

Fluid at rest :pressure considerations

1-Pressure at point: Pascal's law

Pressure acts equally in all directions at any point in the static body of fluid.
By referring to fig. below, shows a small cube of fluid of dimension h located at arbitrary point O within a large static body of fluid (fig.1).

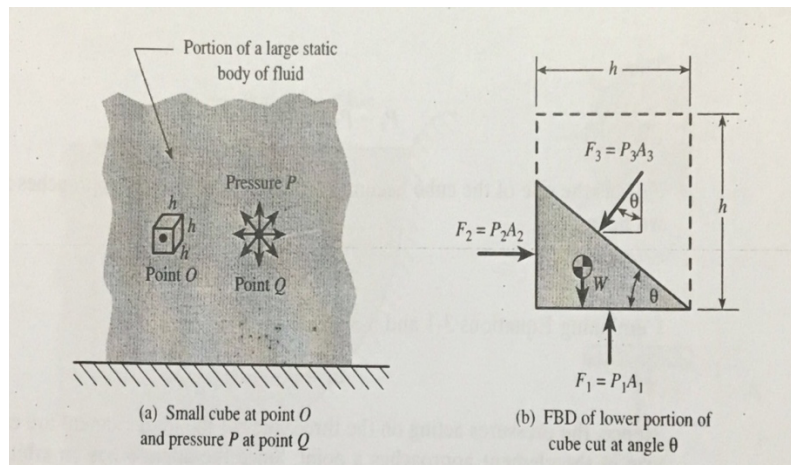


Fig.1

. F_1 is the pressure force due to P_1 upward direction.

. F_2 is the pressure force due to P_2 to the right.

. F_3 is the pressure force due to P_3 normal to the inclined surface (down and left).

. W is the weight of the fluid element.

$$F_2 - F_1 \sin \theta = 0$$

$$P_2 A_2 - P_3 A_3 \sin \theta = 0$$

$$P_2 (h \tan \theta \cdot h) - P_3 (h / \cos \theta \cdot h) \sin \theta = 0$$

$$P_2 = P_1$$

Vertical direction:

$$F_1 - F_3 \cos \theta - W = 0$$

$$P_1 A_1 - P_3 A_3 \cos \theta - \gamma V = 0$$

$$P_1 h^2 - P_3 (h / \cos \theta \cdot h) \cos \theta - \gamma (0.5 h \cdot h \tan \theta) h = 0$$

$$P_1 - P_3 - 0.5 \gamma (h \tan \theta) = 0$$

$$P_1 = P_3. \quad \text{Thus} \quad P_1 = P_2 = P_3$$

2-Derivation of the pressure-depth relationship for liquid

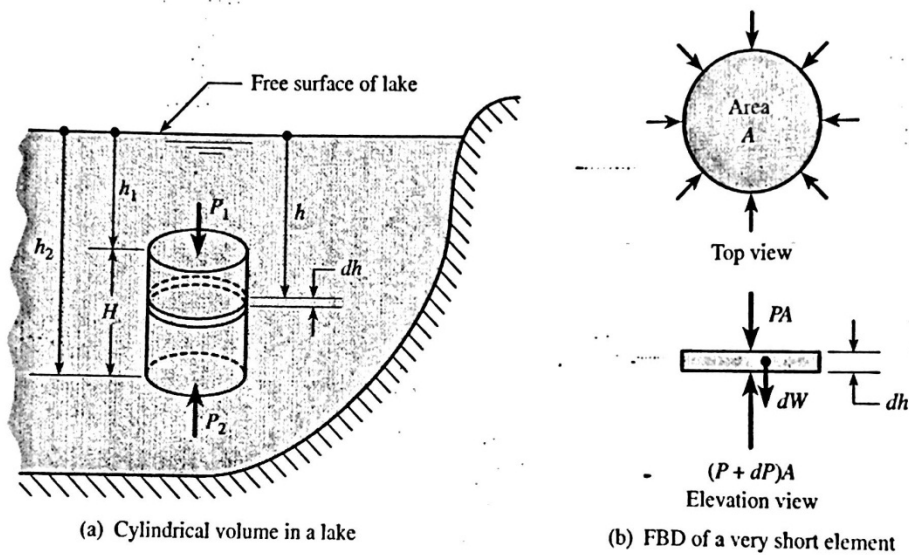


Fig.2

$$(P + dP)A - PA - dW = 0$$

$$PA + dp(A) - PA - \gamma(dV) = 0$$

$$A(dP) - \gamma(A \cdot dh) = 0$$

$$dP = \gamma(dh)$$

after integration

$$P_2 - P_1 = \gamma(h_2 - h_1) = \gamma H$$

$$\underline{P = \gamma h} \quad (\text{the pressure of the liquid on any point below} = \text{weight density} \times \text{the depth above the point})$$

3-What is the absolute pressure and gage pressure (the barometer)

Gage pressure is measured relative to the atmosphere, whereas **absolute pressure** is measured relative to a perfect vacuum such as that which exists in outer space.

Fig.3 below shows graphically depicts the difference between gage and absolute pressure

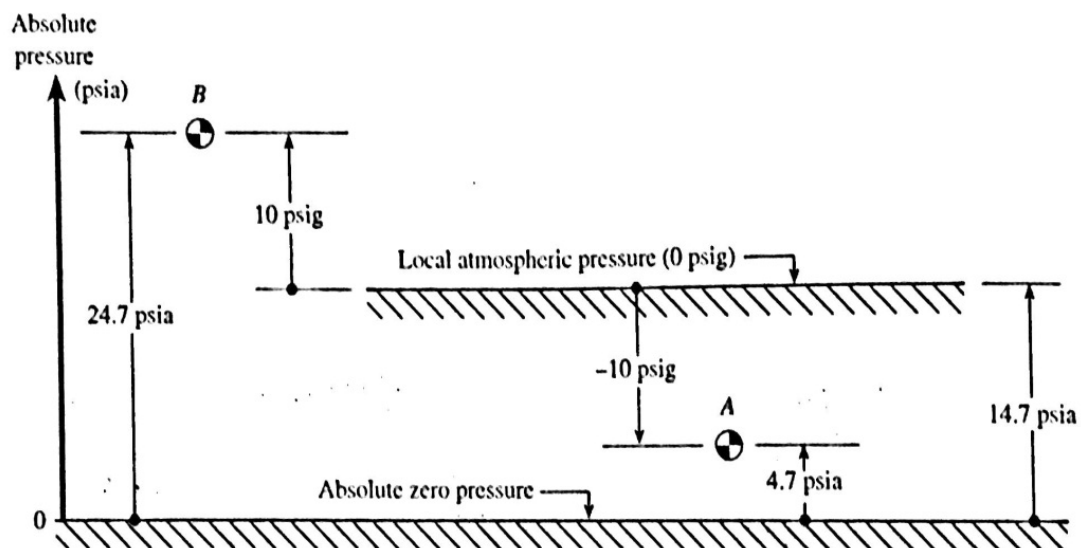


Fig.3

14.7 psi = 76 mm Hg = 101 kpa. = 10 m H₂O.

P_A = 4.7 psia thus P_A is less than atmosphere pressure.

P_B = 24.7 psia thus P_B is greater than atmosphere pressure.

P_A = -10 psig = 10 psi suction = 10 psi vacuum = -10 psi

P_B = 10 psig

Then $P_{abs} = P_{gage} + P_{atm}$.

Manometers

Manometer :It is an instrument used to measure the pressure that act on any point in a liquid

4-Types of Manometers

4-1-Pisometer tube: it is a simple type of manometer, it is a simple tube is open to the atmosphere , at the top and attached at the bottom to a pipe or other container whose liquid pressure is to be measured(fig.4).

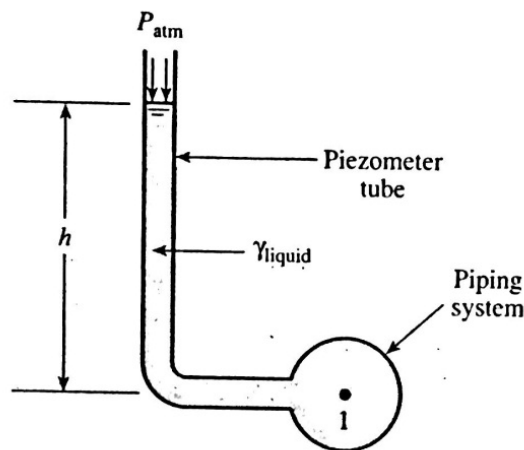


Fig.4

$$P_1 - P_{atm} = \gamma \cdot h$$

4-2-Inclined piezometer: it is for grater accuracy , $P_1 = \gamma h$ (fig.5)

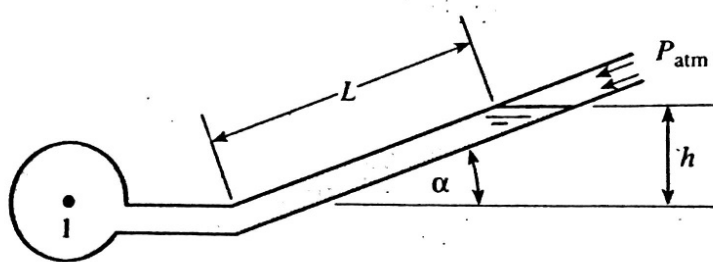


Fig.5

4-3-U-tube manometer: (fig.6) in this design two different fluids are used, and it can measure suction pressure and larger pressure than a piezometer . We start from one end and move toward the other, we use the relationship that starts that the change in pressure equals the specific weight times the change in elevation, we also note that a drop in elevation increases pressure and vice versa:

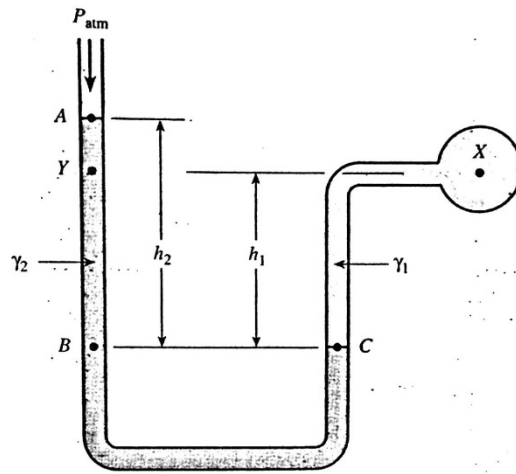


Fig.6

If we use gage pressure $P_A = P_{atm} = 0$ gage

$$P_A + \gamma h_2 - \gamma h_1 = P_x$$

$$P_A = 0 \text{ (atm.)}$$

$$\text{Thus } P_x = \gamma_2 h_2 - \gamma_1 h_1$$

4-4-Differential manometer: it is used to measure the difference in pressure between two points in a fluid system (fig.7).

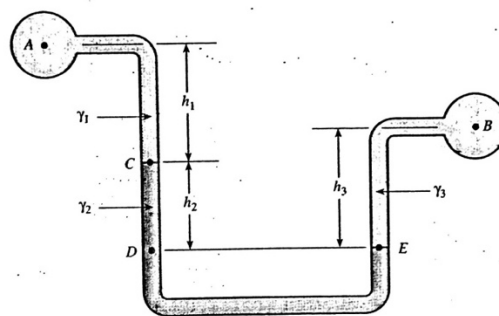


Fig.7

$$P_A + \gamma_1 h_1 + \gamma_2 h_2 - \gamma_3 h_3 = P_B$$

Examples

Ex.1

Find the pressure of the air above the water in the tank (fig.8)

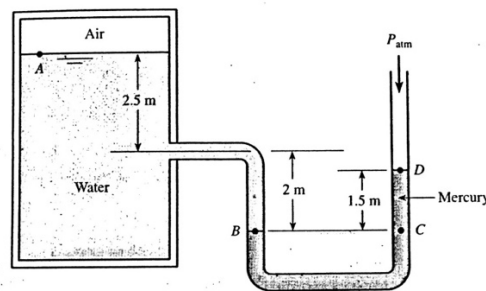


Fig.8

$$P_{air} = P_A$$

$$P_A + 9810 \cdot (2.5 + 2) - 13.6 \cdot 9800 \cdot 1.5 = 0$$

$$P_{air} = 155400 \text{ pa.} \quad \checkmark$$

Ex.2

Water and oil flow in two pipeline (fig.9) using the double U-tube manometer as connected, find the pressure difference $P_A - P_B$, if $\gamma_{oil} = 57 \text{ lb/ft}^3$ (neglect air pressure).

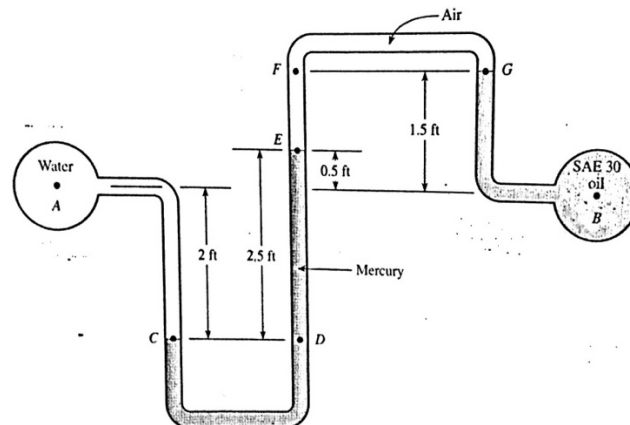


Fig.9

$$P_A + 62.4 \cdot 2 - 13.6 \cdot 62.4 \cdot 2.5 + 57 \cdot 62.4 \cdot 1.5 = P_B$$

$$P_B - P_A = 3338.4 \text{ lb/ft}^2$$

Ex.3

A U-tube manometer is connected to a closed tank (fig.10), the air pressure in the tank is 0.5 psi and the liquid is oil ($\gamma = 54 \text{ lb/ft}^3$), the pressure at point A is 2 psi. Find the depth of oil z and the differential reading h ?

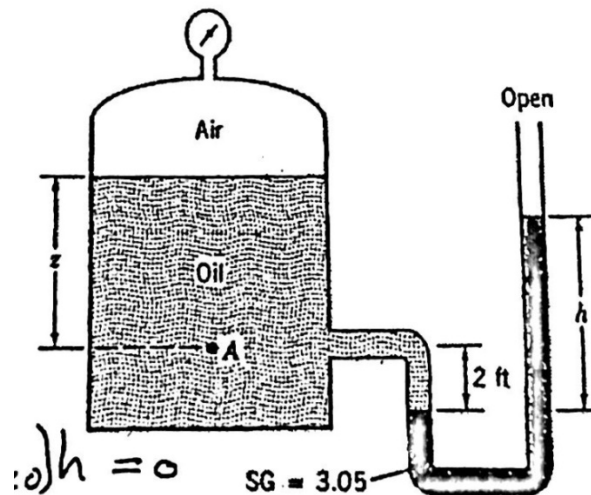


Fig.10

$$P_A = \gamma_{oil} * z + P_{air} \quad , \quad Z = (P_A - P_{air}) / \gamma_{oil}$$

$$= (2 - 0.5) / 54 = 4 \text{ ft.}$$

$$P_A + \gamma_{oil}(2) - S.G.(\gamma_w)h = 0 \quad , \quad h = (P_A + \gamma_{oil}(2)) / (S.G. * \gamma_w)$$

$$= \frac{2 * 144 + 54 * 2}{3.05 * 62.4} = 2.08 \text{ ft.}$$

Ex. 4

- Water is flowing upward through the pipeline, a manometer measures the pressure difference $p_1 - p_2$. Find the difference? (fig. 11)

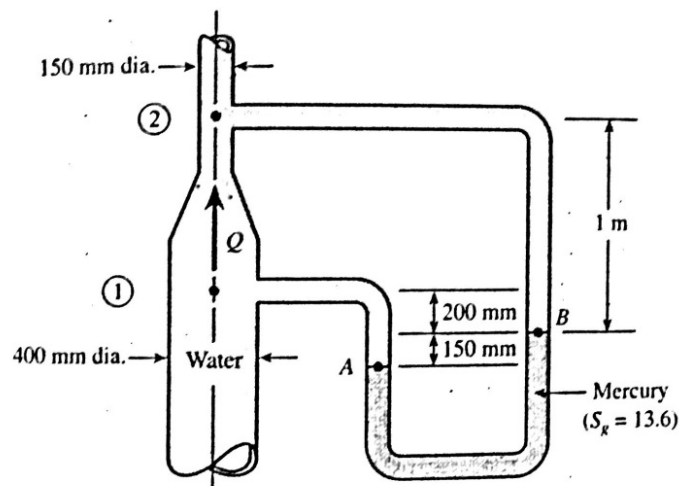


Fig.11

$$P_2 + \gamma (1\text{m}) + \gamma (0.15\text{ m}) - \gamma (0.15+0.2)\text{m} = P_1$$

$$P_2 + 9800 * 1 + 13.6 * 9800 * 0.15 - 9800 * 0.35 = P_1$$

$$P_2 + 26362 = P_1$$

$$P_1 - P_2 = 26362\text{ pa.}$$

Ex. 5

Find the pressure at 1 if $L = 0.5\text{ m}$ and $\alpha = 30^\circ$?

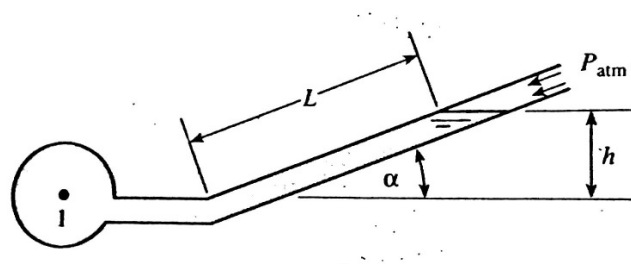


Fig. 12

$$P = \gamma h$$

$$h = L * \sin \alpha = 0.5 * 0.5 = 0.25\text{ m}$$

$$P = 9800 * 0.25 = 2450\text{ pa.}$$

