((CH - 2))

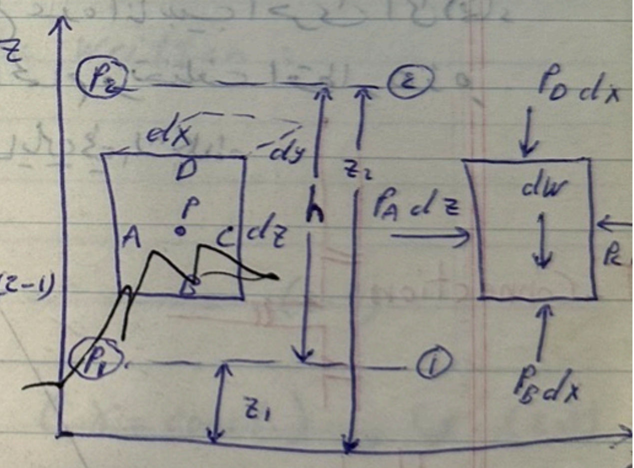
((Fluid Statics))

In static fluids there is no relative motion ($\frac{dV}{dy} = 0$) ~~this~~ between fluid elements, thus no shear stress can exist, Viscosity has no effects.

The basic equation of fluid statics is that relating pressure, density & vertical distance.

Pressure - Density - Height Relationships :-

Applying Newton's first law to the element ($dx\ dz$) and using average pressures on each face :-



$$\Sigma F_x = 0 \Rightarrow P_A \cdot dz - P_C\,dz = 0 \quad \text{--- (2-1)}$$

$$\Sigma F_z = 0 \Rightarrow P_B \cdot dx - P_D\,dx - dW = 0 \quad \text{---- (2.2)}$$

Which P and γ are functions of x and z. Writing P_A, P_B, P_C, and P_D in terms of Pressure (P) @ the center

$P_A = P - \frac{\partial P}{\partial x}\frac{dx}{2}$, $P_C = P + \frac{\partial P}{\partial x}\frac{dx}{2}$

, $P_B = P - \frac{\partial P}{\partial z}\frac{dz}{2}$, $P_D = P + \frac{\partial P}{\partial z}\frac{dz}{2}$,

$dW = \gamma\,dx\,dz$ { where γ = const. as dx & $dz \to 0$ }

Thus, equation (2-1) and (2-2) become :

$$\left[P - \frac{\partial P}{\partial x}\frac{dx}{2}\right] dz - \left[P + \frac{\partial P}{\partial x}\frac{dx}{2}\right] dz = 0$$

$$-\frac{\partial P}{\partial x}\, dx \cdot dz = 0 \implies \boxed{\frac{\partial P}{\partial x} = 0} \quad \text{------} \quad (2.3)$$

((بالاتجاه الأفقي يكون الضغط ثابت))

$$\left[P - \frac{\partial P}{\partial z}\frac{dz}{2}\right] dx - \left\{P + \frac{\partial P}{\partial z}\frac{dz}{2}\right\} dx - \gamma\, dx\, dz = 0$$

$$-\frac{\partial P}{\partial z}\, dz \cdot dx - \gamma\, dx\, dz = 0$$

$$\boxed{\frac{\partial P}{\partial z} = \frac{dP}{dz} = -\gamma} \quad \text{------} \quad (2.3)$$

((بالاتجاه z يكون الضغط متغير فيتناقص الضغط كلما ازداد الارتفاع z))

The second of Eq. (2.3) may be written:

$$\int_{z_2}^{z_1} -dz = \int_{P_2}^{P_1} \frac{dP}{\gamma} \implies$$

$$z_2 - z_1 = \int_{P_2}^{P_1} \frac{dP}{\gamma} \quad \text{------} \quad (2.4)$$

incompressible fluid :- (مائع غير منضغط)

For fluids of constant density ($\gamma = \text{const.}$), Eq. (2.4) becomes:

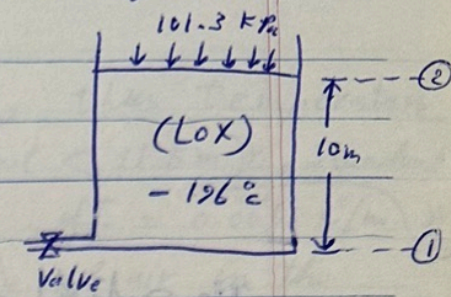
$$\boxed{P_1 - P_2 = \gamma (z_2 - z_1) = \gamma h} \quad \text{------} \quad (2.5)$$

or

$$\frac{P_1}{\gamma} + z_1 = \frac{P_2}{\gamma} + z_2 = \text{const.} \quad \text{------} \quad (2.6)$$

Ex 1 :- The liquid oxygen (LOX) tank of a Saturn moon rocket is partially filled to a depth of 10 m with (LOX) @ −196 °C. The absolute pressure in the vapor above the liquid surface is maintained @ 101.3 kPa calculate the

the pressure @ the inlet valve @ the bottom of the tank ? $\rho = 1206\ kg/m^3$ (من جدول)



$$P_1 - P_2 = \gamma (z_1 - z_2)$$
$$= \gamma h$$

$$P_1 = P_2 + \gamma h$$

$$\gamma_{Lox} = \rho g = 9.81\ (1206)$$
$$= 11.8\ kN/m^3$$

$$P_1 = 101.3 + 10\,(11.8) = 219.3\ kPa$$

✱ For fluids of variable density ($\gamma \neq$ const.) Eq. (2-4) cannot be integrated unless relation between P & γ is known. For gases the polytropic equation :

$$\frac{P}{\gamma^n} = \text{const.} \qquad (2.7)$$

me be used [For adibatic (حالة كظمية) $n = k$, for isothermal $n = 1$]

Since temperature lapse rate (معدل تفاوت درجة حرارة) $(-dT/dz)$ is important in atmosphere, substitute $(\gamma RT/g)$ for P in Eq. (2.7) :-

$$\frac{\gamma RT}{g \gamma^n} = \text{const.}$$

So, temperature lapse rate in atmosphere :-

$$\boxed{-\frac{dT}{dz} = \frac{g}{R}\left(\frac{n-1}{n}\right)} \qquad (2.8)$$

For isothermal atmosphere $\Longrightarrow n = 1 \Longrightarrow -\frac{dT}{dz} = 0$

For adiabatic $\Longrightarrow n = k$ (for air $k = 1.4$)

$$-\frac{dT}{dz} = \frac{9.81}{286.8}\left(\frac{1.4-1}{1.4}\right) = 0.0098\ {}^{\circ}\mathrm{C}/m = 9.8\ {}^{\circ}\mathrm{C}/km$$

In atmosphere $n > 1$ and $-dT/dz > 0$, thus temperature declines with altitude (($n = 1.235$ for level < 11 km)), Standard atmosphere ((@ sea level $T = 15^{\circ}C$, $P = 101.3$ kPa, $-\frac{dT}{dz} = 0.0065\ {}^{\circ}C/m$))

Ex②:- Calculate pressure and weight density of air in the U.S. standard atmosphere @ altitude 10 km ?

from table $\Rightarrow$ $-\frac{dT}{dz} = 0.0065$, $n = 1.235$

$P_1 = 101.3$ kPa, $T_1 = 15^{\circ}c$, $\gamma_1 = 12.01\ N/m^3$

$$\frac{P_1}{\gamma_1^{\,n}} = \frac{P_2}{\gamma_2^{\,n}} \Longrightarrow \frac{P_1}{\gamma_1^{\,1.235}} = \frac{101300}{(12.01)^{1.235}}$$

$$\frac{1}{\gamma_1} = \left(\frac{4702.97}{P_1}\right)^{1/1.235} = \frac{941}{P_1^{\,1/1.235}}$$

$$\frac{dP}{dz} = -\gamma \Longrightarrow -dz = \frac{dP}{\gamma} = \frac{941\, dP}{P^{\,1/1.235}}$$

$$-\int_{z_1}^{z_2} dz = \int_{P_1}^{P_2} 941\,\frac{dP}{P^{0.81}}$$

$$z_1 - z_2 = 0 - 10000 = \left.\frac{941\, P^{0.19}}{0.19}\right]_{101.3}^{P_2}$$

$$\Rightarrow P_2 = 26.3\ \text{kPa}$$

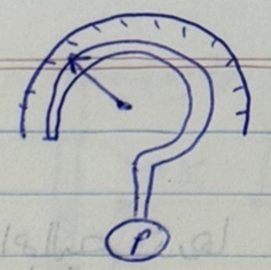
$$-\frac{dT}{dz} = 0.0065\ {}^{\circ}c/m$$

$$T_2 = T_1 + dT \Longrightarrow T_2 = 15^{\circ}c - 0.0065 * 10 * 10^3$$
$$= -50^{\circ}c = 223\ K$$

$$\frac{P_1}{\gamma_1^{\,1.235}} = \frac{P_2}{\gamma_2^{\,1.235}} \longrightarrow \gamma_2 = \gamma_1\left\{\frac{P_2}{P_1}\right\}^{0.18}$$
$$= 4.03\ N/m^3$$

Absolute and Gage Pressures :-

$$\boxed{\text{Absolute pressure} = \text{atmospheric pressure} \begin{array}{l} - \text{Vacuum} \\ + \text{Gage pressure} \end{array}}$$

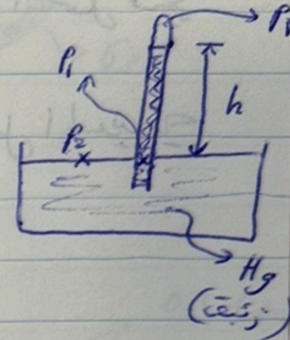


Bourdon gage

$P_1 = P_2$

$P_1 = P_V + h\,\gamma_{Hg}$

$P_V + h\,\gamma_{Hg} = P_2 = P_{atmosphere}$



$-0.17\ Pa + h * 13.57 * 9800 = 0$

$h = 0.0012\ mm\ Hg$

Ex :- A Bourdon gage registers a Vacuum of 310 mm of mercury when the atmospheric (absolute) pressure is 100 KPa. Calculate the corresponding absolute pressure?

$P_{Vac.} = 0.31 * 13.57 * 9800 = 41.3\ KPa$

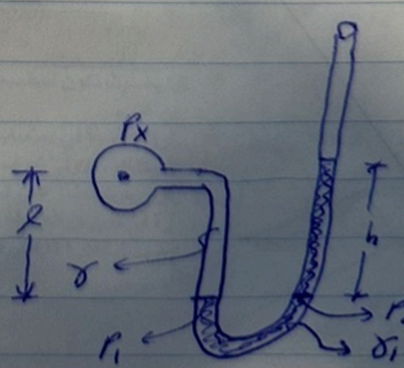
$P_{absolute} = 100 - 41.3 = 58.7\ KPa$

Manometry :-

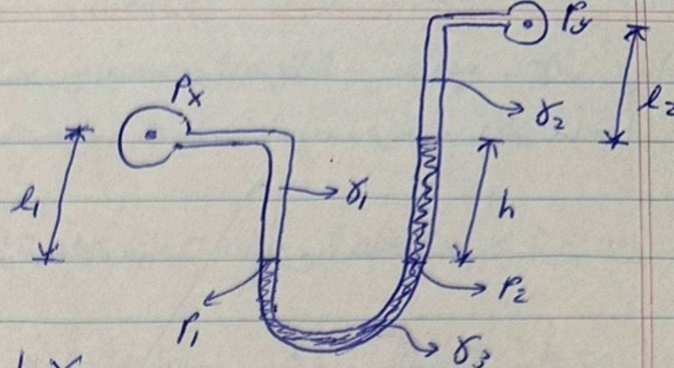
For fig. (a) :-

$P_x + \ell\,\gamma = h\,\gamma_1$

$P_x = h\,\gamma_1 - \ell\,\gamma$



(a)



$P_x + \gamma_1 l_1 = P_1$

$P_y + \gamma_2 . l_2 + h\gamma_3 = P_2$

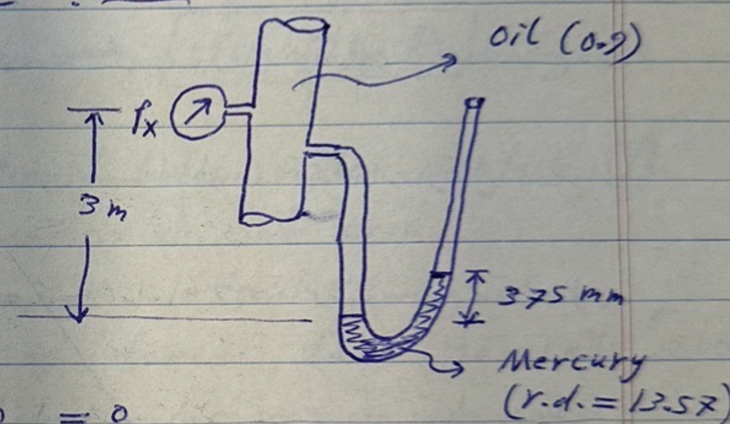
$P_1 = P_2$

$P_x + \gamma_1 l_1 = P_y + \gamma_2 l_2 + h\gamma_3$

$(P_x - P_y) = \gamma_2 l_2 + h\gamma_3 - \gamma_1 l_1$

Ex :-

What will be the gage reading (P_x) ?

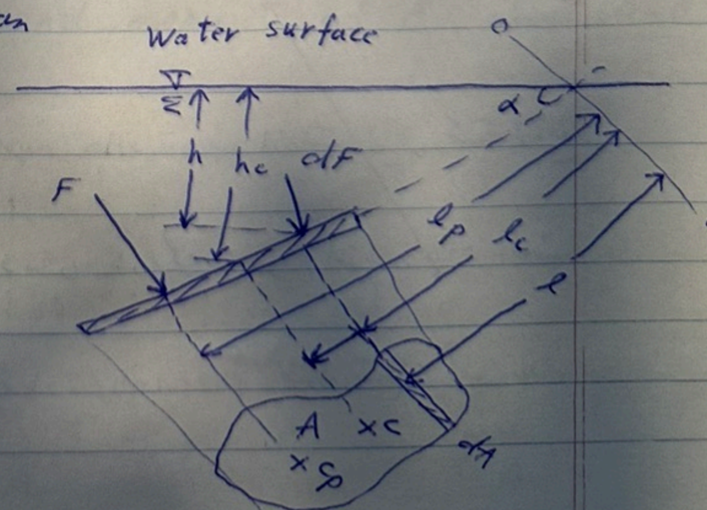


$P_x + (0.9 * 9.8 * 10^3) * 3$

$- 0.375 * 13.57 * 9800 = 0$

$P_x = 23.4$ KPa

Forces on Submerged Plane Surfaces :- القوى على السطوح المستوية المغمورة



The plane area is inclined an angle (α) to the water surface. The force on a differential element (dA) is:

$dF = P\,dA$

(Note that dA is taken so is constant over it)

$h =$ depth below the water surface $= l \sin\alpha$

$l =$ distance from 0-0 to element dA along the plane area.

$dF = \gamma l \sin\alpha \; dA$

$$F = \int_A \gamma l \sin\alpha \, dA = \gamma \sin\alpha \int_A l \, dA$$

Where $\int_A l\,dA =$ first moment of area (A) about 0-0 and equal to ($l_c \cdot A$), where l_c is the distance of centroid of A from 0-0.

$$\therefore \; F = \gamma \sin\alpha \; l_c \; A \Longrightarrow \boxed{F = \gamma h_c A}$$

$h_c =$ the depth of centroid below water surface. The direction of the force is perpendicular to area.

To find the location of force, take moments about 0-0 :

$$dM = l\,dF = l \gamma l \sin\alpha \, dA = \gamma l^2 \sin\alpha \, dA$$

$$M = \gamma \sin\alpha \int_A l^2 \, dA$$

Where $\int_A l^2 \, dA$ is the second moment of A about 0-0.

$$\boxed{M = \gamma \sin\alpha \; I_{0-0}}$$

The location of force is :-

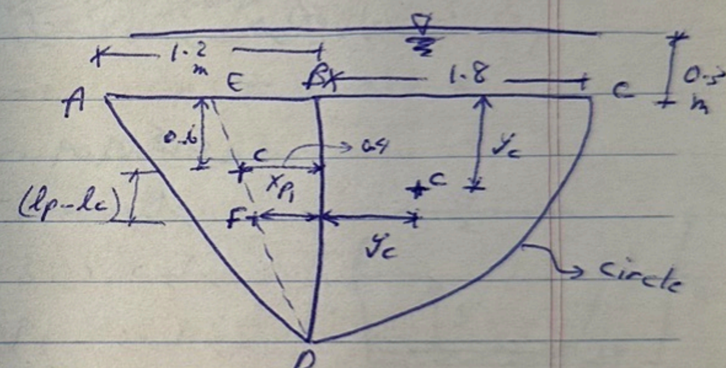
$$l_p = \frac{M}{F} = \frac{\gamma \sin\alpha \; I_{0-0}}{\gamma h_c A} = \frac{\gamma \sin\alpha \; I_{0-0}}{\gamma l_c \sin\alpha \; A}$$

$$= \frac{I_{0-0}}{l_c \cdot A} = \frac{I_c + l_c^2 A}{l_c A} = l_c + \frac{I_c}{l_c A}$$

$$\boxed{\ell_p - \ell_c = \frac{I_c}{\ell_c A}}$$

Where $\ell_p - \ell_c$ = distance between centroid of area and center of pressure (point of application of force).

Ex.:- Calculate magnitude, direction, and location of the total force exerted by the water on one side of this composite area which lies in a vertical plane?



From appendix (6)
for quarter circle

$$\boxed{y_c = \frac{4r}{3\pi}} = \frac{4 * 1.8}{3\pi} = 0.764 \text{ m}$$

$$F = \gamma h_c A$$

$$F)_{triangle} = 9800\,(0.3 + 0.6)\left(\frac{1.2 * 1.8}{2}\right)$$
$$= 9.53 \text{ kN}$$

$$F)_{circle} = 9800\,(0.3 + 0.764)\,\frac{\pi (1.8)^2}{4} = 26.53 \text{ kN}$$

$$\text{Total force} = 9.53 + 26.53 = 36.06 \text{ kN}$$

To find vertical location of force :-

$$\text{For traingle } (\ell_p - \ell_c) = \frac{I_c}{\ell_c A} = \frac{bh^3/36}{(0.3 + 0.6)\frac{bh}{2}}$$

$$= \frac{1.2 * 1.8^3/36}{(0.9)\frac{1.2 * 1.8}{2}} = 0.2 \text{ m}$$

(I) for semi-circle $= \dfrac{\pi d^4}{128}$

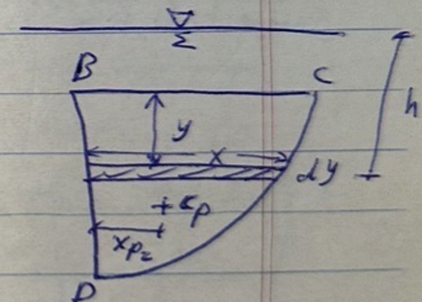
$\therefore$ (I) for quadrant $= \dfrac{\pi d^4}{256} = \dfrac{\pi (2 * 1.8)^4}{256} = 2.061 \; m^4$

$$I = I_c + y_c^2 A \Rightarrow I_c = I - y_c^2 A$$
$$= 2.061 - (0.764)^2 (2.54)$$
$$= 0.576 \; m^4$$

$$l_p - l_c = \frac{I_c}{l_c A} = \frac{0.576}{1.064 * 2.54} = 0.213 \; m$$

$$\frac{0.4 - X_{p_1}}{0.2} = \frac{0.6}{1.8} \longrightarrow X_{p_1} = 0.333 \; m$$

For location of force on quadrant take moment about BD :-



$$dM = \frac{x}{2} \, dF$$
$$= \frac{x}{2} \, \gamma h \, dA$$
$$= \frac{x}{2} \, \gamma h \, x \, dy$$

$$\{ h = 0.3 + y \; , \; x^2 + y^2 = (1.8)^2 \}$$

$$\int dM = \int \gamma \frac{x^2}{2} (0.3 + y) \, dy$$

$$M = \frac{9800}{2} \int_0^{1.8} (1.8^2 - y^2)(y + 0.3) \, dy$$
$$= 18575 \; N.m$$

$$X_{p_2} = \frac{M}{F} = \frac{18575}{26.53 * 10^3} = 0.7 \; m$$

$$36.06 \; X_p = 26.53 * 0.7 - 9.53 * 0.333$$

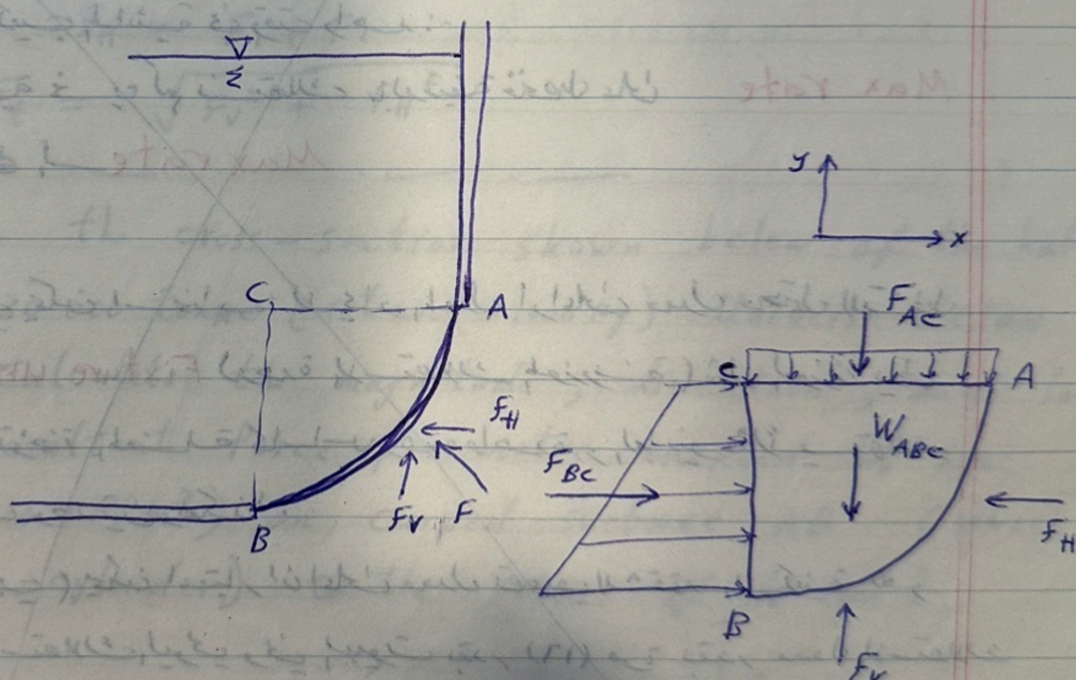
$$X_p = 0.427$$

take moment about (AC) for vertical location:

$$36.06 \; \bar{y} = 26.53 \; (0.764 + 0.213) + 9.53 \; (0.3)$$

$$\bar{y} = 0.93 \text{ m}$$

Forces on Submerged Curved Surface :-



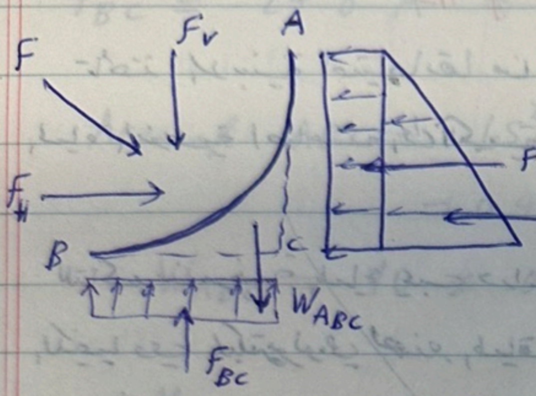
$$\overrightarrow{\Sigma F_x} = 0$$

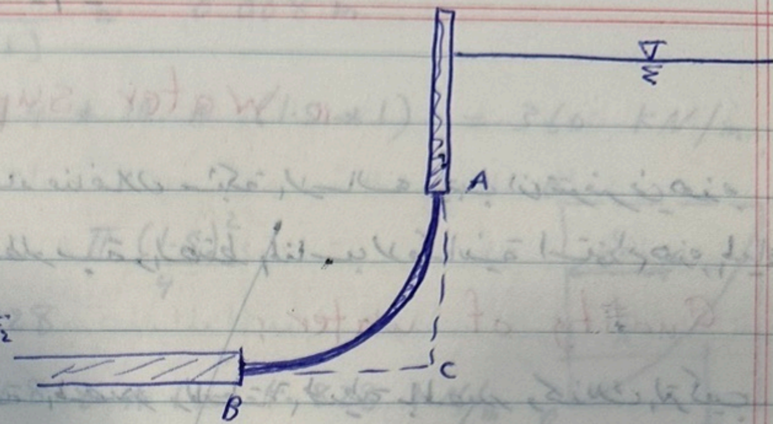
$$F_H = F_{BC}$$

$$\downarrow \Sigma F_y = 0$$

$$F_{AC} + W_{ABC} = F_V$$

$$F = \sqrt{F_H^2 + F_V^2}$$





$$F_V = F_{BC} - W_{ABC}$$

$$F_H = F_1 + F_2$$

$$F_t = \sqrt{F_V^2 + F_H^2}$$

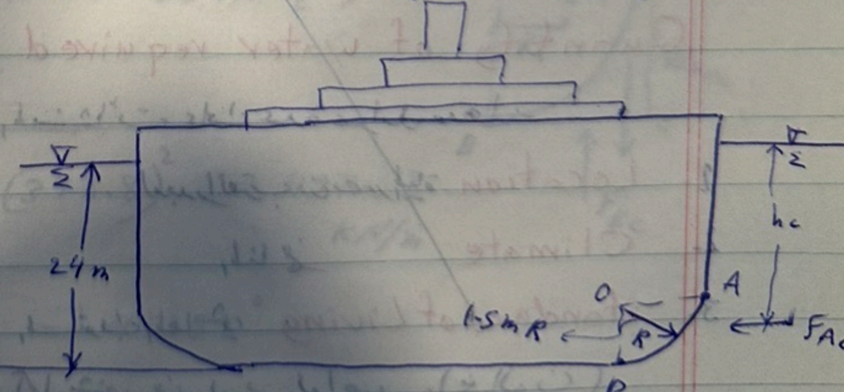
Ex :- Consider the cross-section shown below of the hull of typical 330 000 tonne (1 tonne = 1000 Kg) Universe - class oil tanker. Calculate the magnitude, direction, and location of the resultant force per metre exerted by sea water ($\gamma = 10\ kN/m^3$) on curved surface AB (which is a quarter cylinder) @ the corner of the hull?

$$F_{AC} = \gamma h_c A$$

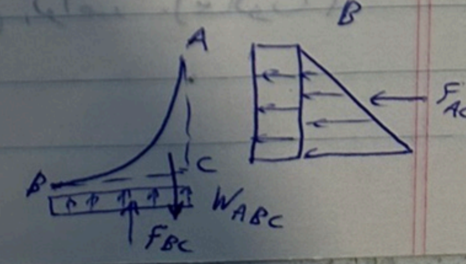
$$= 10 * \left(24 - \frac{1.5}{2}\right)$$

$$(1.5 * 1)$$

$$= 348.8\ kN/m = F_H$$

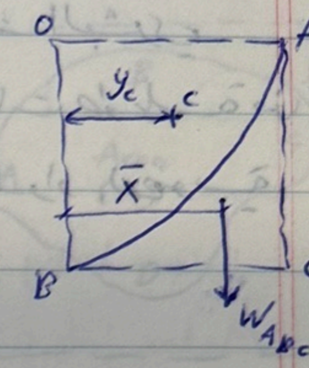


$$l_p - l_c = \frac{I_c}{l_c A} = \frac{bh^3/12}{l_c \cdot A}$$



$$l_p - l_c = \frac{\frac{1.5^3 * 1}{12}}{23.25\ (1.5 * 1)} = 0.008\ m$$

$$F_{BC} = 24\gamma A = 24 * 10 * (1.5 * 1) = 360\ kN/m$$



Area of ABC $= 1.5^2 - \frac{\pi (1.5)^2}{4}$

$= 0.4828$

$$W_{ABC} = 10 * 0.4828 * 1 = 4.828\ kN/m$$

$$\Sigma M_{O-B} = 0$$

$$\frac{\pi r^2}{4} y_c + A_{ABC} * \bar{X} = 1.5^2 \left(\frac{1.5}{2}\right)$$

$$\frac{\pi (1.5)^2}{4} \left(\frac{3 * 1.5}{4\pi}\right) + 0.4828\ \bar{X} = 1.5^2 \left(\frac{1.5}{2}\right)$$

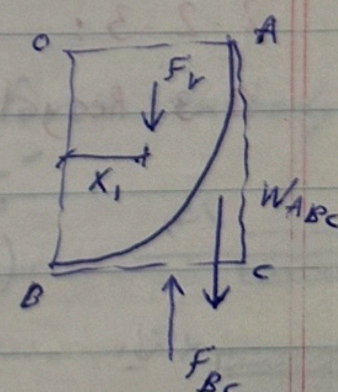
$\bar{X} = 1.165\ m$ to the right of B - O.

$$F_V = F_{BC} - W_{ABC} = 355.2\ kN/m \downarrow$$

Location of Vertical Force :-

$$F_V * X_1 + W_{ABC} * \bar{X} = F_{BC} \left(\frac{1.5}{2}\right)$$

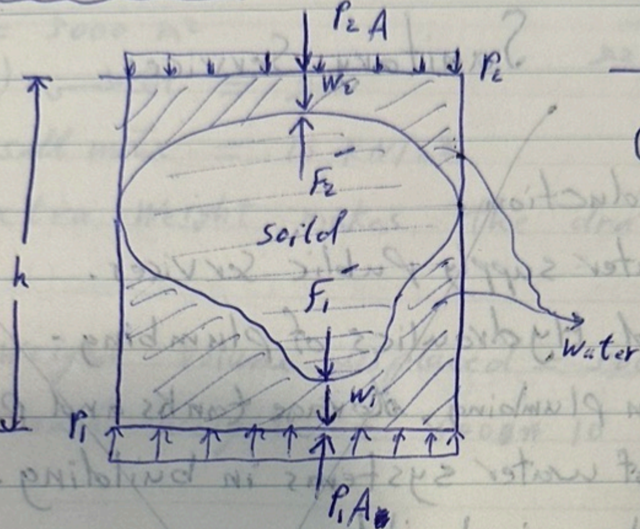
$$X_1 = 0.74\ m$$

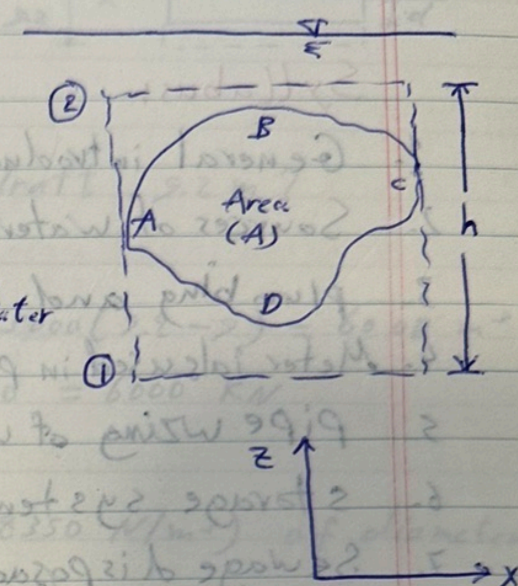


$$F_t = \sqrt{(355.2)^2 + (348.8)^2} = 497.8\ kN/m$$

$$\theta = \tan^{-1} \frac{F_V}{F_H} = 45.52^\circ$$

Buoyancy and Flotation : الطفو والعوم





Free body diagram of Water :-

For upper portion

$\uparrow \Sigma F_z = 0 \implies F_2' - P_2 A - W_2 = 0$

lower portion :-

$\uparrow \Sigma F_z = 0 \implies P_1 A - F_1' - W_1 = 0$

$F_2' = W_2 + P_2 A$

$F_1' = P_1 A - W_1$

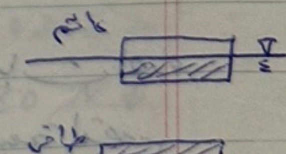
F_B = Buoyancy force = net uppward force on body

= Force up − Force down = $F_1' - F_2'$

$F_B = F_1' - F_2' = (P_1 A - W_1) - (P_2 A + W_2)$

$= (P_1 - P_2) A - (W_1 + W_2)$

$= \gamma h A - (W_1 + W_2)$



The right hand side is the weight of the volume of liquid exactly equal to volume of the body.

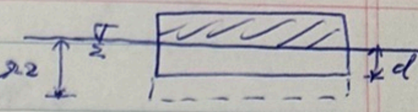
$\therefore$
$$\boxed{\begin{aligned} F_B &= \gamma \text{ (Volume of object)} \\ F_B &= \gamma \text{ (Volume displaced)} \\ F_B &= \text{Weight of object} \end{aligned}}$$

Ex:- A container ship :-

Area = 3000 m^2

draft (d) الغاطس = 9 m

γ for salt water = 10 kN/m^3

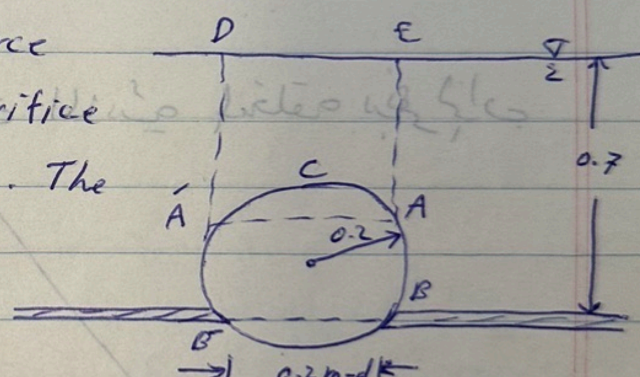


How much extra Weight makes the draft 9.2 m ?

Additional weight volume displaced = 3000 (9.2 − 9) = 600 m^3

Additional weight = 600 * 10 = 6000 kN

Ex:- The Solid Wooden sphere (γ = 8350 N/m^3) of diameter 0.4 m is held in the orifice (0.2 m diameter) by the water. Calculate the force exerted between sphere and orifice plate when the depth is 0.7 m. The sphere will float away if this force becomes zero; can this ever happen ?



Volume of sphere = $\frac{4\pi}{3} R^3 = \frac{4\pi}{3} (0.2)^3$ = 0.0335 m^3

Weight of sphere = 0.0335 * 8350 = 280 N ↓

Volume of sphere outside cylinder (B'BACA') = 0.0218 m^3 (from geometry)

F_B = 0.0218 * 9800 = 214 N ↑

Volume of (A'CAED) = 0.0098 m^3

Force of (CA'CAED) = 9800 * 0.0098 = 96 N ↓

$\therefore$ Net Force (Force between sphere and plate)

$$= 280 + 96 - 214 = 162\ N \downarrow$$

$\because$ $F_{net} = 280 - 214 = 66\ N$

$\therefore$ The force between sphere and plate can never be zero.