((Fluid Mechanics))

((CH-1))

Fundamentals المبادئ الاساسية

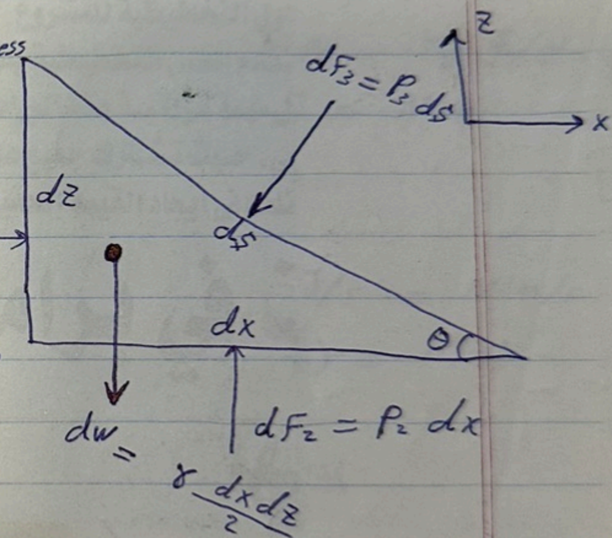
Physical Characteristics of the Fluid State :-

① Space between molecules and mantion of molecules (inter molecular force)

② Fluid as a continuum (متصل).

③ Action under stress. No shear stress in a static fluid.

④ Stresses @ a point in static fluid.



$\Sigma F_x = 0$

$\Rightarrow P_1\, dz - P_3\, ds \sin\theta = 0$

$\Sigma F_z = 0 \Longrightarrow$

$P_2\, dx - \gamma\, dx\, dz/2 - P_3\, ds \cos\theta = 0$

$P_1\, dz - P_3\, ds \cdot \frac{dz}{ds} = 0$

$\Rightarrow \quad P_1 = P_3$

$P_2\, dx - \gamma \frac{dx\, dz}{2} - P_3\, ds\, \frac{dx}{ds} = 0$

$P_2 - \frac{\gamma\, dz}{2} - P_3 = 0$

@ a point $dz \to 0 \Longrightarrow P_2 = P_3$

$\therefore P_1 = P_2 = P_3$

⊛ Pressure is scalar quantity.

⊛ Forces are Vector "

⑤ Pressure imposed on fluids @ rest are transmitted

Dimension and Unites :-

Dimension	Unite
Length (L)	metre (m)
Mass (M)	Kilogram (kg)
time (t)	second (S)
tempeture (T)	kelvin (K) المطلقة
	degree Celsius (°C) المئوية

$$\boxed{K = T + 273}$$

Derived unite :-

Frequency (f)	hertz	$Hz = s^{-1}$
Force (F)	newton	$N = kg\ m/s^2$
energy (E)	joule	$J = N.m$
طاقة Work (W)		=
heat (Q)		=
power (P)	Watt	$J/s = N.m/s$
pressure (p)	pascal (Pa)	$= N/m^2$

F.L.T = Force - length - time
M.L.T = mass - length - time

Multipliers and Sub-multipliers :- الاجزاء والمضاعفات

Kilo	$K = 10^3$	milli	$m = 10^{-3}$
Mega	$M = 10^6$	micro	μ (mu) $= 10^{-6}$
Giga	$G = 10^9$	nono	$n = 10^{-12}$

Density (ρ) :- Mass of fluid contained in unit volume.

Weight density (γ) :- Weight of fluid contained in volume, the gravitational attractive force acting on matter in unit volume.

$$M \cdot g = W \Longrightarrow \boxed{\rho g = \gamma}$$

Unit of ρ (M.L.T) $\Longrightarrow$ $Kg/m^3 = N \cdot s^2/m^4$

γ (F.L.T) $\Longrightarrow$ $N/m^3 = Kg/m^2 \cdot s^2$

Specific Volume (Sv) :- Volume per unit mass. $\left(\frac{m^3}{Kg}\right)$

Relative Density (r.d.) :- The ratio of density a substance to the density of water @ a specificied temperature and pressure.

$$\boxed{r.d. = \frac{\gamma_{sub}}{\gamma_{water}}}$$

* For gases, the equation of state :-

$$\rho = \frac{P}{RT} \quad \text{or} \quad \gamma = \frac{gP}{RT}$$

Where P = absolute pressure (Pa)

T = temperature in (K)

R = Engineering gas constant (N.m/Kg.K)

Consider two gases @ the same temperature and prossure :-

$$R_1 = \frac{P}{\rho_1 T} \quad , \quad R_2 = \frac{P}{\rho_2 T}$$

$$\therefore \frac{R_1}{R_2} = \frac{\rho_2}{\rho_1}$$

According to Avogadro's principle all gases

@ the same temperature and pressure contain the same number of molecular per unit volume for our case, the density of the gas must be proportional to its molecular mass (m).
كتلة الجزيئة

$$\frac{\rho_2}{\rho_1} = \frac{R_1}{R_2} = \frac{m_2}{m_1}$$

$$\therefore m_1 R_1 = m_2 R_2 = \cdots = m_n R_n = \mathcal{R}$$

Where $\mathcal{R}$ = Universal gas constant الثابت العام للغازات
= 8313 N.m/kg.mol.K
جزيئي

Ex:- Find Weight density, specific volume, and density for CO_2 @ temperature 100°C and atmospheris pressure (101.3 kpa)?

Sol:- $mR = \mathcal{R}$

الكتلة الذرية للكاربون = 12 وللأوكسجين = 16

$$\therefore m = 12 + 16 * 2 = 44$$

$$R = \frac{8313}{44} = 189 \text{ N.m/kg.K}$$

$$\rho = \frac{P}{RT} = \frac{101300 \text{ Pa}}{189\ (100+273)} = 1.44 \text{ kg/m}^3$$

$$\gamma = \rho g = 1.44 * 9.81 = 14.1 \text{ N/m}^3$$

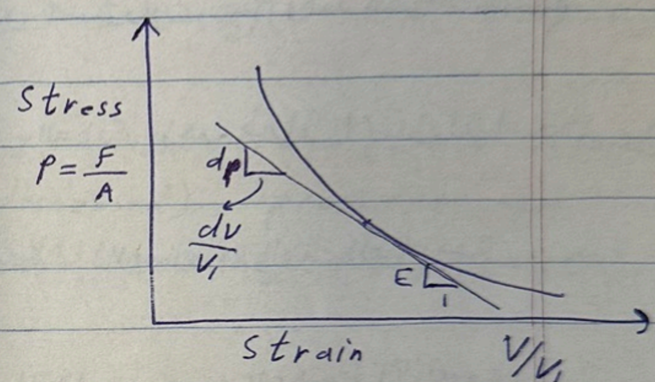
$$S.V. = \frac{1}{\rho} = 0.69 \text{ m}^3/\text{kg}$$

Compressibility and Elasticity :- الانضغاطية والمرونة

صلابة الشكل

Since fluids do not posses rigidity of form, compressibilty defined on the basis of volume modulus :

$$\boxed{E = -\frac{dP}{dV/V_1}} \quad \text{------} \ (1)$$



E = bulk modulus of elasticity (Pa)

V_1 = initial volume

V = Volume

For liquids (E) is usually defined $\frac{V}{V_1} = 1$

For gases :- $\frac{P}{\gamma} = \text{const.}$ (isothermal) --- (2)

متساوي الحرارة

for a frictionless process process no heat exchange

$$\frac{P}{\gamma^k} = \text{const.} \quad \text{-----} (3) \quad \text{(isentropic)}$$

متساوي قصور الحراري

Where $k = \frac{c_p}{c_v}$ = adiabatic exponent

c_p = حرارة الغاز تحت ضغط ثابت

c_v = حرارة الغاز تحت حجم ثابت

$$\therefore \quad \frac{dV}{V} = -\frac{d\gamma}{\gamma} = -\frac{d\rho}{\rho} \quad ----(4)$$

subs. in Eq. (1) $\Longrightarrow$ $$E = \frac{dP}{d\gamma/\gamma} = \frac{dP}{d\rho/\rho} \quad ---(5)$$

$$E = \frac{c\, d\gamma}{d\gamma/\gamma} = \underbrace{c}_{\text{const.}}\gamma = P \quad ----(6)$$

isothermal

$$E = KP \quad ----(7)$$

(isentropic)

اضطراب الضغط

<u>**Pressure Disturbances**</u> :- Wave through elastic fluids cas a Wave of increased (or decreased) pressure and density @ a finite velocity or celerity (سرعة موجية) (a).

$$a = \sqrt{\frac{dP}{d\rho}} = \sqrt{\frac{E}{\rho}} \quad (m/s) \quad ----(8)$$

The disturbances caused by a sound wave moving through a fluid is so small and rapid (مفاجئة) that heat exchange in compression and expansion may be neglected, i.e. isentropic so :-

$$E = KP$$

$$\therefore \quad a = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{KP}{\rho}} \Longrightarrow \boxed{a = \sqrt{KRT}} \quad ----(9)$$

since $\frac{P}{\rho} = RT$

Ex:- Aire @ 15°c and 101.3 kpa is compressed isentropic-cally so that its Volume reduced 50%. Calculate final pressure and temperature and Sonic Velocities before and after compression ?

initial wt. density $\gamma_1 = \dfrac{g P_1}{R T_1}$

$$\gamma_1 = \frac{9.81 * 101300}{286.8\ (273+15)} = 12 \quad N/m^3$$

من جدول

$\gamma_2 = 2\gamma_1 = 24 \quad N/m^3$ (تتضاعف الكثافة لأن الحجم قل الى النصف)

$$\frac{P_1}{\gamma_1^{\,k}} = \frac{P_2}{\gamma_2^{\,k}} \quad \text{(isentropic)} \Longrightarrow P_2 = P_1\left(\frac{\gamma_2}{\gamma_1}\right)^{k}$$

من جدول

$$P_2 = 101.3\,(2)^{1.4} = 267.3 \quad kPa$$

For Temp. :-

$$T_2 = \frac{g P_2}{\gamma_2 R} = \frac{9.81\,(267300)}{24\,(286.8)}$$

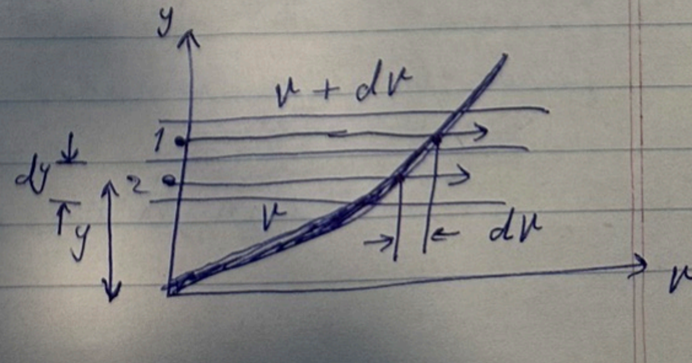
$$= 381 \ K = 108^{\circ}C$$

$$a_1 = \sqrt{KRT_1} = \sqrt{1.4 * 286.8 * 288} = 340 \ m/s$$

$$a_2 = \sqrt{KRT_2} = \sqrt{1.4 * 286.8 * 381} = 391 \ m/s$$

Viscosity :- اللزوجة

Due to cohesion and modecular momentum exchange between fluid layer, thus sheer stress (or tangential stress) appears between moving layers friction.



relative strain $= \dfrac{dS}{dy}$

rate of strain $= \dfrac{dS/dt}{dy}$

$= dv/dy$

For laminar flow (الجريان الطباقي)

$$\boxed{\tau = \mu \frac{dv}{dy}} \quad \text{------(1)}$$

Where τ = Shear stress (Pa)

μ = coefficient of Viscosity (dynamic Viscosity) (Pa . S)

$\frac{dv}{dy}$ = rate of relative strian
= Velocity gradient (S^{-1})

Fluid friction or resistance :-

1. τ and μ are indpendent of pressure.
2. Any τ will cause flow since it causes $\frac{dv}{dy}$ to be different from zero.
3. @ $\tau = 0 \xrightarrow{\text{when}} \frac{dv}{dy} = 0$
4. Velocity profile cannot be tangent to a boundary infinite velocity gradient ($\frac{dv}{dy}$), thus infinite τ.
5. Equation (1) applies to laminar flow.
6. No slip (انزلاق) @ boundary (تمس).

Units of μ :- $\tau = \mu \frac{dv}{dy}$

$$F L^{-2} \doteq \mu \frac{L T^{-1}}{L}$$

$$\mu \doteq F L^{-2} T \doteq \frac{N}{m^2} \cdot S \doteq Pa \cdot S$$

poice = dayne . Sec / cm^2

$$Pa \cdot s = \frac{N}{m^2}\, s = \frac{10^5}{10^4}\, \frac{dyne \cdot s}{cm^2} = 10 \text{ poise}$$

Kinematic Viscosity :- (ν)

$$\boxed{\nu = \frac{\mu}{\rho}} \quad (m^2/s)$$

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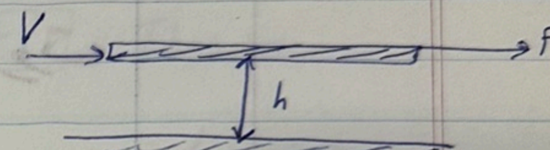
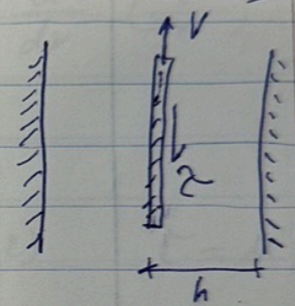
* For constant Velocity (V)

so $\rightarrow$ $\boxed{\frac{dv}{dy} = \frac{V}{h}}$





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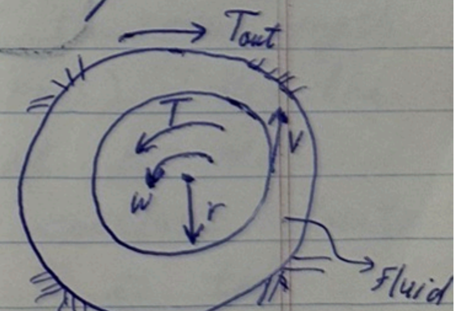
* For coaxial cylinders the torque (T) is equal on both surface.

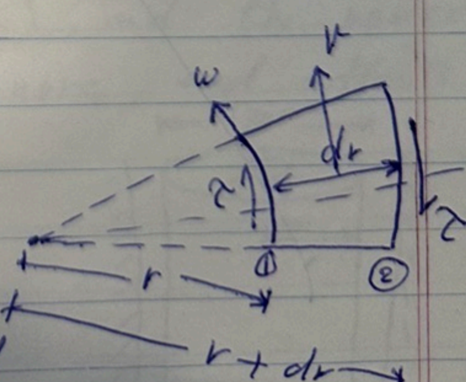


$$T_{out} = T_{inner}$$

$$\tau_o \cdot A_o \cdot r_o = \tau_i \cdot A_i \cdot r_i$$

$$\because \tau_{outer} < \tau_{inner} \Longrightarrow$$

$$\left.\frac{dv}{dy}\right|_o < \left.\frac{dv}{dy}\right|_i$$



Velocity @ outer cylinder is zero and case is different than solid body rotation.

The rate of relative strain of line 1-2 is :-

$$\frac{d\xi/dt}{dr} = \frac{\text{Actual velocity of (2)} - \text{Rigid body velocity of 2 following 1}}{dr}$$

$$= \frac{[(r_1+dr)\omega + (r_1+dr)d\omega] - [(r_1+dr)\omega]}{dr}$$

$$= \frac{(r_1+dr)\,d\omega}{dr} = r_1\frac{d\omega}{dr} + \frac{dr\,d\omega}{dr} = r_1\frac{d\omega}{dr}$$

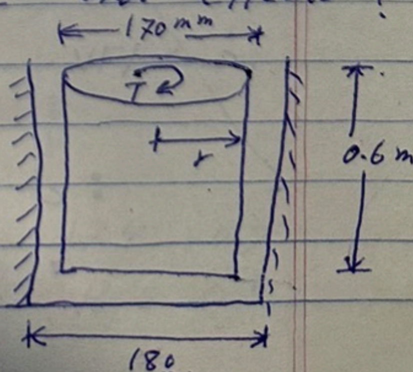
as dr, $d\omega$, and dt vanish in the limit. In terms of the tangential velocity component v, $\boxed{\omega = v/r}$. Thus :-

$$\tau = \mu\,\frac{ds/dt}{dr} = \boxed{\mu r\frac{d\omega}{dr}} = \mu r\frac{d}{dr}\left(\frac{v}{r}\right)$$

$$= \mu r\left[\frac{1}{r}\frac{dv}{dr} - \frac{v}{r^2}\right]$$

$$\tau = \mu\frac{dv}{dr} - \mu\frac{v}{r} \quad \text{---(2)} \qquad \tau = \mu\frac{dv}{dy} \text{--(1)}$$

Ex :- A cylinder 85 mm in radius and 0.6 m in length rotates coxially inside a fixed cylinder of the same length and 90 mm radius. Glycerin ($\mu = 1.48$ Pa.s) fills the space between the cylinders. A torque of 0.7 N.m is applied to the inner cylinder. After constant velocity is attained, calculate the velocity gradients @ the cylinder walls, the resulting (r/min), and the power dissipated by fluid resistance. Ignore end effects ?



Applied torque = resisting torque

$T = -F \cdot r$ [F $\Rightarrow$ shear opposite to applied torque]

$$T = -\tau * \text{area} * r = -\tau\,(2\pi r l)\,r$$

$$\tau = \frac{-T}{2\pi r^2 l} = \mu \cdot r \frac{d\omega}{dr} \quad \text{--- (a)}$$

$$= \mu\, r \frac{d(V/r)}{dr} \quad \text{--- (b)}$$

$$\frac{d\omega}{dr} = \frac{d(V/r)}{dr} = -\frac{T}{2\pi r^3 \mu l}$$

$$\int_{V/r_i}^{0} d(V/r) = -\int_{r_i}^{r_0} \frac{T}{2\pi \mu l} \frac{dr}{r^3}$$

$$\frac{V}{r}\Big|_{\frac{V}{r_i}}^{0} = -\frac{T}{2\pi \mu l}\left(-\frac{1}{2r^2}\right)\Big|_{r_i}^{r_0}$$

$$-\frac{V}{r_i} = \frac{T}{4\pi\mu l}\left[\frac{1}{r_0^2} - \frac{1}{r_i^2}\right]$$

$$V = \frac{-T r_i}{4\mu\pi l}\left[\frac{1}{r_0^2} - \frac{1}{r_i^2}\right]$$

$$= -\frac{0.7\,(0.085)}{4\,(1.48)\,\pi\, 0.6}\left\{\frac{1}{0.09^2} - \frac{1}{0.085^2}\right\}$$

$$= 0.08 \text{ m/s}$$

$$\omega = \frac{V}{r_i} = \frac{0.08}{0.085} = 0.94 \text{ s}^{-1}$$

$$\text{Power } (P) = T \cdot \omega = 0.7\,(0.94) \approx 0.66 \text{ W}$$

from Eq. (2):

$$\frac{dV}{dr} = +\frac{\tau}{\mu} + \frac{V}{r} = \frac{-T}{2\pi\mu l r^2} + \frac{V}{r}$$

$$\left.\frac{dV}{dr}\right)_i = -\frac{T}{2\pi\mu l r_i^2} + \frac{V_i}{r_i} = -17.4 + 0.9$$

$$= -16.5 \text{ s}^{-1}$$

$$\left.\frac{dv}{dr}\right)_o = -15.4 + 0 = -15.4 \ s^{-1}$$

From Eq. (1):- $\quad \tau = \mu \dfrac{dv}{dy}$

$$\frac{T}{2\pi r^2 l} = \mu \frac{dv}{dy} = -\mu \frac{dv}{dr}$$

$$\int_{v_i}^{0} dv = \frac{-T}{2\pi \mu l} \int_{ri}^{r_o} \frac{dr}{r^2} = -0.1255 \int_{ri}^{r_o} \frac{dr}{r^2}$$

$$-v_i = 0.1255 \left[\frac{1}{r_o} - \frac{1}{r_i} \right]$$

$$-v_i = 0.1255 \ (-0.654) = 0.082 \ m/s$$

$$\frac{dv}{dr} = \frac{-T}{2\pi \mu l r^2}$$

$$\left.\frac{dv}{dr}\right|_i = \frac{-0.1255}{0.085^2} = -17.4 \ s^{-1}$$

$$\left.\frac{dv}{dr}\right)_o = \frac{-0.1255}{0.09^2} = -15.5 \ s^{-1}$$