Force considerations

When a body is submerged in a static fluid the fluid pushes on all of the body with pressure . The pressure force acting on the surface of a submerged body determine whether the body will sink or float.

Horizontal plane surfaces submerged in liquids

All points on a horizontal plane surface have the same elevation.(fig.1). So, the pressure is constant :

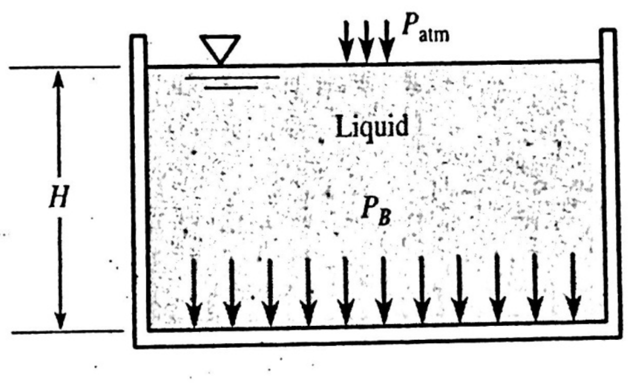

Fig.1

P= γH P :pressure γ: weight density H: the depth from the water surface to the application point.
So F=PA F=γ*V

Submerged vertical plane surfaces

Fig.2, S

hows two views end and right views for a gate as a thin rectangular plate submerged (fig.2):

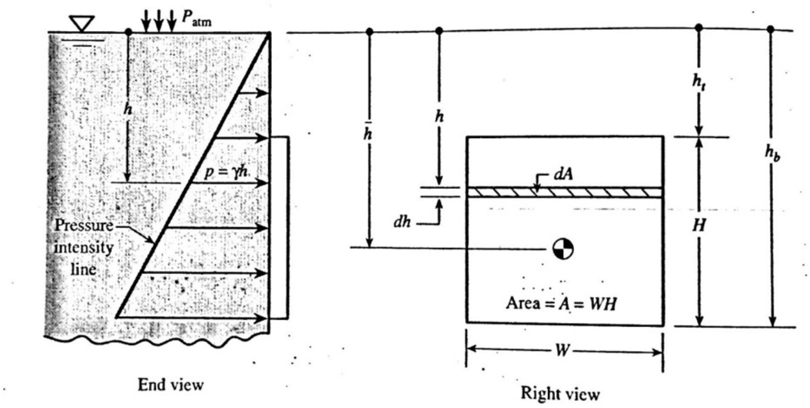

Fig.2

dF=P(dA)=γh(dA)

Fr=$\int_A \gamma h(dA) = \int_{ht}^{hb} \gamma h(W * dh)$

So Fr=$\gamma(((\frac{hb+ht}{2}))((hb - ht)W)$

<u>Fh = γhA</u>
Fh horizontal force (N or lb)
hc: the vertical distance from the water surface to the body centroid.
A : the body section area.

Center of pressure:

$h_{cp} = \frac{Ix}{hA} + h$
from which h_{cp} - hc = e (eccentricity, the deference between the centroid and the center of pressure.
Ix (moment of inertia about the x- axis)

Submerged inclined plane surfaces

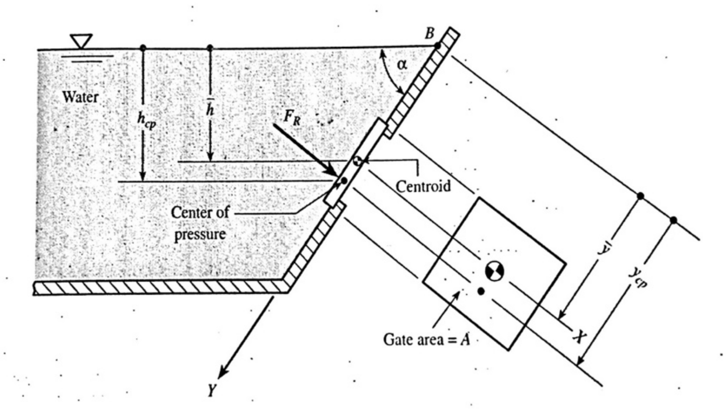

Fig.3

The following three equations employed for inclined plane surfaces:

$$F_h = PA = \gamma\, h\, A$$
$$F_h = \gamma\, y\, A \sin\alpha$$
$$Y_{cp} = \frac{I_x}{yA} + y$$

Examples

Ex.1

Calculate the magnitude and point of application of the resultant force exerted on the square gate (fig.4)?

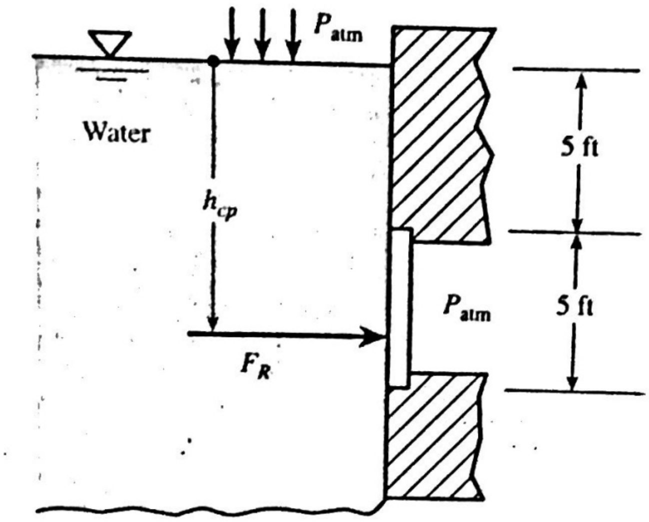

Fig.4

Fr=PA=γhA

=62.4*(5+2.5)(5*5)=11700 lb

$h_{cp}=\frac{Ix}{hA}+h$

$Ix=\frac{BH^3}{12}$=52.1 ft4

$h_{cp}=\frac{52.1}{7.5*25}+7.5=7.78ft$

Ex.2

A rectangular gate of dimensions 6 ft high and 4 ft wide is mounted in a vertical wall of an open rectangular tank(fig.5) the tank is filled with oil . Determine the minimum force Q to keep the gate closed if the gate is hinged at the bottom?

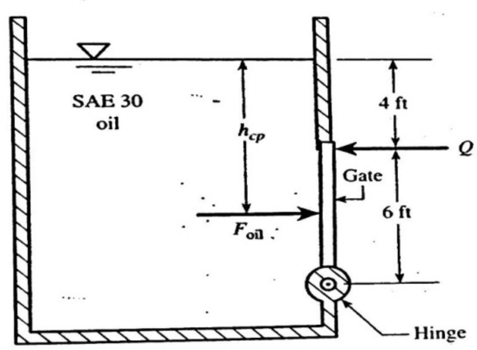

Fig.5

Foil = γh A

=57*7*24=9580 lb

$H_{cp} = \frac{Ix}{hA} + h = \frac{4*6^3/12}{7*24} + 7 = 7.43\ ft$

∑M hinge=0

Foil(10-h_{cp})-Q*6=0

Q = 4100lb.

Ex.3

Find the minimum hight h of water that will cause the rectangular L-shape gate (fig.6)to open . The gate is hinged where the vertical rectangular section and the horizontal section are connected, thus the L-shape gate is free to rotate. The gate has a constant width W is exposed to the atmosphere (neglect the weight of the gate)?

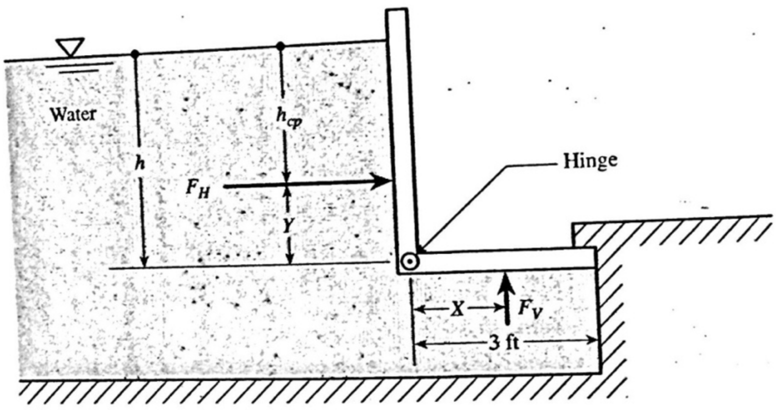

Fig.6

Fh=γ h Av=$\frac{\gamma h^2 W}{2}$

$h_{cp} = \frac{Ix}{h\ Av} + h = \frac{\frac{1}{12}*Wh^3}{\frac{h}{2}*(Wh)} + \frac{h}{2}$ =2/3h

so y=h/3 ft.

Fv=γh A_H=γh(3W)=3γhW

Fv is act at the center of the section thus X=3/2 ft

∑ M hinge=0

Fh(Y) – Fv(X) =0

$\frac{\gamma h^2 W(\frac{h}{3})}{2} - 3\gamma hW\frac{3}{2} = 0$ h=5.2ft

Ex.4

Determine the force acting on the circular gate located in the inclined wall of the open tank (fig.7)the gate diameter is 2 ft ?

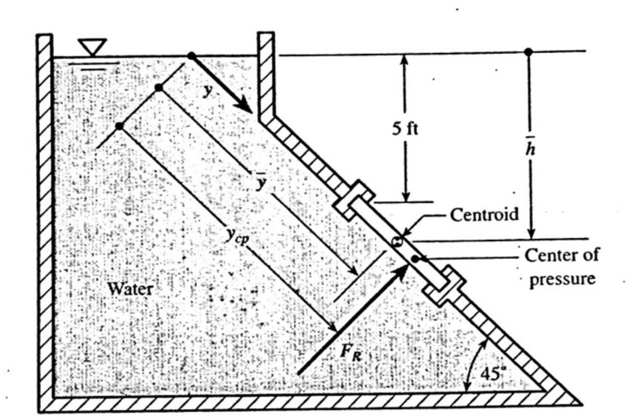

Fig.7

Fh=γ h A

$=62.4*(5+1*\sin 45)*\frac{\pi}{4}(4)^{2} = 1120lb$

$y=\frac{Ix}{yA}+y$

$Y=\frac{h}{\sin 45}$=5.71/sin45=8.08ft

$Ix=\frac{\pi D^{4}}{64} = 0.785\, ft^{4}$

$A=\frac{\pi D^{2}}{4}$=3.14 ft^2

$Y_{cp}=\frac{0.785}{8.08*3.14}+8.08$=8.111 ft.