Fluid in motion

The behavior of fluid in motion is very complex due to the large number of variables involved. Since viscosity introduces a great complexity to fluid flow behavior, we assume the fluid is an ideal one, this means no viscosity and no shear stress as it flows. We also assume incompressible flow which suggests that the density of fluid does not change throughout the system.

Properties of flow:

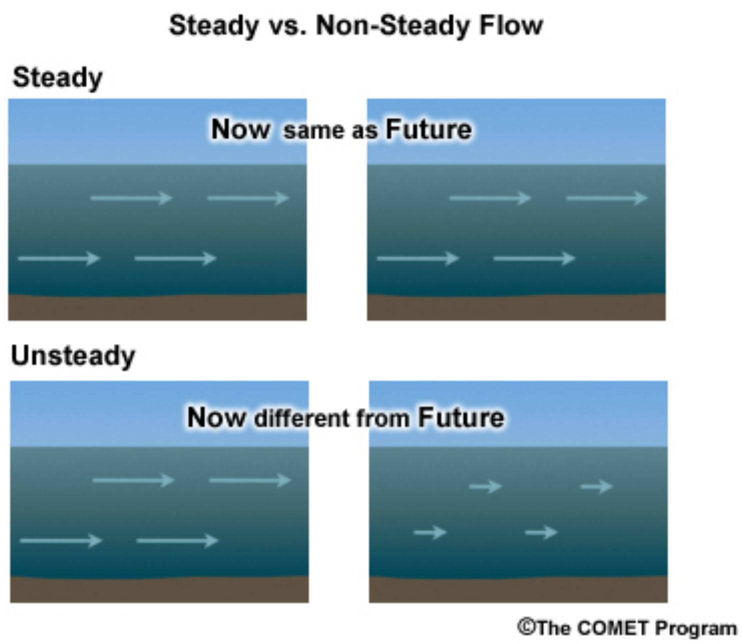
Steady flow: when there is no change in flow properties(Q,v,h) with time.

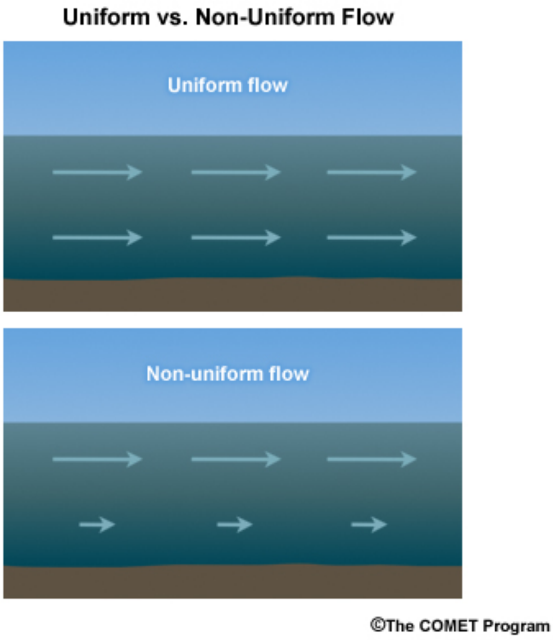
Unsteady flow: when there is the change in flow properties with time.

Uniform flow: when there is no change in fluid properties with distance.

Non-uniform flow(2D): flow with two directions (x,y) and velocity distribution become parabola section with maximum value at the center and minimum at the wall of pipe.

Three-dimensions flow(3D): flow with 3 directions (x, y, z) as flow through orifice.





The continuity equation

This theorem states that the mass can neither be created nor destroyed. For steady flow, the rate at which mass enters a control volume equals the rate at which mass leaves this volume. in fig.1, fluid is flowing from left to right the pipe has two different sizes (A1 and A2) the volume between 1 and 2 is the control volume (CV), and the rate at which the mass enters equals the rate at which the mass leaves CV.

1-Maas flow rate

From fig.1 let M represent the rate at which mass enters or leaves the CV, we have M1=M2, thus for a steady flow, the mass flow rate at the inlet to the CV equals the mass flow rate at the exit from the CV.

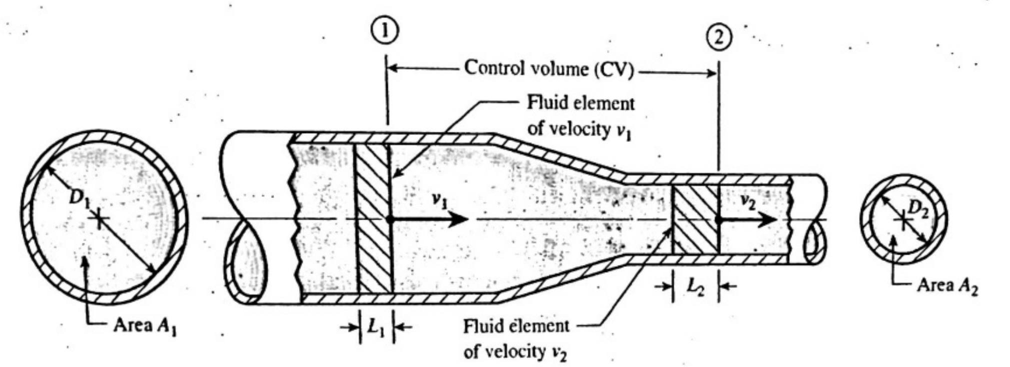


Fig.1

$M_1 = \frac{m_1}{t} = \frac{\rho_1 A_1 L_1}{t} = \rho_1 A_1 v_1$
$M_2 = \frac{m_2}{t} = \frac{\rho_2 A_2 L_2}{t} = \rho_2 A_2 v_2$
$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

2- **Weight flow rate**

Since specific weight equals times the acceleration of gravity ($\gamma = \rho * g$)

The continuity equation shows that we also have equality of weight flow rates entering and leaving the CV, W1=W2

$\gamma_1 A_1 v_1 = \gamma_1 A_1 v_1$

3-Volume flow rate

for steady incompressible flow, the volume flow rate is also constant represented by Q(flow), because flow =volume per time.

Q1=Q2
$\frac{Vol.1}{t} = \frac{vol.2}{t}$
$\frac{vol_1}{vol_2} = \frac{A_2}{A_1} = (\frac{D_2}{D_1})^2$

D is the diameter of a circular section.

Or Q = v*A (v is velocity)

Examples:

Ex1

For the fig.1 the following data are given: D_1=4 in. D_2=2in. v_1=4ft/s

Find the volume flow rate, fluid velocity at sec. 2, weight flow rate, and mass flow rate?

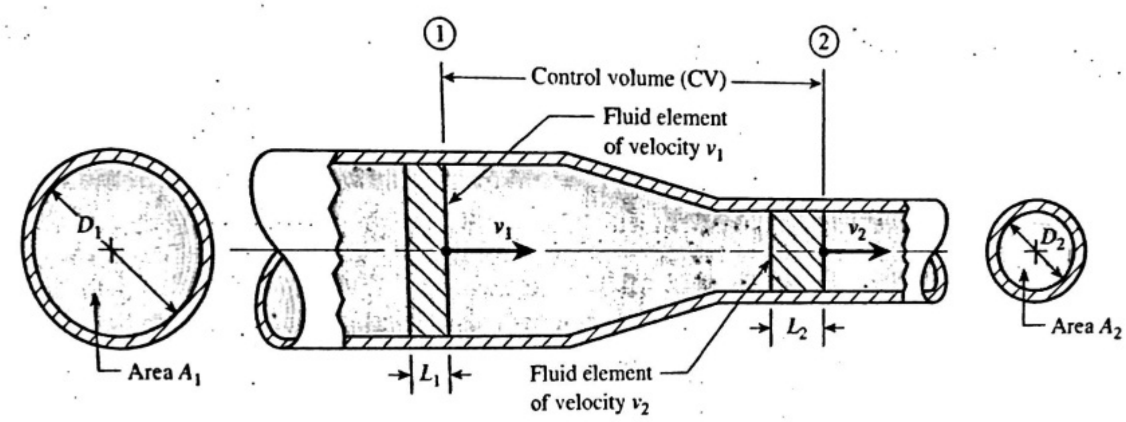


Fig.1

$Q=Q_1=A_1v_1$

$A_1=\frac{\pi}{4}D^2=\frac{\pi}{4}(4/12)^2=0.0873\ ft^2$

$Q = (0.0873)\ (4) = 0.349\ ft^3/s$

b- $v_2=v_1\ (D_1/D_2)^2$

$=4(4/2)^2=16 ft/s$

c-$W=W_1=\gamma\ A_1\ v_1=\gamma\ Q_1=62.4*0.349=21.8$ lb/s

d-$M=M_1=\rho A_1v_1=\rho Q_1=1.94*0.349=0.677$ slug/s

Ex2

A pipeline of 300 mm diameter carrying water at an average velocity 4.5 m/s branches into two pipes of 150 mm, 200mm dia., if the average velocity in the 150mm is 5/8 of the velocity in the main pipe, find the average velocity of flow in 200 mm, and the total flow rate in the system by l/s?

$Q=AV=Q1+Q2$

$AV=A1V1+A2V2$

$\frac{1}{4}\pi\ (0.3)^2*\ 4.5=\frac{1}{4}\pi\ (0.15)^2*\frac{5}{8}*\ 4.5+\frac{1}{4}\pi(0.2)^2*V_2$

$V_2=8.54$ m/s

Total flow $Q=\frac{1}{4}\pi\ (0.3)^2*4.5$

$=0.318\ m^3/s.$ $=318$ l/s

Ex3

The velocity of a liquid (s.g)=1.4, in a 150mm pipeline is 0.8 m/s. Calculate the rate of flow in l/s , m^3/s, kg/s and KN/s?

$Q=VA=0.8(\frac{\pi}{4})(\frac{150}{1000})^2=0.01414\ m^3/s= 14.141\ l/s$

M=ρ*Q= (1.4*1000) * 0.01414=19.79 kg/s

W=γ*Q= (1.4*9.8) * 0.01414=0.194 KN/s

Ex. 4

Two streams of water enter the mixing chamber if the inlet velocity is 80 kg/sec. and , find the outlet mass flow? And outlet weight flow?

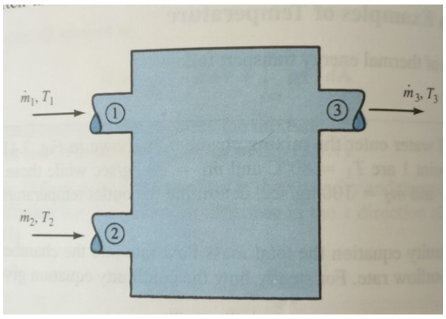


Fig. 2

Total mass flow rate= m_1+m_2

80 +100 =180 kg/sec.

W = mass * g

180 * g =180 * 9.8 = 1764 N/sec

Bernoulli's equation

Bernoulli's equation is based on the conservation of energy law, which states that energy can be neither created nor destroyed. The total energy possessed by a given mass of fluid can be considered to consist of three types: potential, kinetic, and flow energy.

Fig.3 shows fluid flowing from left to right, the total energy by a given weight w of fluid entering CV at sta.1 and the same weight of fluid leaving CV at sta.2.

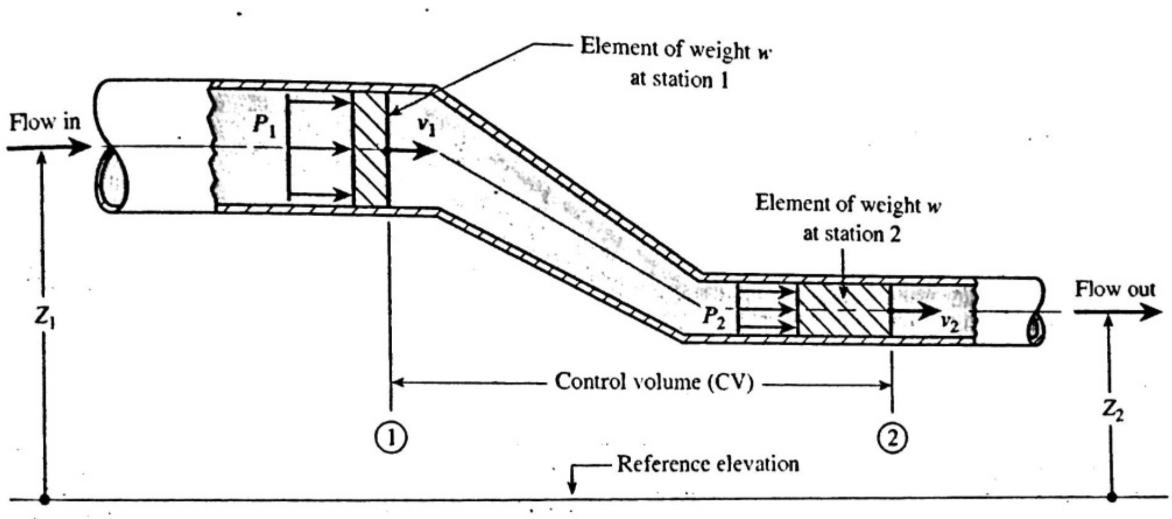


Fig.3

1-Potential energy: الطاقة الكامنة

The fluid element of weight w has a potential energy due to its elevation Z related to a reference plane.

PE= wz

2-Kinetic energy: الطاقة الحركية

The fluid element of weight w moving with a velocity

KE=$\frac{1}{2}$ mv^2

3-Flow energy; طاقه الجريان

It is the amount of work that pressure accomplishes by pushing the element of weight w at sta.1 into the CV or pushing the element of weight w at sta.2 out of CV.

FE= $\frac{Pw}{\gamma}$

Statement of Bernoullis equation:

Daniel Bernoulli an eighteenth-century Swiss scientist, formulated his equation by noting that the total energy possessed by the fluid in CV does not change concerning time.

Total energy in element at 1 = total energy in element at 2

(PE+KE+FE)1=(PE+KE+FE)2

$wZ_1+\frac{1}{2}\frac{w}{g}v_1^2+\frac{P_1 w}{\gamma}= wZ_2+\frac{1}{2}\frac{w}{g}v_2^2+\frac{P_2}{\gamma}w$ divide by w

$Z_1+\frac{v_1^2}{2g}+\frac{P_1}{\gamma}= Z_2+\frac{v_2^2}{2g}+\frac{P_2}{\gamma}$

$$\frac{P_1}{\gamma}+\frac{v_1^2}{2g}+Z_1=\frac{P_2}{\gamma}\ \frac{v_2^2}{2g}+Z_2$$

$\frac{P}{\gamma}$: pressure head.

$\frac{v_2^2}{2g}$: *velocity head*

Z: elevation head.

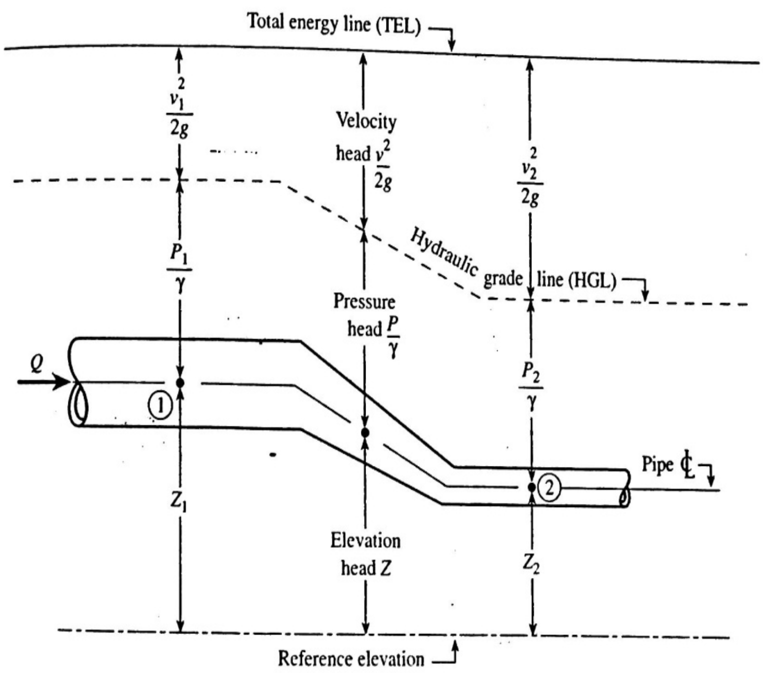


Fig.4

Ex5

For the pipe of fig.5 find P2 if the following data are given:p1= 20 psi, D1 =2 in and D2=1.5 in , Q=200 gpm of water?

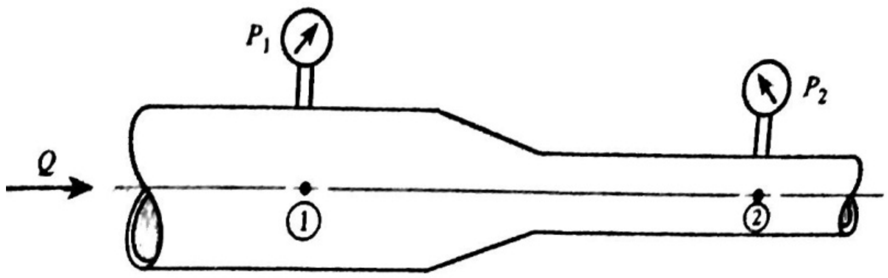


Fig.5

$$\frac{P_1}{\gamma} + \frac{{v_1}^2}{2g} + Z_1 = \frac{P_2}{\gamma}\ \frac{{v_2}^2}{2g} + Z_2$$

$$V1 = \frac{Q}{A1} = \frac{200*1*231*1}{\frac{\pi}{4}\left(\frac{2}{12}\right)*60*1*1728}$$

=20.4 ft/s

$v2 = v1(D1/D2)^2$

$$\frac{20*144}{62.4} + \frac{20.4}{2(32.2)} = \frac{P2}{62.4} + \frac{36.2^{\ 2}}{2(32.2)} + 46.2 + 6.46 = \frac{P2}{62.4} + 20.3$$

Solving for P at 2 yields:

P2= 2020 lb/ft^2 gage=14 psig.

Torricillis theorem

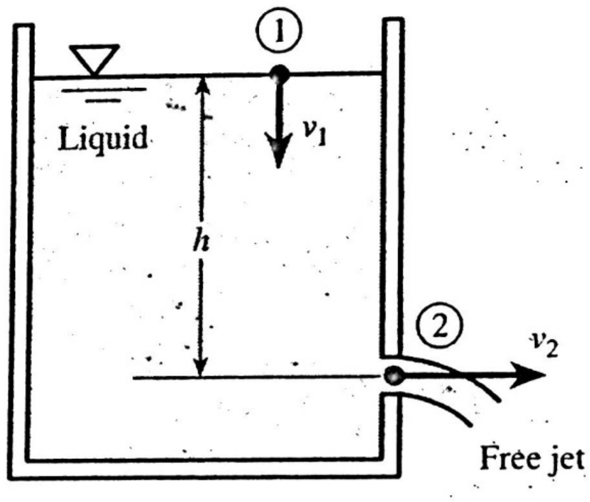


Fig.6

The velocity of a free jet of fluid is equal to:

$\frac{P_1}{\gamma} + \frac{v_1^{\,2}}{2g} + Z_1 = \frac{P_2}{\gamma}\ \frac{v_2^{\,2}}{2g} + Z_2$
Solving the eq. for V2 : $V2=\sqrt{2gh}$

Because:

1-P1=P2 atmosphere pressure

2-v1 negligible cross sec. A1 very large compared with A2 so

$v1=\frac{A2}{A1}\,v2$ (very small number)

3-Z2 can be taken as a zero number.

EX6

For the system in fig.6, h=36 ft and the diameter of the side opening is 2 in, find the jet velocity and the volume flow rate in gpm?

V2=$\sqrt{2gh}$

=$\sqrt{2 * 32.2 * 36}$ =48.3 ft/s

Q=A2v2=$\frac{\pi}{4}$ $(2)^2$*48.3*12^3*$\frac{1}{231\ in^3}$*60

=473gpm

Ex.7

In fig. 7 how do the magnitude of the velocity of the three jets compare with each other?

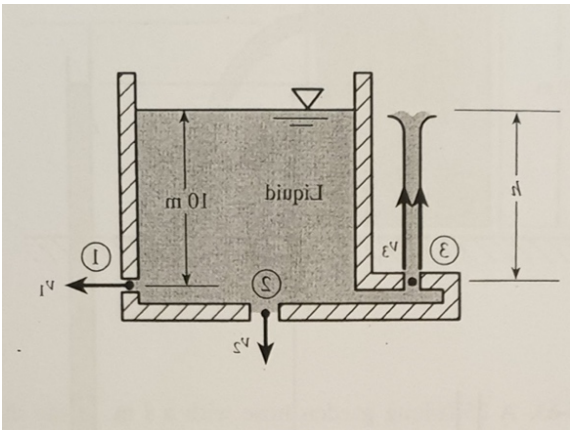


Fig. 7

From the torticollis theorem

V=$\sqrt{2\ g\ h}$

V1=v2=v3= $\sqrt{2 * 9.8 * 10}$ = 14 m/s

The siphon

It is a device that is used to cause a liquid to flow from one container in an upward direction downward into a second. as shown in Fig. 6,

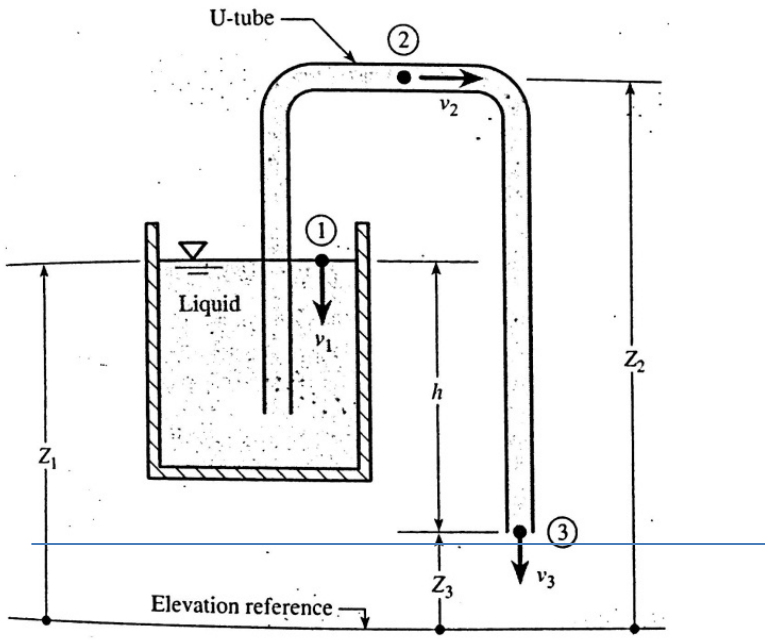


Fig.9

1-Point 1 lies on the free surface of the container.

2-Point 2 lies in the U-tube at its highest elevation.

3-Point 3 lies in the U-tube at the lowest elevation

4-The output at 3 is a free jet.

If we apply Bernoullis eq. for points 1 &3

$P_1=P_3$, $v1\approx 0$, $Z_1-Z_3=h$	
$V_3=\sqrt{2gh}$	So $Q_3 = A_3 V_3$
$P2=\gamma(Z1-Z2)+\gamma(-v_2^2/2g)$	

Ex9

Water is siphoned from a large storage tank through a 50 mm diameter hose(fig.10). Find the maximum height H of a building over which the water can be siphoned. If P2= (-98.66) KPa

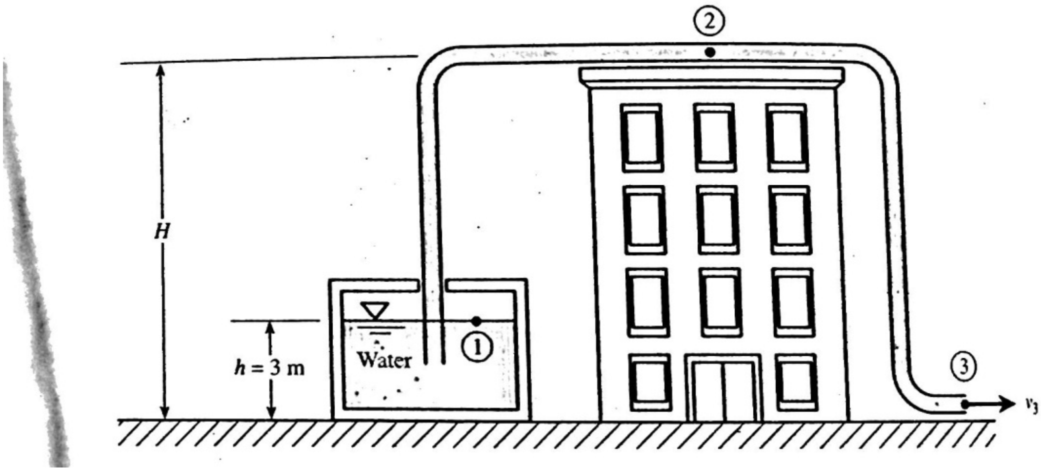


Fig.10

$$\frac{P_1}{\gamma} + \frac{v_1^{\,2}}{2g} + Z_1 = \frac{P3}{\gamma} + \frac{v\,3^2}{2g} + Z_3$$

$V3=\sqrt{2gh}$

$V3=\sqrt{2*9.8*3}$ =7.67 m/s

V2=v3

$$\frac{P_1}{\gamma} + \frac{v_1^{\,2}}{2g} + Z_1 = \frac{P_2}{\gamma}\ \frac{v_2^{\,2}}{2g} + Z_2$$

$$0+0+3= -\frac{98.66*1000}{9800} + \frac{7.67^\wedge 2}{2*9.8} + H$$

H = 10.06 M

Ex10

The siphon in fig. 11 is filled with water and discharged at 150 L/s, find the velocity in the pipe, and the pressure at point 2?

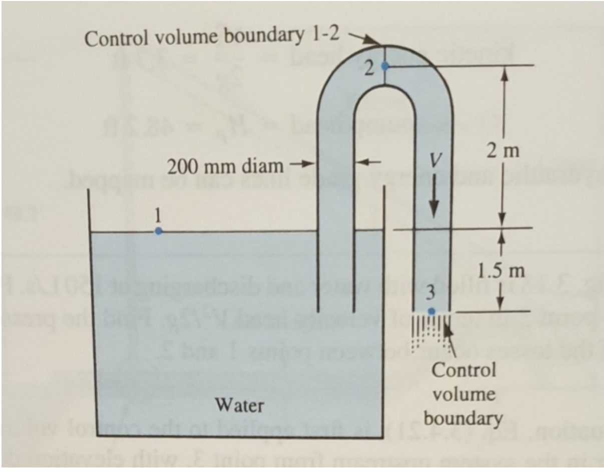


Fig.11

Q = V*A

V = 0.15 / 0.2^2* $\frac{\pi}{2}$ = 4.77 m/s

$$\frac{P_1}{\gamma} + \frac{v_1^{\,2}}{2g} + Z_1 = \frac{P_2}{\gamma} \frac{v_2^{\,2}}{2g} + Z_2$$

$$0+0+0 = \frac{P_2}{\gamma} \frac{4.77^2}{2*9.8} + 2$$

P = 31010.03 Pa

Ex. 11

Water is flowing upward through the pipeline (fig.12), a manometer measured the difference P1-P2. Find the volume of the flow rate?

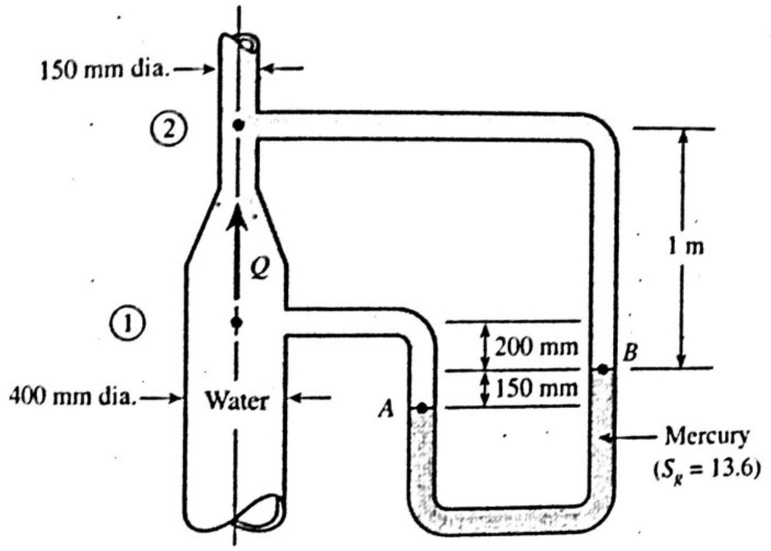


Fig.12

P2+ 9800*1.00 + 13.6 *9800*0.15 – 9800 * 0.350 = P1

P1-P2=26370 pa

$$Z_1+\frac{v_1^{\,2}}{2g}+\frac{P_1}{\gamma}=Z_2+\frac{v_2^{\,2}}{2g}+\frac{P_2}{\gamma}$$

A1v1=A2v2

V2=A1v1/A2

$=(\frac{400}{150})^{2}$*v1=7.11 v1

$\frac{v_2^{\,2}-v_1^{\,2}}{2g}$=(Z1-Z2) + (P1-P2)/γ

$\frac{(7.11v_1)^2-v_1^{\,2}}{2*9.81}=-(1-0.2)+\frac{26370}{9800}$

$\frac{49.6v_1^{\,2}}{19.6}$=-0.8+2.69=1.89 m

V1=0.89 m/s

Q1=A1v1=$\frac{\pi}{4}(0.4)^2$*0.86=0.108 m^3/s

The pitot-static tube:

Fig 13. Shows a pitot tube installed in a pipeline along with a piezometer, both are pressure measuring devices, Unlike, piezometer, pitot tube continues inward toward the centerline of the pipe. As a result, the fluid that enters the inside of the pitot tube comes to a stop (stagnates), in contrast, the piezometer tube allows the fluid to travel past without any change of velocity.

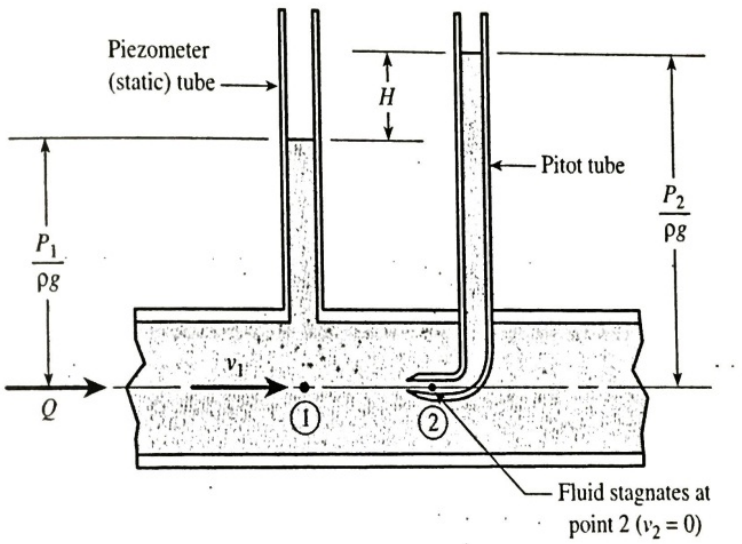


Fig.13

$$\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + Z_1 = \frac{P_2}{\gamma}\ \frac{v_2^2}{2g} + Z_2$$

Z1 =Z2 and V2 = 0

$$H = \frac{V^2}{2g}$$

Ex. 12

Determine the flow rate through the pipe?

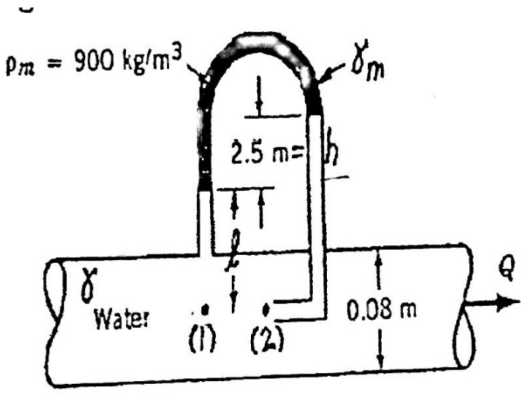


Fig. 14

Z1=Z2 v2=0

$$\frac{v_1^{\,2}}{2g}+\frac{P_1}{\gamma}=\frac{P_2}{\gamma}$$

$$V1=\sqrt{\frac{2g\,(P2-P1)}{\gamma}}$$

P1-γ L-γ $_m$h+γ(L+h)=P2

P2-P1= $(\gamma-\ \gamma m)h$

$$\text{So v1=}\sqrt{\frac{2g\left(\gamma-\gamma m\right)h}{\gamma}}$$

$$=\sqrt{\frac{2*9.81(9800-900*9.8)*2.5}{9800}}$$

=2.2. m/s

Q=A1V1=$\frac{\pi}{4}(0.08)^2$ (2.2)=0.0111 m^3/s.

Verturi, nozzle, and orifice flow meters:

Three additional used flowmeters measuring that operate on the principles of Bernoullis eq., venture, nozzle, and orifice flow meters. They are based on the principle that as the velocity of flow increases, a drop in pressure occurred. The pressure drop measurement can be used to indicate the flow rate.

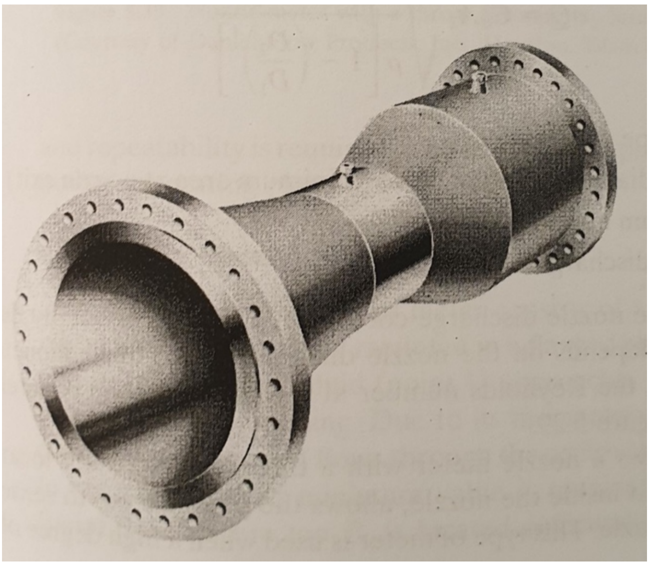


Fig. 16

Figs16 and 177, shows an actual venturi meter, and a venturi meter is installed to the pipeline whose flow rate is measured, venture meter consists of a converging section followed by a constant diameter section (throat) followed by a diverging section.

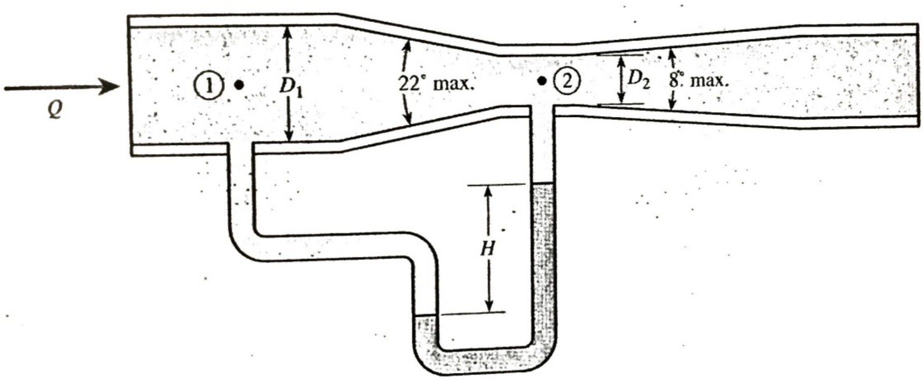


Fig. 17

$$\frac{P_1}{\gamma} + \frac{v_1^{\ 2}}{2g} + Z_1 = \frac{P_2}{\gamma}\ \frac{v_2^{\ 2}}{2g} + Z_2$$

Z1=Z2

A1v1=A2v2

V1=A2*V2/A1

$$Z_1+\frac{A2*V2/A1^{2}}{2g}+\frac{P_1}{\gamma}=Z_2+\frac{v_2^{2}}{2g}+\frac{P_2}{\gamma}$$

$$\frac{P_1}{\gamma}-\frac{P_2}{\gamma}=\frac{v_2^{2}}{2g}-\frac{A2*\frac{V2}{A1}^{2}}{2g}$$

$$\frac{P_1}{\gamma}-\frac{P_2}{\gamma}=V2^{2}\left(\frac{1}{2g}-\frac{\frac{A2}{A1}^{2}}{2g}\right)$$

$$V_2^{2}=\frac{P_1}{\gamma}-\frac{P_2}{\gamma}\Big/\left(\frac{1}{2g}-\frac{\frac{A2}{A1}^{2}}{2g}\right)$$

So V2 $ideal=\sqrt{\frac{2(P1-P2)}{\rho\left(1-\left(\frac{A2}{A1}\right)^{2}\right)}}$

To calculate the flow coefficient Cv=$\frac{V2\ actual}{V2\ ideal}$

Q=V2 actual *A2= Cv* V2 ideal* A2

$$\boxed{Q=Cv\ A2\sqrt{\frac{2(P1-P2)}{\rho\left(1-\left(\frac{A2}{A1}\right)^{2}\right)}}=Cv\ A2\sqrt{\frac{2(P1-P2)}{\rho\left(1-\left(\frac{D2}{D1}\right)^{2}\right)}}}$$

Fig.18, shows the nozzle meter contains a nozzle has a flange on its upstream face.

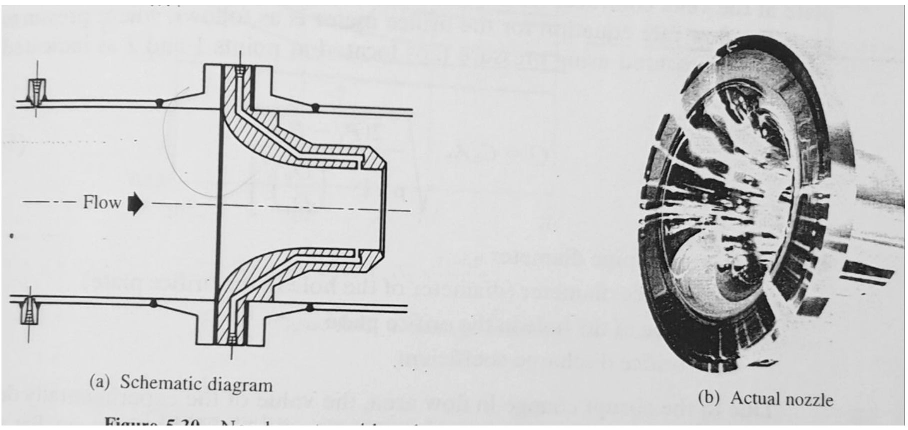
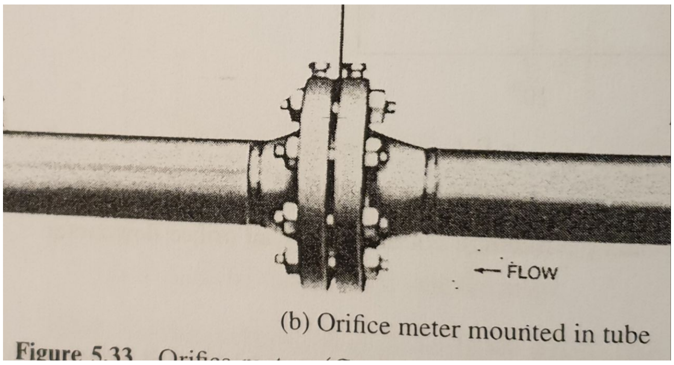


Fig.18

$$Q = Cn\ A2 \sqrt{\frac{2(P1-P2)}{\rho\left(1-\left(\frac{D2}{D1}\right)^{2}\right)}}$$ Cn discharge coefficient.

Fig.19, shows the orifice meter it is a simple flow meter consist of a circular plate containing a sharp edge, the upstream fluid approach the orifice it must turn inward to inter the orifice. The fluid jet cannot immediately change directions as its flow, thus the jet area continues to contract until its minimum value is obtained downstream:



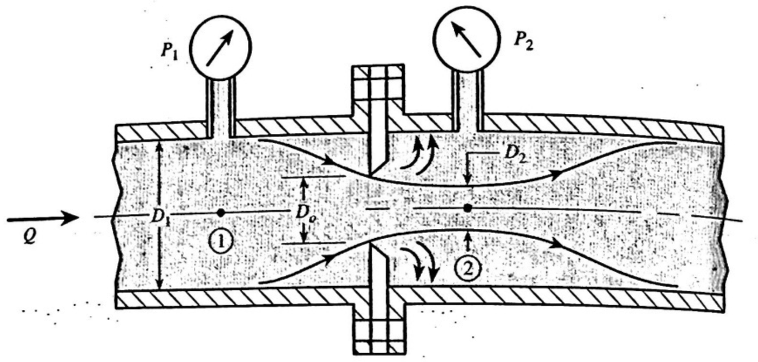


Fig.19

$$Q = C0\ A0 \sqrt{\frac{2(P1-P2)}{\rho\left(1-\left(\frac{D0}{D1}\right)^{2}\right)}}$$ A0: area of the hole in the orifice plate.

Ex 14:

The venture meter of fig.17, has the following data: D1 3in, D2 0.75 in, Cv 0.98, fluid in the manometer is mercury, P2-P1=10 psi, ~~H=20 in Hg,~~ find the flow rate in gallons per minute ?

$$Q = Cv\ A2\sqrt{\frac{2(P1-P2)}{\rho(1-\left(\frac{A2}{A1}\right)^2)}}$$

$A2 = \frac{\pi}{4}(\frac{0.75}{12})^2 = 0.00307\ ft^2$

ρwater =1.94 slugs/ft^3

P1-P2=10*144=1440 lb/ft^2

$\left(\frac{A2}{A1}\right)^2 = (\frac{0.75}{3})^4 = 0.00391$

$Q = 0.98 * 0.00307 * \sqrt{\frac{2*1440}{1.94(1-0.00391)}} = 0.116\ ft^3/s$

$Q = 0.116*1728*\frac{1}{231}*60 = 52.1$ gpm

Ex. 15

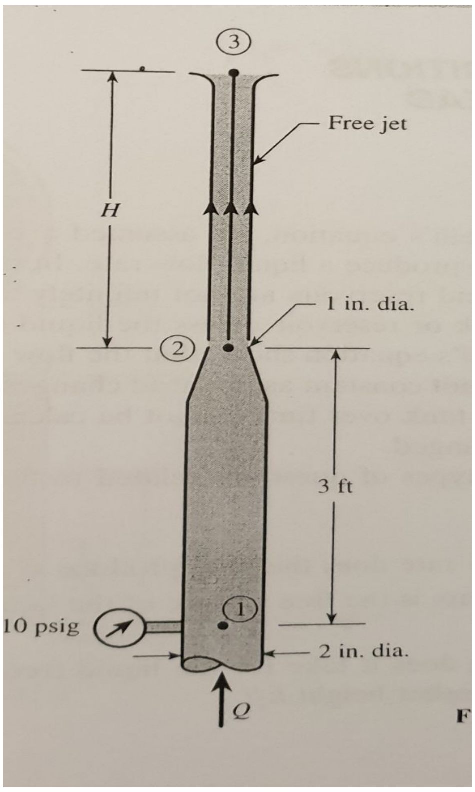
An orifice flow meter consists of a 100 mm dia. pipe with a 50 mm dia. sharp edge orifice. When water flows through the orifice a U- tube manometer is connected indicating a differential head of 350 mm mercury, if the discharge coefficient is 0.65, find the flow?

$$Q = C0\ A0\sqrt{\frac{2(P1-P2)}{\rho(1-\left(\frac{D0}{D1}\right)^2)}}$$

$$Q = 0.62 * 0.0025\ \frac{\pi}{4}\ \frac{\sqrt{2*(0.35*13.6*9800)}}{\sqrt{1000*(1-\frac{0.0025}{0.01})}}$$

Q= 0.0135 m3/s

Extra examples:



Q1-For the figure above find Q?

Q2-Water flows through a venturi meter an inlet dia. = 100 mm and throat dia. =50 mm , P at inlet = 70 kpa. And at the throat 75 mm mercury, if the discharge coefficient = 0.95 , find the flow?