial F2}{\partial Q3} \\ \frac{\partial F3}{\partial Q1} & \frac{\partial F3}{\partial Q2} & \frac{\partial F3}{\partial Q3} \end{bmatrix}^{-1} \begin{bmatrix} F1 \\ F2 \\ F3 \end{bmatrix}$$

Knowing the corrections, the discharges are improved

$$\begin{bmatrix} Q1 \\ Q2 \\ Q3 \end{bmatrix}_{new} = \begin{bmatrix} Q1 \\ Q2 \\ Q3 \end{bmatrix}_{Old} + \begin{bmatrix} \Delta Q1 \\ \Delta Q2 \\ \Delta Q3 \end{bmatrix}$$

The process is repeated again until the values of ΔQi reduce to a very small values.

Example 1: The pipe network of two loops as shown in Figure below has to be analyzed by Newton Raphson method for pipe flows for given pipe lengths L and pipe diameters D. The nodal inflow at node 1 and nodal outflow at node 3 are shown in the figure. Assume a constant friction factor f = 0.02.

Solution

To apply continuity equation for initial pipe discharges, the discharges in pipes 1 and 5 equal to 0.1 m^3/s are assumed. The obtained discharges are

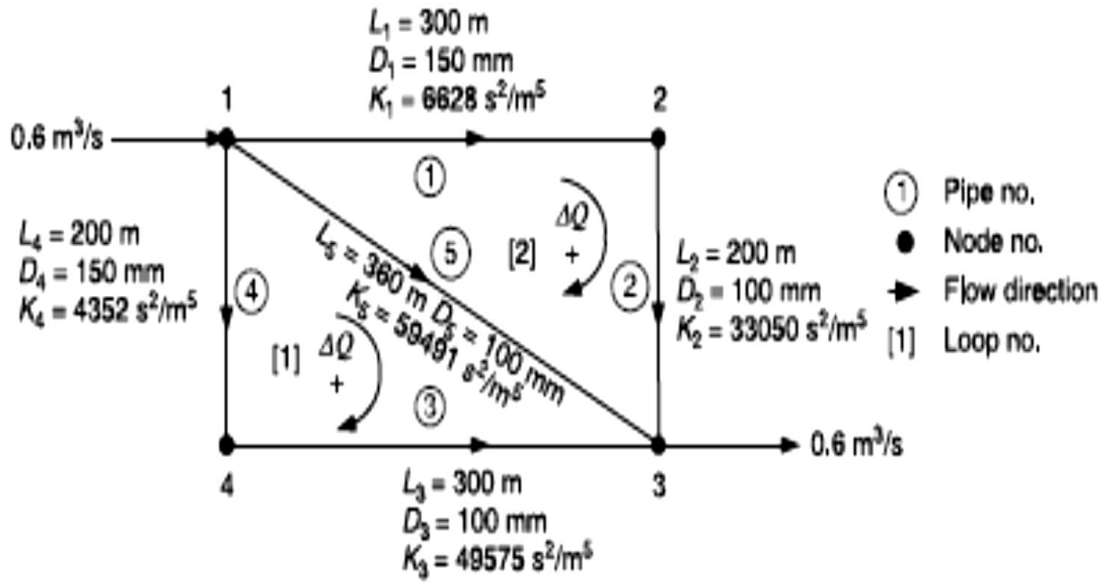
$Q1 = 0.1\ m^3/s$

$Q2 = 0.1\ m^3/s$

$Q3 = 0.4\ m^3/s$

$Q4 = 0.4\ m^3/s$

$Q5 = 0.1\ m^3/s$



The nodal discharge

$F1 = -Q1 - Q4 - Q5 + 0.6 = 0$, With $\frac{\partial F_1}{\partial Q_1} = -1$, $\frac{\partial F_1}{\partial Q_2} = 0$, $\frac{\partial F_1}{\partial Q_3} = 0$, $\frac{\partial F_1}{\partial Q_4} = -1$ & $\frac{\partial F_1}{\partial Q_5} = -1$

$F2 = Q1 - Q2 = 0$, With $\frac{\partial F_2}{\partial Q_1} = 1$, $\frac{\partial F_2}{\partial Q_2} = -1$, $\frac{\partial F_2}{\partial Q_3} = 0$, $\frac{\partial F_1}{\partial Q_4} = 0$ & $\frac{\partial F_1}{\partial Q_5} = 0$

$F3 = Q2 + Q3 + Q5 - 0.6 = 0$, With $\frac{\partial F_3}{\partial Q_1} = 0$, $\frac{\partial F_3}{\partial Q_2} = 1$, $\frac{\partial F_3}{\partial Q_3} = 1$, $\frac{\partial F_3}{\partial Q_4} = 0$ & $\frac{\partial F_3}{\partial Q_5} = 1$

head loss function $F4 = K5\ Q5|Q5| - K3\ Q3|Q3| - K4\ Q4|Q4|$

With $\frac{\partial F_4}{\partial Q_1} = \frac{\partial F_4}{\partial Q_2} = 0$, $\frac{\partial F_4}{\partial Q_3} = -2\,K3\,|Q3|$, $\frac{\partial F_4}{\partial Q_4} = -2\,K4\,|Q4|$ & $\frac{\partial F_4}{\partial Q_5} = 2\,K5\,|Q5|$

$F5 = K1\ Q1|Q1| + K2\ Q2|Q2| - K5\ Q5|Q5|$

With $\frac{\partial F_5}{\partial Q_1} = 2\,K1\,|Q1|$, $\frac{\partial F_5}{\partial Q_2} = 2\,K2\,|Q2|$, $\frac{\partial F_5}{\partial Q_3} = 0$, $\frac{\partial F_5}{\partial Q_4} = 0$ & $\frac{\partial F_5}{\partial Q_5} = -2\ K5\,|Q5|$

$$\begin{bmatrix} \Delta Q1 \\ \Delta Q2 \\ \Delta Q3 \\ \Delta Q4 \\ \Delta Q5 \end{bmatrix} = - \begin{bmatrix} \frac{\partial F_1}{\partial Q_1} & \frac{\partial F_1}{\partial Q_2} & \frac{\partial F_1}{\partial Q_3} & \frac{\partial F_1}{\partial Q_4} & \frac{\partial F_1}{\partial Q_5} \\ \frac{\partial F_2}{\partial Q_1} & \frac{\partial F_2}{\partial Q_2} & \frac{\partial F_2}{\partial Q_3} & \frac{\partial F_2}{\partial Q_4} & \frac{\partial F_2}{\partial Q_5} \\ \frac{\partial F_3}{\partial Q_1} & \frac{\partial F_3}{\partial Q_2} & \frac{\partial F_3}{\partial Q_3} & \frac{\partial F_3}{\partial Q_4} & \frac{\partial F_3}{\partial Q_5} \\ \frac{\partial F_4}{\partial Q_1} & \frac{\partial F_4}{\partial Q_2} & \frac{\partial F_4}{\partial Q_3} & \frac{\partial F_4}{\partial Q_4} & \frac{\partial F_4}{\partial Q_5} \\ \frac{\partial F_5}{\partial Q_1} & \frac{\partial F_5}{\partial Q_2} & \frac{\partial F_5}{\partial Q_3} & \frac{\partial F_5}{\partial Q_4} & \frac{\partial F_5}{\partial Q_5} \end{bmatrix}^{-1} \begin{bmatrix} F1 \\ F2 \\ F3 \\ F4 \\ F5 \end{bmatrix}$$

$$\begin{bmatrix} \Delta Q1 \\ \Delta Q2 \\ \Delta Q3 \\ \Delta Q4 \\ \Delta Q5 \end{bmatrix} = - \begin{bmatrix} -1 & 0 & 0 & -1 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & -2\,K3\,|Q3| & -2\,K4\,|Q4| & 2\,K5\,|Q5| \\ 2\,K1\,|Q1| & 2\,K2\,|Q2| & 0 & 0 & -2\,K5\,|Q5| \end{bmatrix}^{-1} \begin{bmatrix} F1 \\ F2 \\ F3 \\ F4 \\ F5 \end{bmatrix}$$

With initial values of ***Q1 = 0.1 , Q2 = 0.1 , Q3 = 0.4 , Q4 = 0.4 , Q5 = 0.1, the matrix is:***

$$\begin{bmatrix} \Delta Q1 \\ \Delta Q2 \\ \Delta Q3 \\ \Delta Q4 \\ \Delta Q5 \end{bmatrix} = - \begin{bmatrix} -1 & 0 & 0 & -1 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & -39660.89 & -3481.89 & 11898.27 \\ 1305.71 & 6610.15 & 0 & 0 & -11898.27 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -8033.64 \\ -199.12 \end{bmatrix}$$

All details of the solution are explained in the following table

Newton Raphson Method						Example 1

Pipoe no.	L	f	D	Ri	Ki
Pipe 1	300	0.02	0.15	0	6528.542
Pipe 2	200	0.02	0.1	0	33050.743
Pipe 3	300	0.02	0.1	0	49576.114
Pipe 4	200	0.02	0.15	0	4352.361
Pipe 5	360	0.02	0.1	0	59491.337

Iteration 1				Matrix of Derivatives					Inverse Matrix					ΔQ
Q1	0.1	F1	0.00	-1.00	0.00	0.00	-1.00	-1.00	-0.0437	0.8806	0.49724	1E-05	6E-05	0.11227
Q2	0.1	F2	0.00	1.00	-1.00	0.00	0.00	0.00	-0.0437	-0.1194	0.49724	1E-05	6E-05	0.11227
Q3	0.4	F3	0.00	0.00	1.00	1.00	0.00	1.00	0.0727	0.0891	0.17194	-2E-05	-1E-05	-0.17023
Q4	0.4	F4	-8033.64	0.00	0.00	-39660.89	-3481.89	11898.27	-0.9273	-0.9109	-0.82806	-2E-05	-1E-05	-0.17023
Q5	0.1	F5	-199.12	1305.71	6610.15	0.00	0.00	-11898.27	-0.029	0.0303	0.33081	8E-06	-5E-05	0.057957

Iteration 2				Matrix of Derivatives					Inverse Matrix					ΔQ
Q1	0.21227	F1	0.00	-1.00	0.00	0.00	-1.00	-1.00	-0.0314	0.8678	0.35741	2E-05	4E-05	0.010503
Q2	0.21227	F2	0.00	1.00	-1.00	0.00	0.00	0.00	-0.0314	-0.1322	0.35741	2E-05	4E-05	0.010503
Q3	0.22977	F3	0.00	0.00	1.00	1.00	0.00	1.00	0.0594	0.1029	0.32304	-3E-05	-2E-05	-0.0358
Q4	0.22977	F4	-1362.86	0.00	0.00	-22782.53	-2000.11	18794.16	-0.9406	-0.8971	-0.67696	-3E-05	-2E-05	-0.0358
Q5	0.15796	F5	299.04	2771.62	14031.33	0.00	0.00	-18794.16	-0.0281	0.0293	0.31955	1E-05	-2E-05	0.025302

Iteration 3				Matrix of Derivatives					Inverse Matrix					ΔQ
Q1	0.2228	F1	0.00	-1.00	0.00	0.00	-1.00	-1.00	-0.0304	0.8668	0.3467	0.0000	0.0000	0.0018
Q2	0.2228	F2	0.00	1.00	-1.00	0.00	0.00	0.00	-0.0304	-0.1332	0.3467	0.0000	0.0000	0.0018
Q3	0.1940	F3	0.00	0.00	1.00	1.00	0.00	1.00	0.0551	0.1075	0.3729	0.0000	0.0000	-0.0016
Q4	0.1940	F4	-31.05	0.00	0.00	-19232.42	-1688.44	21804.62	-0.9449	-0.8925	-0.6271	0.0000	0.0000	-0.0016
Q5	0.1833	F5	-33.72	2908.76	14725.60	0.00	0.00	-21804.62	-0.0246	0.0257	0.2804	0.0000	0.0000	-0.0001

Iteration 4			
Q1	0.2245	*F1*	0.00
Q2	0.2245	*F2*	0.00
Q3	0.1923	*F3*	0.00
Q4	0.1923	*F4*	-0.14
Q5	0.1831	*F5*	0.12

Matrix of Derivatives				
-1.00	0.00	0.00	-1.00	-1.00
1.00	-1.00	0.00	0.00	0.00
0.00	1.00	1.00	0.00	1.00
0.00	0.00	-19071.77	-1674.34	21789.09
2931.62	14841.32	0.00	0.00	-21789.09

Inverse Matrix				
-0.0302	0.8666	0.3440	0.0000	0.0000
-0.0302	-0.1334	0.3440	0.0000	0.0000
0.0548	0.1077	0.3754	0.0000	0.0000
-0.9452	-0.8923	-0.6246	0.0000	0.0000
-0.0246	0.0257	0.2806	0.0000	0.0000

ΔQ	Qnew
0.0000	0.2245
0.0000	0.2245
0.0000	0.1923
0.0000	0.1923
0.0000	0.1831

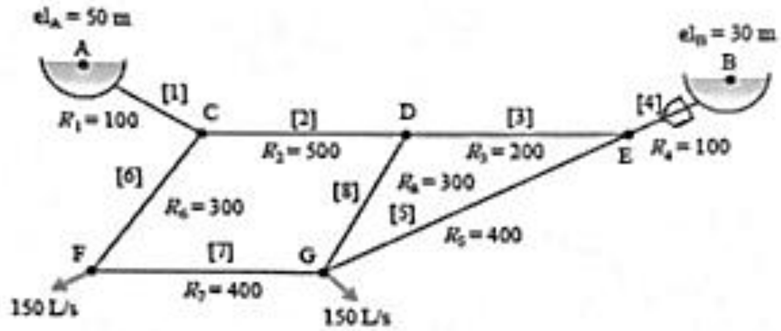
Iteration 5			
Q1	0.2245	*F1*	0.00
Q2	0.2245	*F2*	0.00
Q3	0.1923	*F3*	0.00
Q4	0.1923	*F4*	0.00
Q5	0.1831	*F5*	0.00

Matrix of Derivatives				
-1.00	0.00	0.00	-1.00	-1.00
1.00	-1.00	0.00	0.00	0.00
0.00	1.00	1.00	0.00	1.00
0.00	0.00	-19071.53	-1674.32	21789.59
2931.60	14841.21	0.00	0.00	-21789.59

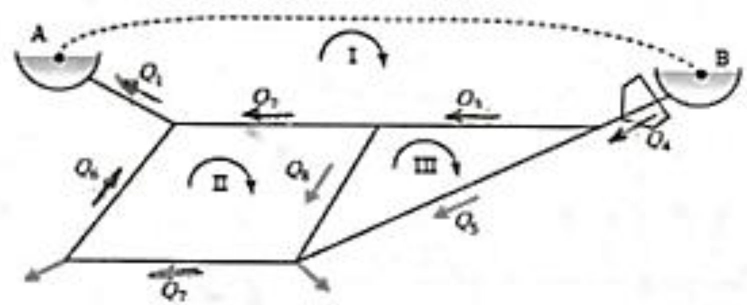
Inverse Matrix				
-0.0302	0.8666	0.3440	0.0000	0.0000
-0.0302	-0.1334	0.3440	0.0000	0.0000
0.0548	0.1077	0.3754	0.0000	0.0000
-0.9452	-0.8923	-0.6246	0.0000	0.0000
-0.0246	0.0257	0.2806	0.0000	0.0000

ΔQ	Qnew
0.0000	0.2245
0.0000	0.2245
0.0000	0.1923
0.0000	0.1923
0.0000	0.1831

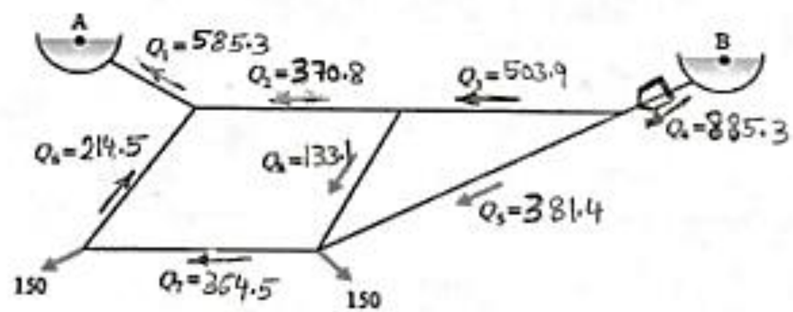
Example 2: Re solve example 3 considering a pump located at pipe 4 with
$Hp = 250 – 0.4\,Q – 0.1\,Q^2$



(a)



(b)



(c)

The nodal discharge

$F1 = FC = -Q1 + Q2 + Q6 = 0$

$F2 = FD = -Q2 + Q3 - Q8 = 0$

$F3 = FE = -Q3 + Q4 - Q5 = 0$

$F4 = FG = +Q5 - Q7 + Q8 – 0.15 = 0$

$F5 = FF = -Q6 + Q7 – 0.15 = 0$

head loss functions

$$F6 = +k_4Q_4|Q_4| + k_3Q_3|Q_3| + k_2Q_2|Q_2| + k_1Q_1|Q_1| + 20 - (250 - 0.4|Q_4| - 0.1\,Q_4^2)$$

$$F7 = -k_2Q_2|Q_2| + k_8Q_8|Q_8| + k_7Q_7|Q_7| + k_6Q_6|Q_6|$$

$$F8 = -k_3Q_3|Q_3| + k_5Q_5|Q_5| - k_8Q_8|Q_8|$$

The derivatives are

$$\frac{\partial F_1}{\partial Q_1} = -1,\ \frac{\partial F_1}{\partial Q_2} = +1,\ \frac{\partial F_1}{\partial Q_6} = +1,\ Others = 0$$

$$\frac{\partial F_2}{\partial Q_2} = -1,\ \frac{\partial F_2}{\partial Q_3} = 1,\ \frac{\partial F_2}{\partial Q_8} = -1,\ Others = 0$$

$$\frac{\partial F_3}{\partial Q_3} = -1,\ \frac{\partial F_3}{\partial Q_4} = 1\ \ \&\ \frac{\partial F_3}{\partial Q_5} = -1,\ Others = 0$$

$$\frac{\partial F_4}{\partial Q_5} = +1,\ \frac{\partial F_4}{\partial Q_7} = -1,\ \frac{\partial F_4}{\partial Q_8} = 1,\ Others = 0$$

$$\frac{\partial F_5}{\partial Q_6} = -1,\ \frac{\partial F_5}{\partial Q_7} = 1,\ Others = 0$$

$$\frac{\partial F_6}{\partial Q_1} = 2k_1|Q_1|,\ \frac{\partial F_6}{\partial Q_2} = 2k_2|Q_2|,\ \frac{\partial F_6}{\partial Q_3} = 2k_3|Q_3|,$$

$$\frac{\partial F_6}{\partial Q_4} = 2k_4|Q_4| - (-0.4 - 0.2|Q4|),\ Others = 0$$

$$\frac{\partial F_7}{\partial Q_2} = -2k_2|Q_2|,\ \frac{\partial F_7}{\partial Q_6} = 2k_6|Q_6|,\ \frac{\partial F_7}{\partial Q_7} = 2k_7|Q_7|,\ \frac{\partial F_7}{\partial Q_8} = 2k_8|Q_8|$$

$$\frac{\partial F_8}{\partial Q_3} = -2k_3|Q_3|,\ \frac{\partial F_8}{\partial Q_5} = +2k_5|Q_5|,\ \frac{\partial F_8}{\partial Q_8} = -2k_8|Q_8|$$

$$
\begin{bmatrix} \Delta Q1 \\ \Delta Q2 \\ \Delta Q3 \\ \Delta Q4 \\ \Delta Q5 \\ \Delta Q6 \\ \Delta Q7 \\ \Delta Q8 \end{bmatrix}
= -
\begin{bmatrix}
\frac{\partial F_1}{\partial Q_1} & \frac{\partial F_1}{\partial Q_2} & \frac{\partial F_1}{\partial Q_3} & \frac{\partial F_1}{\partial Q_4} & \frac{\partial F_1}{\partial Q_5} & \frac{\partial F_1}{\partial Q_6} & \frac{\partial F_1}{\partial Q_7} & \frac{\partial F_1}{\partial Q_8} \\
\frac{\partial F_2}{\partial Q_1} & \frac{\partial F_2}{\partial Q_2} & \frac{\partial F_2}{\partial Q_3} & \frac{\partial F_2}{\partial Q_4} & \frac{\partial F_2}{\partial Q_5} & \frac{\partial F_2}{\partial Q_6} & \frac{\partial F_2}{\partial Q_7} & \frac{\partial F_2}{\partial Q_8} \\
\frac{\partial F_3}{\partial Q_1} & \frac{\partial F_3}{\partial Q_2} & \frac{\partial F_3}{\partial Q_3} & \frac{\partial F_3}{\partial Q_4} & \frac{\partial F_3}{\partial Q_5} & \frac{\partial F_3}{\partial Q_6} & \frac{\partial F_3}{\partial Q_7} & \frac{\partial F_3}{\partial Q_8} \\
\frac{\partial F_4}{\partial Q_1} & \frac{\partial F_4}{\partial Q_2} & \frac{\partial F_4}{\partial Q_3} & \frac{\partial F_4}{\partial Q_4} & \frac{\partial F_4}{\partial Q_5} & \frac{\partial F_4}{\partial Q_6} & \frac{\partial F_4}{\partial Q_7} & \frac{\partial F_4}{\partial Q_8} \\
\frac{\partial F_5}{\partial Q_1} & \frac{\partial F_5}{\partial Q_2} & \frac{\partial F_5}{\partial Q_3} & \frac{\partial F_5}{\partial Q_4} & \frac{\partial F_5}{\partial Q_5} & \frac{\partial F_5}{\partial Q_6} & \frac{\partial F_5}{\partial Q_7} & \frac{\partial F_5}{\partial Q_8} \\
\frac{\partial F_6}{\partial Q_1} & \frac{\partial F_6}{\partial Q_2} & \frac{\partial F_6}{\partial Q_3} & \frac{\partial F_6}{\partial Q_4} & \frac{\partial F_6}{\partial Q_5} & \frac{\partial F_6}{\partial Q_6} & \frac{\partial F_6}{\partial Q_7} & \frac{\partial F_6}{\partial Q_8} \\
\frac{\partial F_7}{\partial Q_1} & \frac{\partial F_7}{\partial Q_2} & \frac{\partial F_7}{\partial Q_3} & \frac{\partial F_7}{\partial Q_4} & \frac{\partial F_7}{\partial Q_5} & \frac{\partial F_7}{\partial Q_6} & \frac{\partial F_7}{\partial Q_7} & \frac{\partial F_7}{\partial Q_8} \\
\frac{\partial F_8}{\partial Q_1} & \frac{\partial F_8}{\partial Q_2} & \frac{\partial F_8}{\partial Q_3} & \frac{\partial F_8}{\partial Q_4} & \frac{\partial F_8}{\partial Q_5} & \frac{\partial F_8}{\partial Q_6} & \frac{\partial F_8}{\partial Q_7} & \frac{\partial F_8}{\partial Q_8}
\end{bmatrix}^{-1}
\begin{bmatrix} F1 \\ F2 \\ F3 \\ F4 \\ F5 \\ F6 \\ F7 \\ F8 \end{bmatrix}
$$

With initial values of ***Q1 = 0.6, Q2 = 0.5 , Q3 = 0.6, Q4 = 0.9 , Q5 = 0.3, Q6= 0.1 , Q7= 0.25 & Q8= 0.1 the matrix is:***

$$
\begin{bmatrix} \Delta Q1 \\ \Delta Q2 \\ \Delta Q3 \\ \Delta Q4 \\ \Delta Q5 \\ \Delta Q6 \\ \Delta Q7 \\ \Delta Q8 \end{bmatrix}
= -
\begin{bmatrix}
-1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & -1 & 1 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & -1 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\
120 & 500+240 & 180.58 & 0 & 0 & 0 & 0 & \\
0 & -500 & 0 & 0 & 0 & 60 & 200 & 60 \\
0 & 0 & -240 & 0 & 240 & 0 & 0 & -60
\end{bmatrix}^{-1}
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 84.441 \\ -94 \\ -39 \end{bmatrix}
$$

All details of the solution are explained in the following table

Newton Raphson Method

Example 2

Pipoe no.	*Pump Position*	*L*	*f*	*D*	*Ri*	*Ki*
Pipe 1		-----	-----	-----	100	100
Pipe 2		-----	-----	-----	500	500
Pipe 3		-----	-----	-----	200	200
Pipe 4	1	-----	-----	-----	100	100
Pipe 5		-----	-----	-----	400	400
Pipe 6		-----	-----	-----	300	300
Pipe 7		-----	-----	-----	400	400
Pipe 8		-----	-----	-----	300	300

Iteration 1

Q1	0.6	*F1*	0.00
Q2	0.5	*F2*	0.00
Q3	0.6	*F3*	0.00
Q4	0.9	*F4*	0.00
Q5	0.3	*F5*	0.00
Q6	0.1	*F6*	84.4410
Q7	0.25	*F7*	-94.00
Q8	0.1	*F8*	-39.00

Matrix of Derivatives

-1.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00
0.00	-1.00	1.00	0.00	0.00	0.00	0.00	-1.00
0.00	0.00	-1.00	1.00	-1.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	1.00	0.00	-1.00	1.00
0.00	0.00	0.00	0.00	0.00	-1.00	1.00	0.00
120.000	500.00	240.00	180.58	0.00	0.00	0.00	0.00
0.00	-500.00	0.00	0.00	0.00	60.00	200.00	60.00
0.00	0.00	-240.00	0.00	240.00	0.00	0.00	-60.00

Inverse Matrix

-0.80	-0.50	-0.30	-0.51	-0.73	0.00	0.00
0.07	-0.21	-0.11	-0.15	0.02	0.00	0.00
0.10	0.31	-0.15	0.20	0.12	0.00	0.00
0.20	0.50	0.70	0.49	0.27	0.00	0.00
0.10	0.19	-0.16	0.29	0.15	0.00	0.00
0.13	-0.29	-0.20	-0.37	-0.75	0.00	0.00
0.13	-0.29	-0.20	-0.37	0.25	0.00	0.00
0.03	-0.48	-0.04	0.35	0.10	0.00	0.00

Iteration 2

Q1	0.59421	*F1*	0.00
Q2	0.37706	*F2*	0.00
Q3	0.51154	*F3*	0.00
Q4	0.89421	*F4*	0.00
Q5	0.38267	*F5*	0.00
Q6	0.21715	*F6*	9.1290
Q7	0.36715	*F7*	2.41
Q8	0.13449	*F8*	0.81

Matrix of Derivatives

-1.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00
0.00	-1.00	1.00	0.00	0.00	0.00	0.00	-1.00
0.00	0.00	-1.00	1.00	-1.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	1.00	0.00	-1.00	1.00
0.00	0.00	0.00	0.00	0.00	-1.00	1.00	0.00
118.842	377.06	204.62	179.42	0.00	0.00	0.00	0.00
0.00	-377.06	0.00	0.00	0.00	130.29	293.72	80.69
0.00	0.00	-204.62	0.00	306.13	0.00	0.00	-80.69

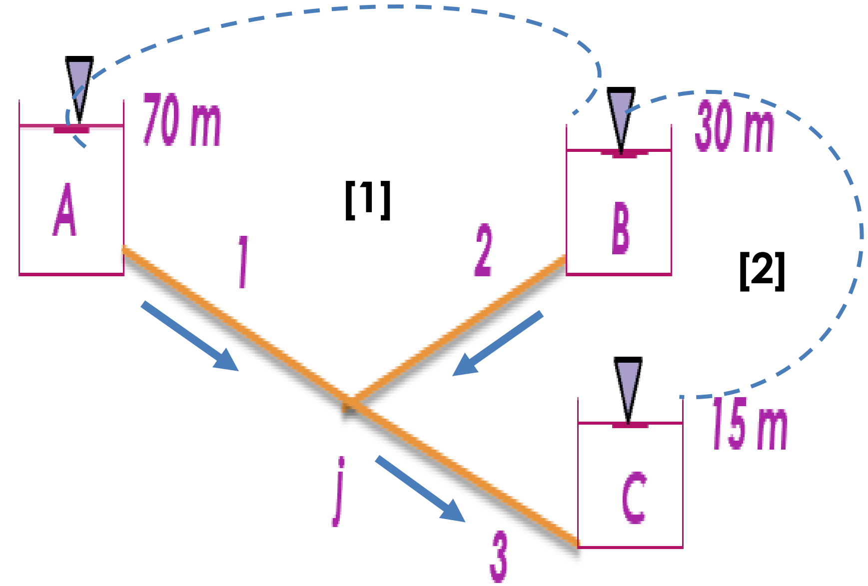
Inverse Matrix

-0.81	-0.48	-0.29	-0.49	-0.71	0.00	0.00
0.10	-0.29	-0.15	-0.21	0.00	0.00	0.00
0.11	0.36	-0.17	0.23	0.15	0.00	0.00
0.19	0.52	0.71	0.51	0.29	0.00	0.00
0.08	0.15	-0.12	0.27	0.14	0.00	0.00
0.09	-0.19	-0.13	-0.28	-0.72	0.00	0.00
0.09	-0.19	-0.13	-0.28	0.28	0.00	0.00
0.01	-0.35	-0.02	0.45	0.15	0.00	0.00

Iteration 3				Matrix of Derivatives								Inverse Matrix								ΔQ	Qnew
Q1	0.57717	*F1*	0.00	-1.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	-0.81	-0.49	-0.29	-0.49	-0.71	0.00	0.00	0.00	-0.0002	0.5770
Q2	0.37083	*F2*	0.00	0.00	-1.00	1.00	0.00	0.00	0.00	0.00	-1.00	0.10	-0.29	-0.15	-0.21	0.01	0.00	0.00	0.00	0.0000	0.3708
Q3	0.50325	*F3*	0.00	0.00	0.00	-1.00	1.00	-1.00	0.00	0.00	0.00	0.11	0.36	-0.17	0.23	0.15	0.00	0.00	0.00	-0.0001	0.5032
Q4	0.87717	*F4*	0.00	0.00	0.00	0.00	0.00	1.00	0.00	-1.00	1.00	0.19	0.51	0.71	0.51	0.29	0.00	0.00	0.00	-0.0002	0.8770
Q5	0.37392	*F5*	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	1.00	0.00	0.08	0.15	-0.12	0.27	0.14	0.00	0.00	0.00	-0.0001	0.3738
Q6	0.20633	*F6*	0.0913	115.433	370.83	201.30	176.01	0.00	0.00	0.00	0.00	0.09	-0.20	-0.14	-0.28	-0.72	0.00	0.00	0.00	-0.0002	0.2062
Q7	0.35633	*F7*	0.06	0.00	-370.83	0.00	0.00	0.00	123.80	285.07	79.45	0.09	-0.20	-0.14	-0.28	0.28	0.00	0.00	0.00	-0.0002	0.3562
Q8	0.13241	*F8*	0.02	0.00	0.00	-201.30	0.00	299.14	0.00	0.00	-79.45	0.01	-0.35	-0.02	0.45	0.14	0.00	0.00	0.00	-0.0001	0.1324

Iteration 4				Matrix of Derivatives								Inverse Matrix								ΔQ	Qnew
Q1	0.57696	*F1*	0.00	-1.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	-0.81	-0.49	-0.29	-0.49	-0.71	0.00	0.00	0.00	0.0000	0.5770
Q2	0.37080	*F2*	0.00	0.00	-1.00	1.00	0.00	0.00	0.00	0.00	-1.00	0.10	-0.29	-0.15	-0.21	0.01	0.00	0.00	0.00	0.0000	0.3708
Q3	0.50316	*F3*	0.00	0.00	0.00	-1.00	1.00	-1.00	0.00	0.00	0.00	0.11	0.36	-0.17	0.23	0.15	0.00	0.00	0.00	0.0000	0.5032
Q4	0.87696	*F4*	0.00	0.00	0.00	0.00	0.00	1.00	0.00	-1.00	1.00	0.19	0.51	0.71	0.51	0.29	0.00	0.00	0.00	0.0000	0.8770
Q5	0.37380	*F5*	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	1.00	0.00	0.08	0.15	-0.12	0.27	0.14	0.00	0.00	0.00	0.0000	0.3738
Q6	0.20616	*F6*	0.0000	115.391	370.80	201.26	175.97	0.00	0.00	0.00	0.00	0.09	-0.20	-0.14	-0.28	-0.72	0.00	0.00	0.00	0.0000	0.2062
Q7	0.35616	*F7*	0.00	0.00	-370.80	0.00	0.00	0.00	123.69	284.93	79.42	0.09	-0.20	-0.14	-0.28	0.28	0.00	0.00	0.00	0.0000	0.3562
Q8	0.13236	*F8*	0.00	0.00	0.00	-201.26	0.00	299.04	0.00	0.00	-79.42	0.01	-0.35	-0.02	0.45	0.14	0.00	0.00	0.00	0.0000	0.1324

Example 3: For the piping system shown below, determine the flow distribution and piezometric heads at the junction J using Newton Raphson method of solution



<u>Solution</u>

head loss function

$$F1 = Q1 + Q2 - Q3$$

head loss function

$$F2 = +k_2 Q_2|Q_2| - k_1 Q_1|Q_1| + 40$$

$$F3 = -k_3 Q_3|Q_3| - k_2 Q_2|Q_2| + 15$$

The derivatives are

$$\frac{\partial F_1}{\partial Q_1} = 1 , \quad \frac{\partial F_1}{\partial Q_2} = 1 \quad \& \quad \frac{\partial F_1}{\partial Q_3} = -1$$

$$\frac{\partial F_2}{\partial Q_1} = -2\,K1\,|Q1| , \quad \frac{\partial F_2}{\partial Q_2} = 2\,K2\,|Q2| , \quad \frac{\partial F_2}{\partial Q_3} = 0$$

$$\frac{\partial F_3}{\partial Q_1} = 0 , \quad \frac{\partial F_3}{\partial Q_2} = -2\,K2\,|Q2| , \quad \frac{\partial F_3}{\partial Q_3} = -2\,K3\,|Q3|$$

Pipe	L (m)	D (m)	f	$\sum K_f$
1	1500	0.3	0.01	0
2	1500	0.3	0.01	0
3	1500	0.3	0.01	0

<u>From Res. A to Junction j</u>

$$\rightarrow h_{f1} = K_1\,Q1^2 \rightarrow h_{f1} = 510.042\;Q1^2 = 70 - hj$$

$$\rightarrow hj = 70 - 510.042\;Q1^2 = 70 - 510.042(0.2685)^2 = 33.230\;m$$

Solving the above equations such that :

$$\begin{bmatrix} \Delta Q1 \\ \Delta Q2 \\ \Delta Q3 \end{bmatrix} = -\begin{bmatrix} \frac{\partial F1}{\partial Q1} & \frac{\partial F1}{\partial Q2} & \frac{\partial F1}{\partial Q3} \\ \frac{\partial F2}{\partial Q1} & \frac{\partial F2}{\partial Q2} & \frac{\partial F2}{\partial Q3} \\ \frac{\partial F3}{\partial Q1} & \frac{\partial F3}{\partial Q2} & \frac{\partial F3}{\partial Q3} \end{bmatrix}^{-1} \begin{bmatrix} F1 \\ F2 \\ F3 \end{bmatrix}$$

$$\begin{bmatrix} \Delta Q1 \\ \Delta Q2 \\ \Delta Q3 \end{bmatrix} = -\begin{bmatrix} 1 & 1 & -1 \\ -2\,K1\,|Q1| & 2\,K2\,|Q2| & 0 \\ 0 & -2\,K2\,|Q2| & -2\,K3\,|Q3| \end{bmatrix}^{-1} \begin{bmatrix} F1 \\ F2 \\ F3 \end{bmatrix}$$

Assuming initial pipe discharge in pipes, Q1 = 0.2 , Q2 = 0.2 & Q3 = 0.4 .

$$\begin{bmatrix} \Delta \boldsymbol{Q1} \\ \Delta \boldsymbol{Q2} \\ \Delta \boldsymbol{Q3} \end{bmatrix} = -\begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{-1} \\ \mathbf{-204.02} & \mathbf{204.02} & \mathbf{0} \\ \mathbf{0} & \mathbf{-204.02} & \mathbf{-408.03} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} \\ \mathbf{40} \\ \mathbf{-87.01} \end{bmatrix}$$

All details of the solution are explained in the following table

Newton Raphson Method

Example 3

Pipoe no.	Pump position	L	f	D	Ri	Ki
Pipe 1		1500	0.01	0.3	0	510.042
Pipe 2		1500	0.01	0.3	0	510.042
Pipe 3		1500	0.01	0.3	0	510.042

Iteration 1				Matrix of Derivatives			Inverse Matrix			ΔQ	Qnew
Q1	0.2000	*F1*	0.00	1.00	1.00	-1.00	0.4000	-0.0029	-0.0010	0.0323	0.232
Q2	0.2000	*F2*	40.00	-204.02	204.02	0.00	0.4000	0.0020	-0.0010	-0.1637	0.036
Q3	0.4000	*F3*	-87.01	0.00	-204.02	-408.03	-0.2000	-0.0010	-0.0020	-0.1314	0.269

Iteration 2				Matrix of Derivatives			Inverse Matrix			ΔQ	Qnew
Q1	0.2323	*F1*	0.00	1.00	1.00	-1.00	0.1209	-0.0037	-0.0004	0.0388	0.271
Q2	0.0363	*F2*	13.14	-237.01	37.01	0.00	0.7745	0.0033	-0.0028	-0.1065	-0.070
Q3	0.2686	*F3*	-22.47	0.00	-37.01	-274.02	-0.1046	-0.0004	-0.0033	-0.0676	0.201

Iteration 3					Matrix of Derivatives				Inverse Matrix				ΔQ		Qnew
Q1	0.2712	*F1*	0.00		1.00	1.00	-1.00		0.1609	-0.0030	-0.0008		-0.0025		0.269
Q2	-0.0702	*F2*	-0.01		-276.60	71.58	0.00		0.6219	0.0022	-0.0030		-0.0093		-0.080
Q3	0.2010	*F3*	-3.09		0.00	-71.58	-205.02		-0.2171	-0.0008	-0.0038		-0.0118		0.189

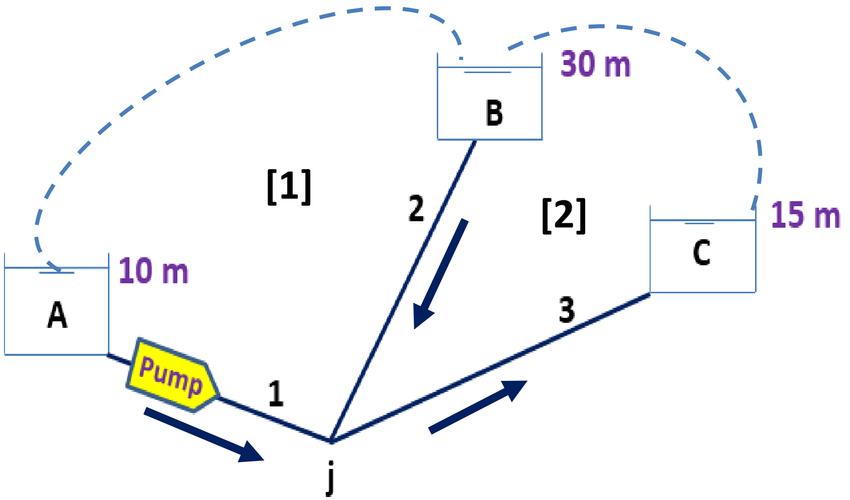
Iteration 4					Matrix of Derivatives				Inverse Matrix				ΔQ		Qnew
Q1	0.2687	*F1*	0.00		1.00	1.00	-1.00		0.1724	-0.0030	-0.0009		-0.0002		0.269
Q2	-0.0795	*F2*	-0.05		-274.09	81.12	0.00		0.5826	0.0021	-0.0030		0.0000		-0.080
Q3	0.1892	*F3*	-0.03		0.00	-81.12	-192.97		-0.2449	-0.0009	-0.0039		-0.0001		0.189

Iteration 5					Matrix of Derivatives				Inverse Matrix				ΔQ		Qnew
Q1	0.2685	*F1*	0.00		1.00	1.00	-1.00		0.1725	-0.0030	-0.0009		0.0000		0.269
Q2	-0.0795	*F2*	0.00		-273.92	81.10	0.00		0.5825	0.0021	-0.0030		0.0000		-0.080
Q3	0.1890	*F3*	0.00		0.00	-81.10	-192.82		-0.2450	-0.0009	-0.0039		0.0000		0.189

Note that for Q2= - 0.0795, it means that the flow must be in the opposite direction

Example 4: For the piping system shown below, determine the flow distribution and piezometric heads at the junction J using Newton Raphson method . Power = 20 Kw

Solution



The nodal discharge

$$F1 = Q1 + Q2 - Q3$$

head loss function

$$F2 = k_2 Q_2|Q_2| - k_1 Q_1|Q_1| + \frac{20}{9.8\,|Q_1|} - 20$$

$$F3 = -k_3 Q_3|Q_3| - k_2 Q_2|Q_2| + 15$$

Pipe	L (m)	D (m)	f	$\sum K_f$
1	50	0.15	0.02	2
2	100	0.10	0.015	1
3	300	0.10	0.025	1

The derivatives are

$$\frac{\partial F_1}{\partial Q_1} = 1\,, \quad \frac{\partial F_1}{\partial Q_2} = 1 \quad \& \quad \frac{\partial F_1}{\partial Q_3} = -1$$

$$\frac{\partial F_2}{\partial Q_1} = -2\,K1\,|Q1| - \frac{20}{\gamma\, Q1^2}, \quad \frac{\partial F_2}{\partial Q_2} = 2\,K2\,|Q2|\,, \quad \frac{\partial F_2}{\partial Q_3} = 0$$

$$\frac{\partial F_3}{\partial Q1} = 0\,, \quad \frac{\partial F_3}{\partial Q_2} = -2\,K2\,|Q2|\,, \quad \frac{\partial F_3}{\partial Q_3} = -2\,K3\,|Q3|$$

For pipe 1 $\quad Z1 + H_p = h_{f1} + H_j$

$$10 + \frac{20}{9.8\,Q1} = 1414.517\,Q1^2 + H_j \rightarrow H_j = 10 + \frac{20}{9.8\,Q1} - 1414.517\,Q1^2$$

$$= 10 + \frac{20}{9.8 \,x\, 0.0538} - 1414.517\,(0.0538)^2 = 43.839 \text{ m}$$

Solving the above equations such that :

$$\begin{bmatrix} \Delta Q1 \\ \Delta Q2 \\ \Delta Q3 \end{bmatrix} = -\begin{bmatrix} \frac{\partial F1}{\partial Q1} & \frac{\partial F1}{\partial Q2} & \frac{\partial F1}{\partial Q3} \\ \frac{\partial F2}{\partial Q1} & \frac{\partial F2}{\partial Q2} & \frac{\partial F2}{\partial Q3} \\ \frac{\partial F3}{\partial Q1} & \frac{\partial F3}{\partial Q2} & \frac{\partial F3}{\partial Q3} \end{bmatrix}^{-1} \begin{bmatrix} F1 \\ F2 \\ F3 \end{bmatrix}$$

$$\begin{bmatrix} \Delta Q1 \\ \Delta Q2 \\ \Delta Q3 \end{bmatrix} = -\begin{bmatrix} 1 & 1 & -1 \\ -2\,K1\,|Q1| - \frac{20}{\gamma\,Q1^2} & 2\,K2\,|Q2| & 0 \\ 0 & -2\,K2\,|Q2| & -2\,K3\,|Q3| \end{bmatrix}^{-1} \begin{bmatrix} F1 \\ F2 \\ F3 \end{bmatrix}$$

Assuming initial pipe discharge in pipes, Q1 = 0.05 , Q2 = 0.05 & Q3 = 0.1 .

$$\begin{bmatrix} \Delta Q1 \\ \Delta Q2 \\ \Delta Q3 \end{bmatrix} = -\begin{bmatrix} 1 & 1 & -1 \\ -957.78 & 1322.44 & 0 \\ 0 & -1322.44 & -12559.28 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 50.34 \\ -646.03 \end{bmatrix}$$

All details of the solution are explained in the following table

Newton Raphson Method

Example 4

Pipoe no.	Pump Position	L	f	D	Ri	Ki
Pipe 1		65	0.02	0.15	0	1414.5174
Pipe 2		107	0.015	0.1	0	13224.429
Pipe 3		304	0.025	0.1	0	62796.412

Iteration 1

				Matrix of Derivatives			Inverse Matrix			ΔQ	Qnew
Q1	0.0500	*F1*	0.00	1.00	1.00	-1.00	0.5554	-0.0005	0.0000	-0.0052	0.045
Q2	0.0500	*F2*	50.34	-957.78	1322.44	0.00	0.4022	0.0004	0.0000	-0.0418	0.008
Q3	0.1000	*F3*	-646.03	0.00	-1322.44	-12559.28	-0.0424	0.0000	-0.0001	-0.0470	0.053

Iteration 2

				Matrix of Derivatives			Inverse Matrix			ΔQ	Qnew
Q1	0.0448	*F1*	0.00	1.00	1.00	-1.00	0.1547	-0.0007	0.0000	0.0137	0.058
Q2	0.0082	*F2*	23.60	-1143.58	216.01	0.00	0.8188	0.0007	-0.0001	-0.0368	-0.029
Q3	0.0530	*F3*	-162.06	0.00	-216.01	-6652.25	-0.0266	0.0000	-0.0001	-0.0232	0.030

Iteration 3				Matrix of Derivatives			Inverse Matrix			ΔQ	Qnew
Q1	0.0585	*F1*	0.00	1.00	1.00	-1.00	0.4527	-0.0007	-0.0001	-0.0042	0.054
Q2	-0.0287	*F2*	-0.81	-762.26	758.39	0.00	0.4551	0.0006	-0.0001	-0.0032	-0.032
Q3	0.0298	*F3*	-29.90	0.00	-758.39	-3742.91	-0.0922	-0.0001	-0.0002	-0.0073	0.022

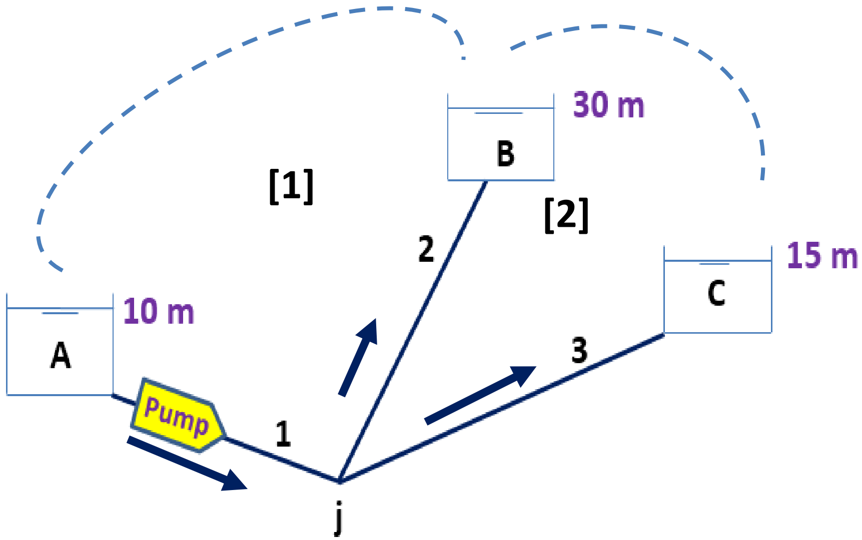
Iteration 4				Matrix of Derivatives			Inverse Matrix			ΔQ	Qnew
Q1	0.0543	*F1*	0.00	1.00	1.00	-1.00	0.4337	-0.0007	-0.0002	-0.0005	0.054
Q2	-0.0318	*F2*	0.04	-846.28	841.75	0.00	0.4361	0.0005	-0.0002	-0.0005	-0.032
Q3	0.0225	*F3*	-3.26	0.00	-841.75	-2819.83	-0.1302	-0.0002	-0.0003	-0.0010	0.021

Iteration 5				Matrix of Derivatives			Inverse Matrix			ΔQ	Qnew
Q1	0.0538	*F1*	0.00	1.00	1.00	-1.00	0.4310	-0.0007	-0.0002	0.0000	0.054
Q2	-0.0323	*F2*	0.00	-857.25	855.60	0.00	0.4318	0.0005	-0.0002	0.0000	-0.032
Q3	0.0215	*F3*	-0.06	0.00	-855.60	-2694.22	-0.1371	-0.0002	-0.0003	0.0000	0.021

Iteration 6				Matrix of Derivatives			Inverse Matrix			ΔQ	Qnew
Q1	0.0538	*F1*	0.00	1.00	1.00	-1.00	0.4309	-0.0007	-0.0002	0.0000	0.054
Q2	-0.0324	*F2*	0.00	-857.49	855.84	0.00	0.4318	0.0005	-0.0002	0.0000	-0.032
Q3	0.0214	*F3*	0.00	0.00	-855.84	-2691.82	-0.1373	-0.0002	-0.0003	0.0000	0.021

Note that for Q2= - 0.0324, it means that the flow must be in the opposite direction

Note that : Another assumption
If the direction of flow was assumed as follows :
Then the solution will take the following form:



Solution

The nodal discharge

$$F1 = Q1 - Q2 - Q3$$

head loss function

$$F2 = -k_2Q_2|Q_2| - k_1Q_1|Q_1| + \frac{20}{9.8\,|Q_1|} - 20$$

$$F3 = -k_3Q_3|Q_3| + k_2Q_2|Q_2| + 15$$

The derivatives are

$$\frac{\partial F_1}{\partial Q_1} = 1\,,\quad \frac{\partial F_1}{\partial Q_2} = -1 \quad \& \quad \frac{\partial F_1}{\partial Q_3} = -1$$

$$\frac{\partial F_2}{\partial Q_1} = -2\,K1\,|Q1| - \frac{20}{\gamma\,Q1^2}\,,\quad \frac{\partial F_2}{\partial Q_2} = -2\,K2\,|Q2|\,,\quad \frac{\partial F_2}{\partial Q_3} = 0$$

$$\frac{\partial F_3}{\partial Q_1} = 0\,,\quad \frac{\partial F_3}{\partial Q_2} = +2\,K2\,|Q2|\,,\quad \frac{\partial F_3}{\partial Q_3} = -2\,K3\,|Q3|$$

Pipe	L (m)	D (m)	f	$\sum K_f$
1	50	0.15	0.02	2
2	100	0.10	0.015	1
3	300	0.10	0.025	1

For pipe 1 $\quad Z1 + H_p = h_{f1} + H_j$

$$10 + \frac{20}{9.8\,Q1} = 1414.517\,Q1^2 + H_j \rightarrow H_j = 10 + \frac{20}{9.8\,Q1} - 1414.517\,Q1^2$$

$$= 10 + \frac{20}{9.8\,x\,0.0538} - 1414.517\,(0.0538)^2 = 43.839\text{ m}$$

Solving the above equations such that :

$$\begin{bmatrix} \Delta Q1 \\ \Delta Q2 \\ \Delta Q3 \end{bmatrix} = -\begin{bmatrix} \frac{\partial F1}{\partial Q1} & \frac{\partial F1}{\partial Q2} & \frac{\partial F1}{\partial Q3} \\ \frac{\partial F2}{\partial Q1} & \frac{\partial F2}{\partial Q2} & \frac{\partial F2}{\partial Q3} \\ \frac{\partial F3}{\partial Q1} & \frac{\partial F3}{\partial Q2} & \frac{\partial F3}{\partial Q3} \end{bmatrix}^{-1} \begin{bmatrix} F1 \\ F2 \\ F3 \end{bmatrix}$$

$$\begin{bmatrix} \Delta Q1 \\ \Delta Q2 \\ \Delta Q3 \end{bmatrix} = -\begin{bmatrix} 1 & -1 & -1 \\ -2\,K1\,|Q1| - \frac{20}{\gamma\, Q1^2} & -2\,K2\,|Q2| & 0 \\ 0 & +2\,K2\,|Q2| & -2\,K3\,|Q3| \end{bmatrix}^{-1} \begin{bmatrix} F1 \\ F2 \\ F3 \end{bmatrix}$$

Assuming initial pipe discharge in pipes, Q1 = 0.1 , Q2 = 0.06 & Q3 = 0.04 .

$$\begin{bmatrix} \Delta Q1 \\ \Delta Q2 \\ \Delta Q3 \end{bmatrix} = -\begin{bmatrix} 1 & -1 & -1 \\ -486.99 & -1586.93 & 0 \\ 0 & 1586.93 & -5023.71 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -61.34 \\ -37.87 \end{bmatrix}$$

All details of the solution are explained in the following table

Newton Raphson Method

Example4

Another Assumption

Pipoe no.	*Pump Position*	*L*	*f*	*D*	*Ri*	*Ki*
Pipe 1		65	0.02	0.15	0	1414.5174
Pipe 2		107	0.015	0.1	0	13224.429
Pipe 3		304	0.025	0.1	0	62796.412

Iteration 1

	Q		F	Matrix of Derivatives			Inverse Matrix			ΔQ	Qnew
Q1	0.1000	*F1*	0.00	1.00	-1.00	-1.00	0.7123	-0.0006	-0.0001	-0.0416	0.058
Q2	0.0600	*F2*	-61.34	-486.99	-1586.93	0.00	-0.2186	-0.0004	0.0000	-0.0259	0.034
Q3	0.0400	*F3*	-37.87	0.00	1586.93	-5023.71	-0.0691	-0.0001	-0.0002	-0.0157	0.024

Iteration 2

	Q		F	Matrix of Derivatives			Inverse Matrix			ΔQ	Qnew
Q1	0.0584	*F1*	0.00	1.00	-1.00	-1.00	0.4769	-0.0007	-0.0002	-0.0046	0.054
Q2	0.0341	*F2*	-5.26	-763.68	-902.20	0.00	-0.4037	-0.0005	0.0001	-0.0019	0.032
Q3	0.0243	*F3*	-6.65	0.00	902.20	-3049.95	-0.1194	-0.0002	-0.0003	-0.0027	0.022

Iteration 3

	Q		F	Matrix of Derivatives			Inverse Matrix			ΔQ	Qnew
Q1	0.0538	*F1*	0.00	1.00	-1.00	-1.00	0.4301	-0.0007	-0.0002	0.0000	0.054
Q2	0.0322	*F2*	0.16	-858.42	-851.88	0.00	-0.4334	-0.0005	0.0002	0.0001	0.032
Q3	0.0215	*F3*	-0.42	0.00	851.88	-2705.62	-0.1365	-0.0002	-0.0003	-0.0001	0.021

Iteration 4

	Q		F	Matrix of Derivatives			Inverse Matrix			ΔQ	Qnew
Q1	0.0538	*F1*	0.00	1.00	-1.00	-1.00	0.4309	-0.0007	-0.0002	0.0000	0.054
Q2	0.0324	*F2*	0.00	-857.48	-855.84	0.00	-0.4318	-0.0005	0.0002	0.0000	0.032
Q3	0.0214	*F3*	0.00	0.00	855.84	-2691.84	-0.1373	-0.0002	-0.0003	0.0000	0.021

Note that the solution is just needs to 4 Its. Only to provide the same results.