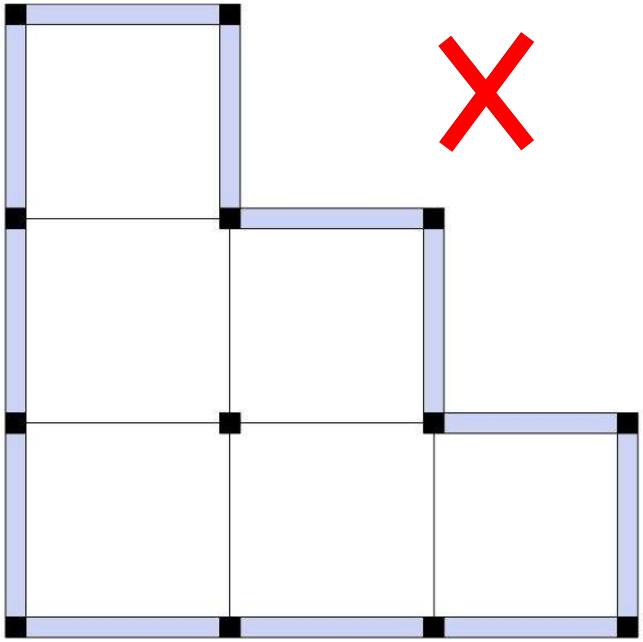
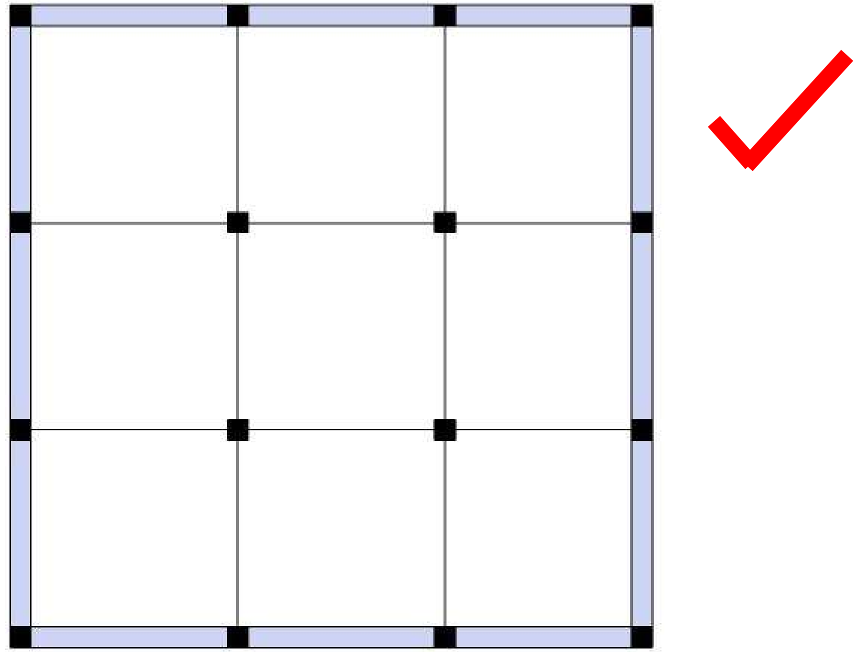
Limitations of Direct Design Method ACI (8.10.2)

1- There shall be a minimum 3 continuous spans in each direction.



Not continuous in each direction



Continuous in each direction

2- the panels shall be rectangular, with a ratio of longer to shorter span center to center of supports within a panel not greater than 2 $\left(\frac{L}{S} \leq 2\right)$

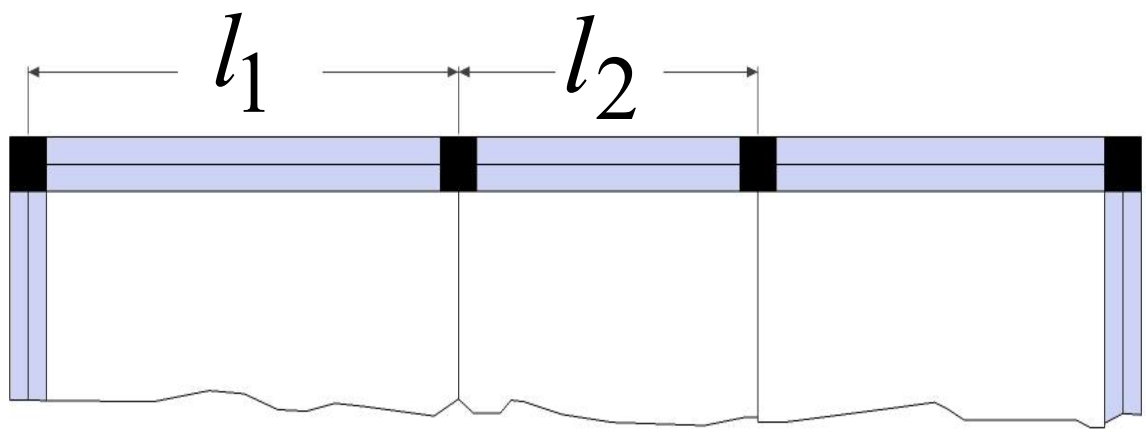
3- Successive span lengths in each direction shall not differ more that 1/3 longer span.

$$l_1 - l_2 \le \frac{1}{3} l_1$$

$$l_1 - \frac{1}{3} l_1 \le l_2$$

$$\frac{2}{3} l_1 \le l_2$$

$$l_1 \le 1.5 l_2$$



Exercise:

$$l_1 = 6$$

$$l_2 = 3.5$$

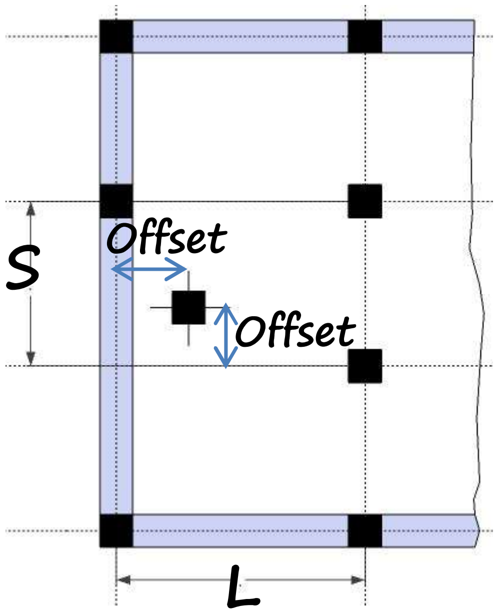
$$6 - 3.5 = 2.5 > \frac{6}{3} = 2$$

Invalid

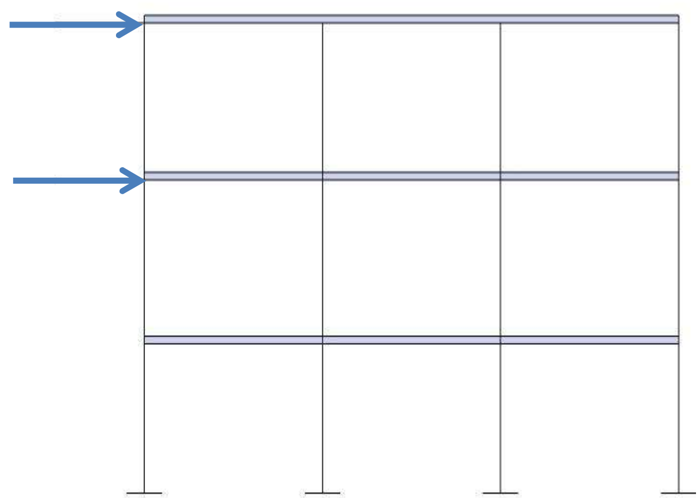
4- Columns may be offset a max 10 % of the span in the direction of offset from either axis between C.L of successive columns.

Offset ≤ 0.1 L , Offset ≤ 0.1 S

5- All loads shall be due to gravity only and uniformly distributed over an entire panel. Unfactored live load shall not exceed 2 times unfactored dead load. (L.L ≤ 2D.L)



Note 4



Note 5

6- If beams are used on column lines, check the following formula for every panel:

$$0.2 \le \frac{(\alpha_f)_1 L_2^2}{(\alpha_f)_2 L_1^2} \le 5$$

Where $(\alpha f)_1$ average flexural stiffness in direction (1)

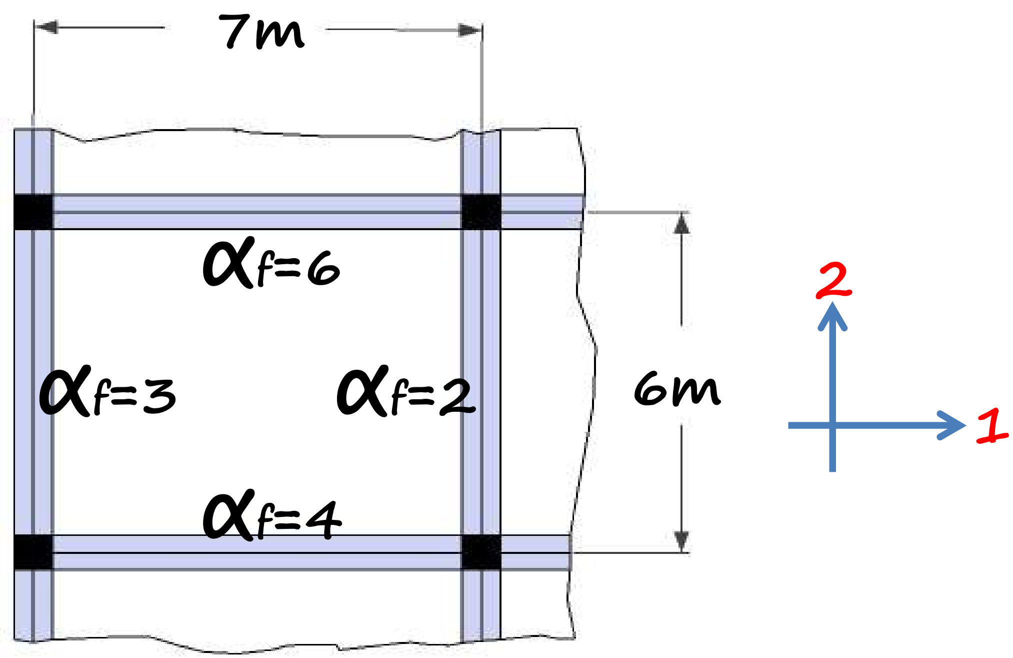
Where $(\alpha f)_2$ average flexural stiffness in direction (2)

Exercise:

$$(\alpha_f)_1 = \frac{6+4}{2} = 5$$

$$(\alpha_f)_2 = \frac{3+2}{2} = 2.5$$

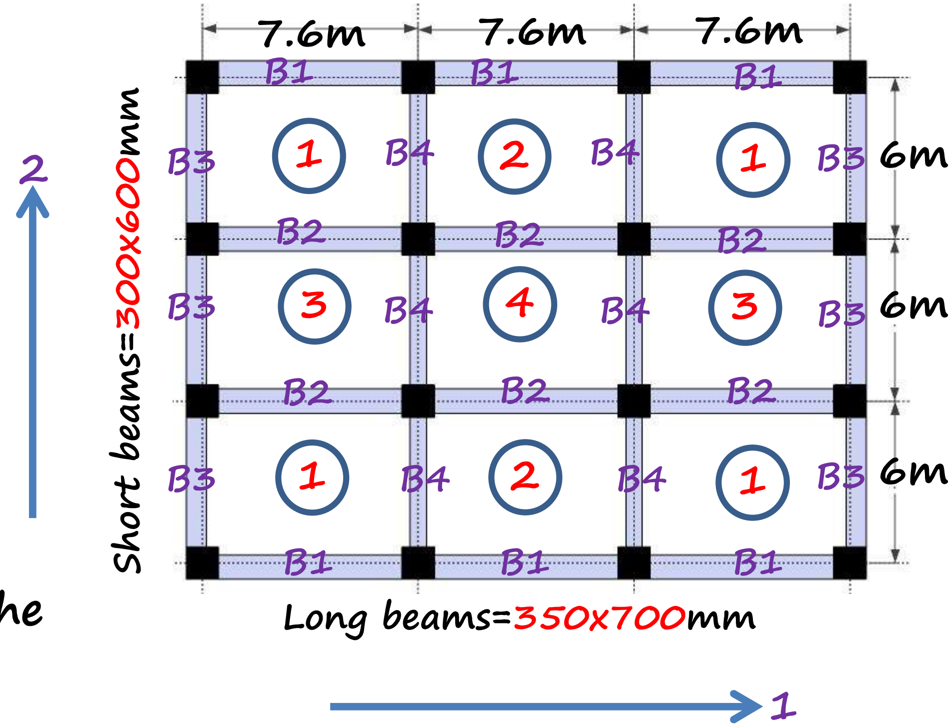
$$0.2 < \frac{5 \times 6^2}{2.5 \times 7^2} = 1.469 < 5$$



Example (6)

Two way slab with beams, L.L=5.75 kN/m², t=165mm, f'c=21MPa, fy=420 MPa, cols.= 375x375 mm, check the six limitations of the direct design method.

Sol.



1. There are 3 continuous spans in each direction. (O.K)

2. The panel is rectangular with ratio of:
   (L /S)=(7.6/6)=1.27< 2 (O.K)

3. $l_1 - l_2 \le \frac{1}{3} l_1$ (the successive spans in each direction are equal). (O.K)

4. All the columns in each direction are on the same centerline. (offset=0) (O.K)

5. The loads are by gravity only and over an entire panel.
(L.L / D.L)=(5.75/0.165x24)=1.45 < 2 (O.K)

6. If beams are used on the columns lines, the relative stiffness ratio must be between 0.2 and 5. (Check)

$$0.2 \le \frac{(\alpha_f)_1 L_2^2}{(\alpha_f)_2 L_1^2} \le 5$$

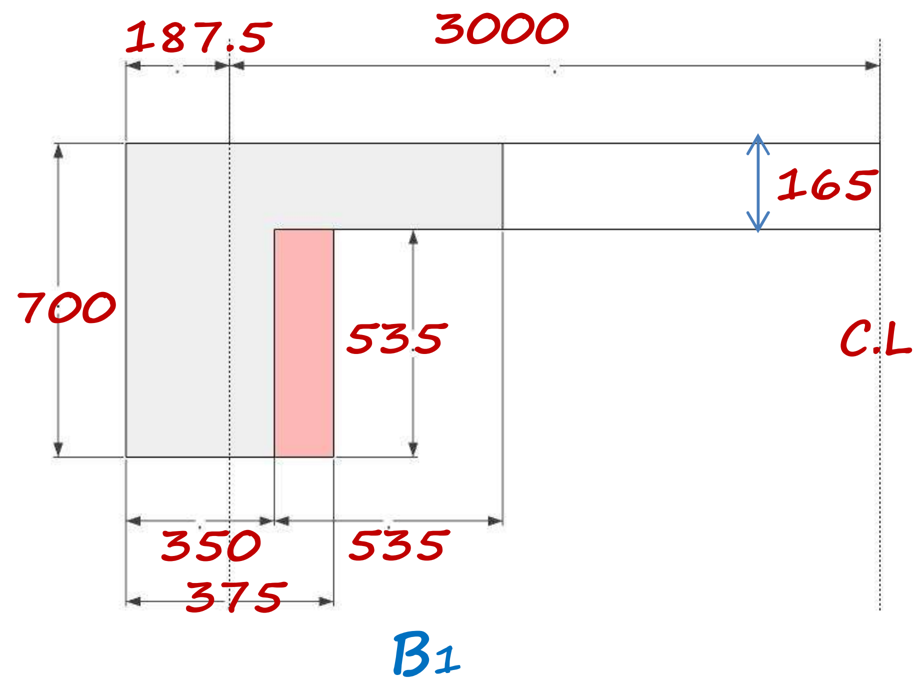
α_f for B_1: (edge beam)

$$I_b \cong 1.5\times\frac{bh^3}{12} = 1.5\times\frac{350\times700^3}{12}$$

$$= 1.5\times10^{10}\,mm^4$$

$$I_s = \frac{bh^3}{12} = \frac{3187.5\times165^3}{12} = 1.19\times10^9\,mm^4$$

$$(\alpha_f)_{B_1} = \frac{I_b}{I_s} = \frac{1.5\times10^{10}}{1.19\times10^9} = 12.57$$



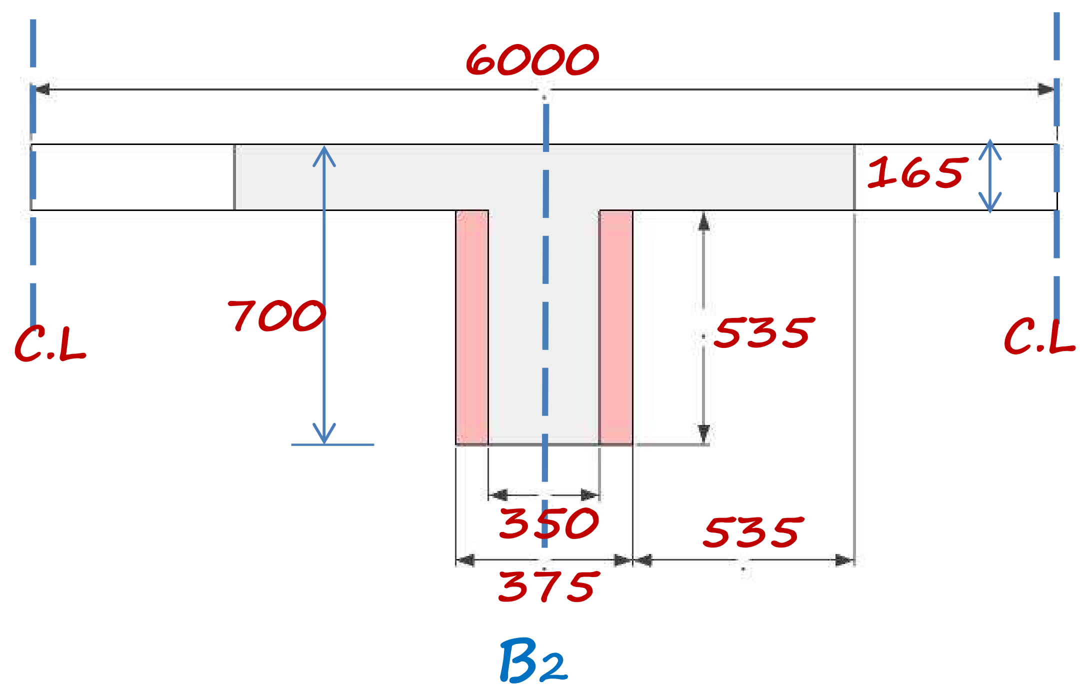
α_f for B_2: (Interior beam)

$$I_b \cong 2\times\frac{bh^3}{12} = 2\times\frac{350\times700^3}{12}$$

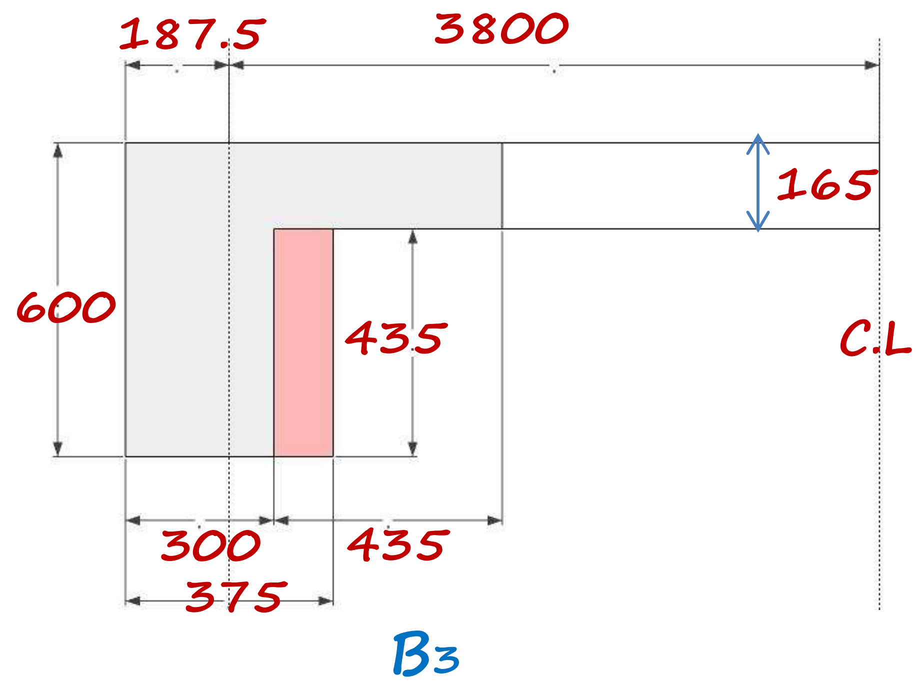
$$= 2\times10^{10}\,mm^4$$

$$I_s = \frac{bh^3}{12} = \frac{6000\times165^3}{12} = 2.24\times10^9\,mm^4$$

$$(\alpha_f)_{B_2} = \frac{I_b}{I_s} = \frac{2\times10^{10}}{2.24\times10^9} = 8.9$$



α_f for B3: (edge beam)



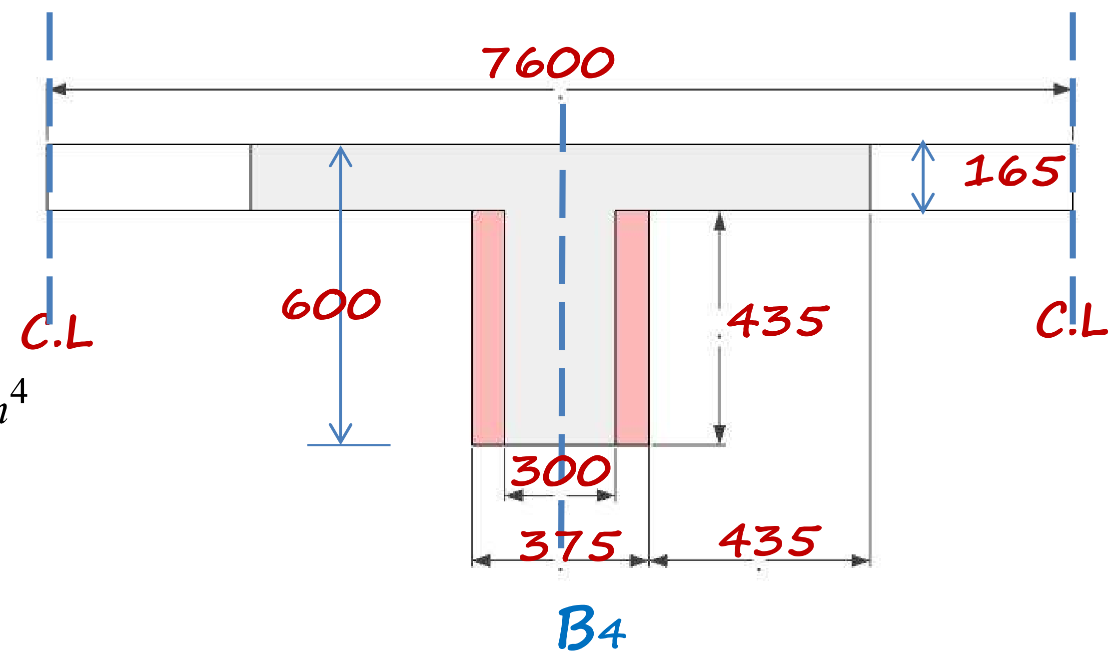
$$I_b \cong 1.5 \times \frac{bh^3}{12} = 1.5 \times \frac{300 \times 600^3}{12}$$

$$= 8.1 \times 10^9 \, mm^4$$

$$I_s = \frac{bh^3}{12} = \frac{3987.5 \times 165^3}{12} = 1.49 \times 10^9 \, mm^4$$

$$(\alpha_f)_{B_3} = \frac{I_b}{I_s} = \frac{8.1 \times 10^9}{1.49 \times 10^9} = 5.4$$

α_f for B4: (Interior beam)



$$I_b \cong 2 \times \frac{bh^3}{12} = 2 \times \frac{300 \times 600^3}{12}$$

$$= 10.8 \times 10^9 \, mm^4$$

$$I_s = \frac{bh^3}{12} = \frac{7600 \times 165^3}{12} = 2.84 \times 10^9 \, mm^4$$

$$(\alpha_f)_{B_4} = \frac{I_b}{I_s} = \frac{10.8 \times 10^9}{2.84 \times 10^9} = 3.79$$

Panel 1:

$$\frac{(\alpha_f)_1 L_2^2}{(\alpha_f)_2 L_1^2} = \frac{\left(\frac{12.57+8.9}{2}\right)\times 6^2}{\left(\frac{5.4+3.79}{2}\right)\times 7.6^2} = 1.44$$

$0.2 < 1.44 < 5$ O.K

Panel 2:

$$\frac{(\alpha_f)_1 L_2^2}{(\alpha_f)_2 L_1^2} = \frac{\left(\frac{12.57+8.9}{2}\right)\times 6^2}{\left(\frac{3.79+3.79}{2}\right)\times 7.6^2} = 1.76$$

$0.2 < 1.76 < 5$ O.K

Panel 3:

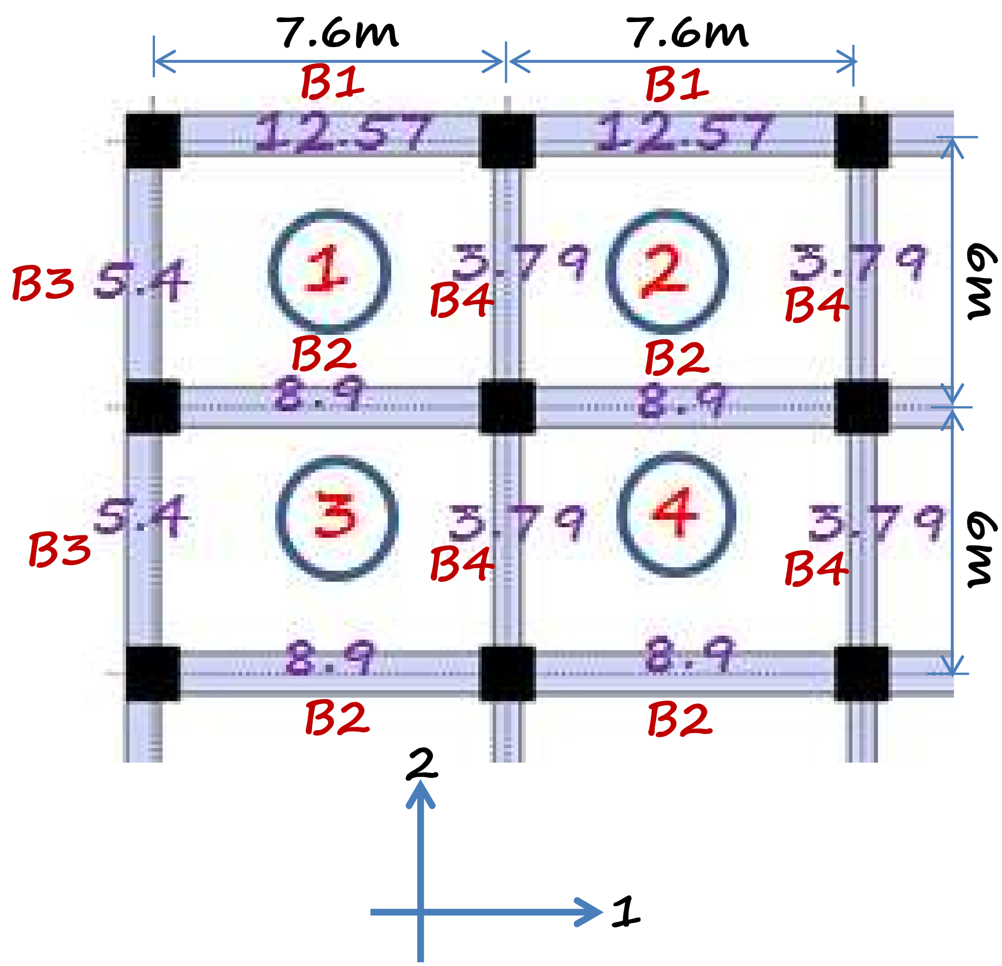
$$\frac{(\alpha_f)_1 L_2^2}{(\alpha_f)_2 L_1^2} = \frac{\left(\frac{8.9+8.9}{2}\right)\times 6^2}{\left(\frac{5.4+3.79}{2}\right)\times 7.6^2} = 1.19$$

$0.2 < 1.19 < 5$ O.K

Panel 4:

$$\frac{(\alpha_f)_1 L_2^2}{(\alpha_f)_2 L_1^2} = \frac{\left(\frac{8.9+8.9}{2}\right)\times 6^2}{\left(\frac{3.79+3.79}{2}\right)\times 7.6^2} = 1.46$$

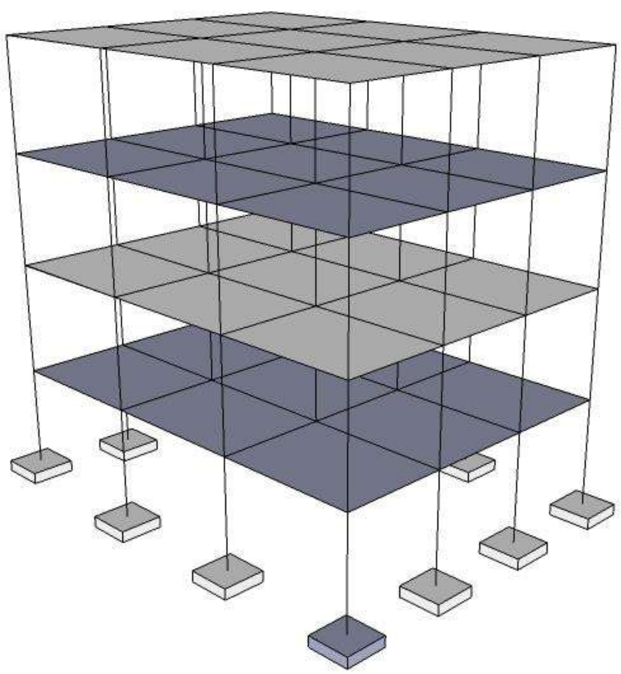
$0.2 < 1.46 < 5$ O.K



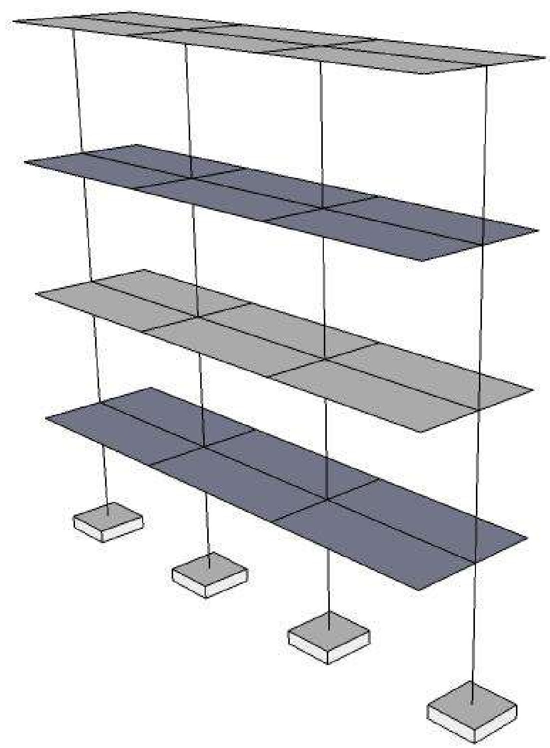
Direct Design Method (D.D.M) (ACI 8.10)

General Procedure for Design of Two Way Slab System by D.D.M

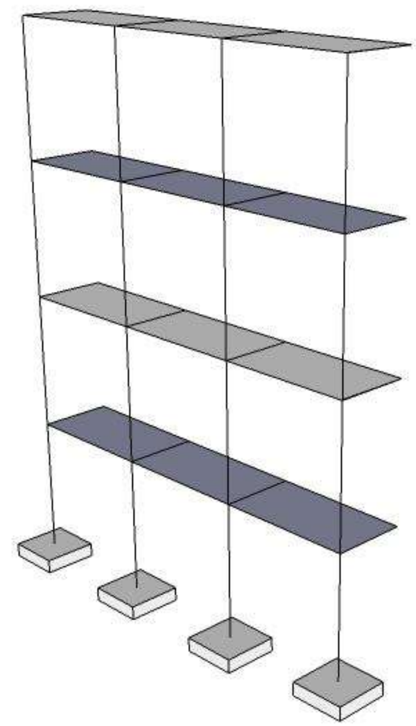
1- Transform the 3-dimensional building into simpler 2-dimensional frames by cutting the structure with imaginary vertical planes along the centerlines of the planes (in each direction).



3-D building

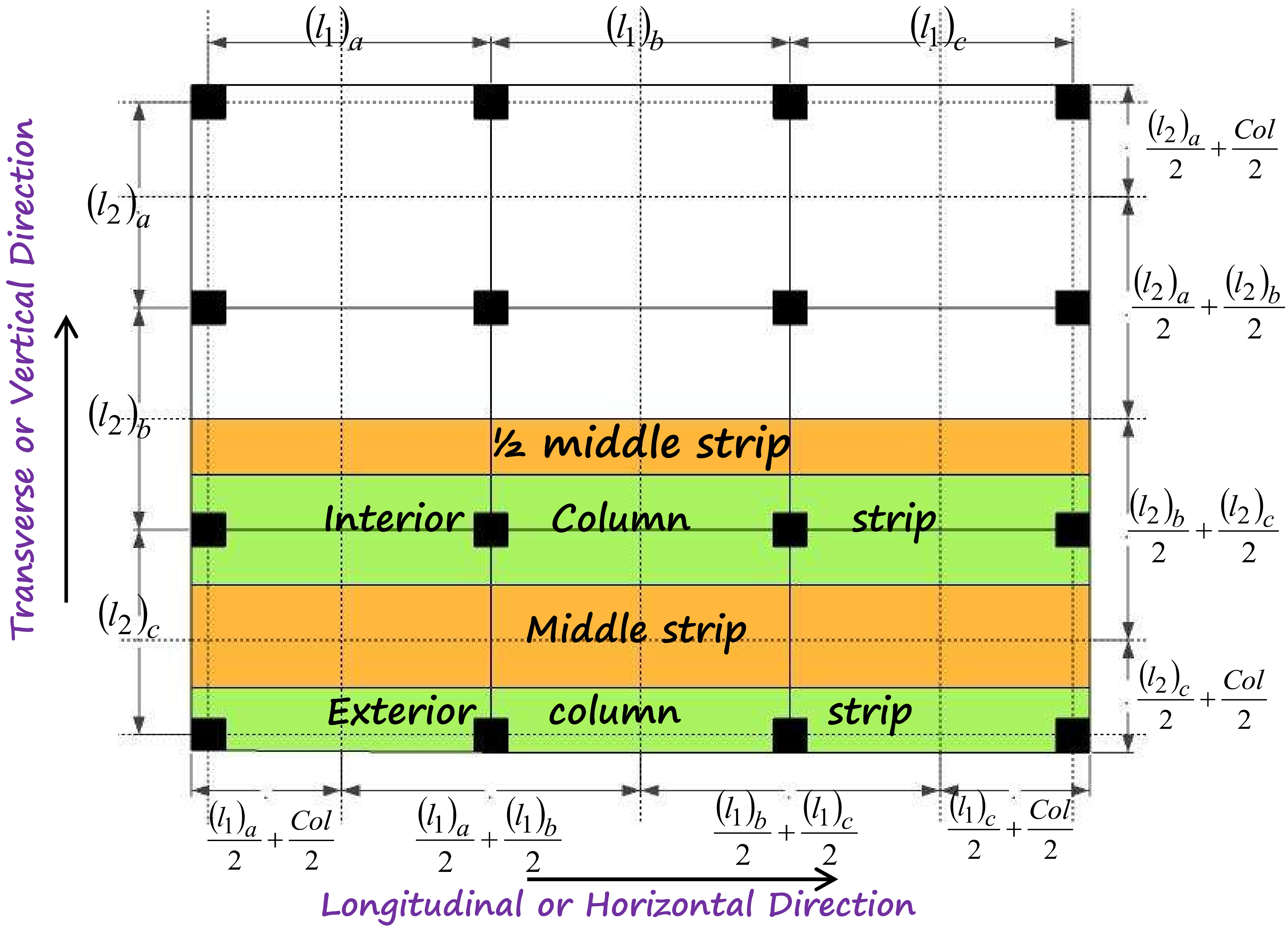


Interior Frame



Edge or Exterior Frame

2-Divide the slabs into panels by drawing centerlines along the columns. Check the D.D.M limitations



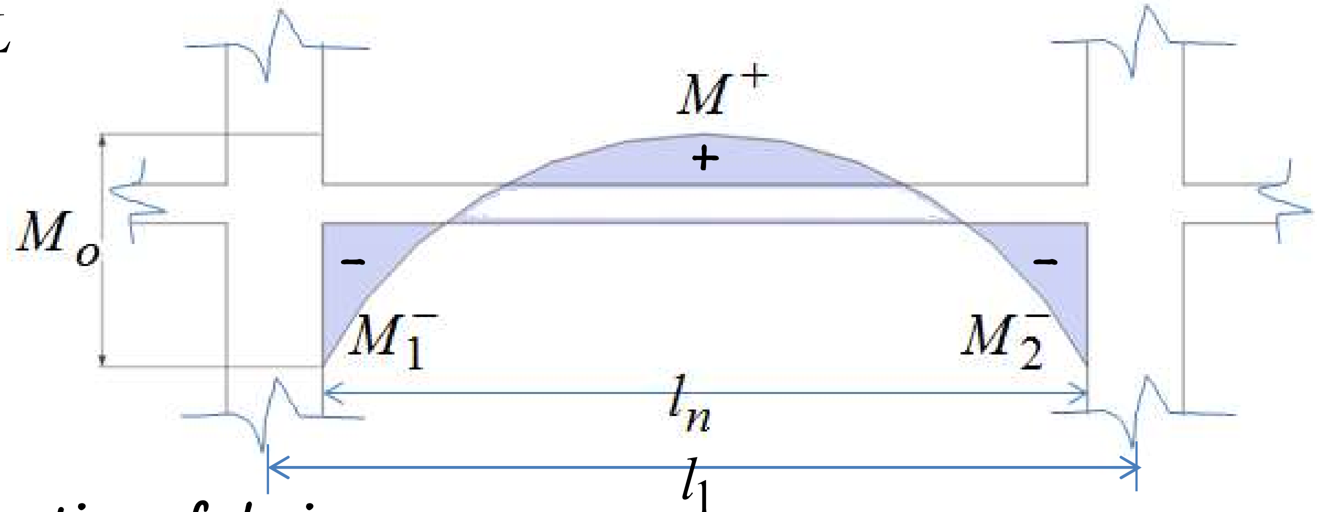
3-For each frame calculate the total span moment (M_o) for the end and the interior spans. ACI (8.10.3)

For uniform loading, the total design moment (M_o) for a span of the design strip is calculated by the simple static moment expression:

$$M_o = M^+ + \left| \frac{M_1^- + M_2^-}{2} \right| \quad , \quad M_o = \frac{Wu \times l_2 \times l_n^2}{8}$$

Where:

$$W_u = 1.2D.L + 1.6L.L$$



Important Notes:

l_1: the span c/c in the direction of design.

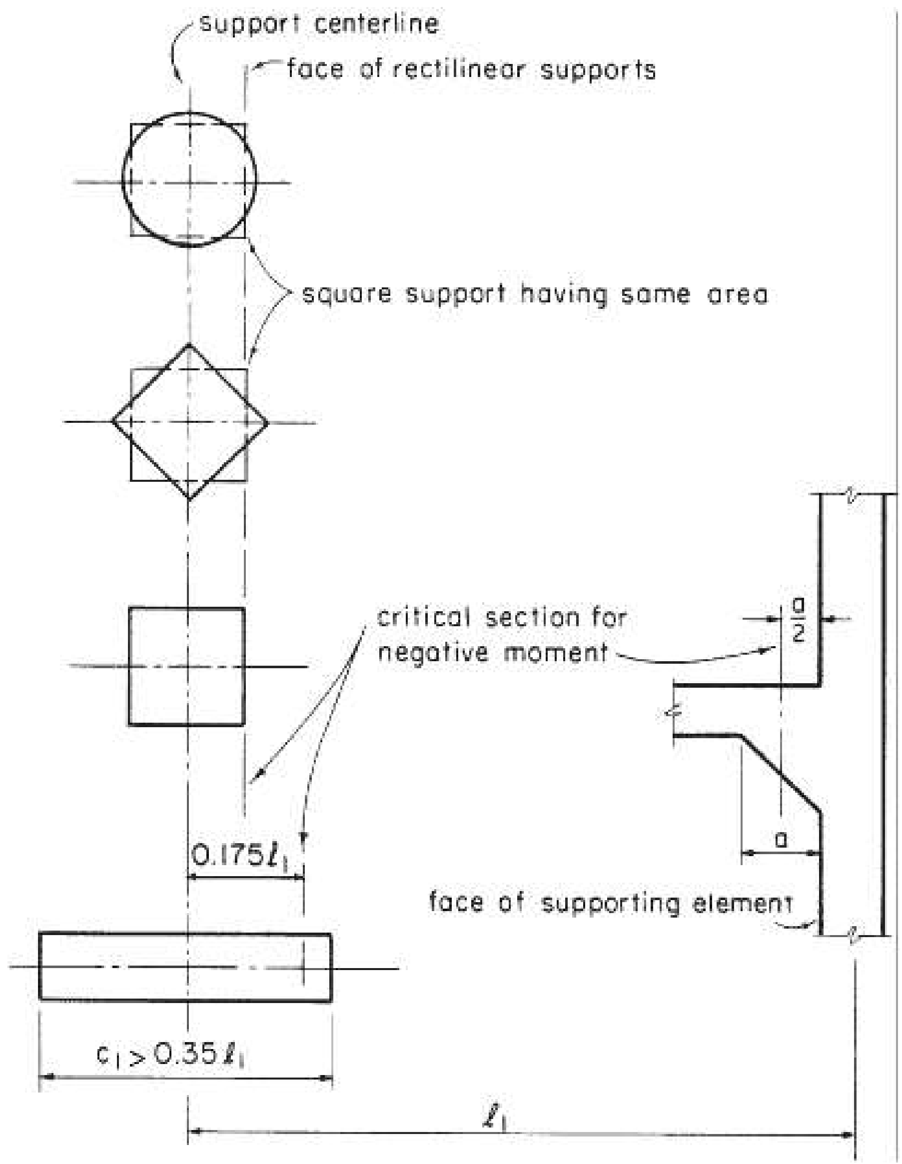
l_2 : the span c/c in the direction perpendicular on the direction of design. Or the width of frame.

Interior Frame: c/c of adjacent panels.

Edge Frame: $(l_2/2)+(Col/2)$

l_n : is a distance from face to face of columns, capitals, brackets or walls even if beams are exist and shall not be less than 0.65 l_1

Critical Sections for Negative Design Moment



(a) Interior & Exterior Column or Wall Supports

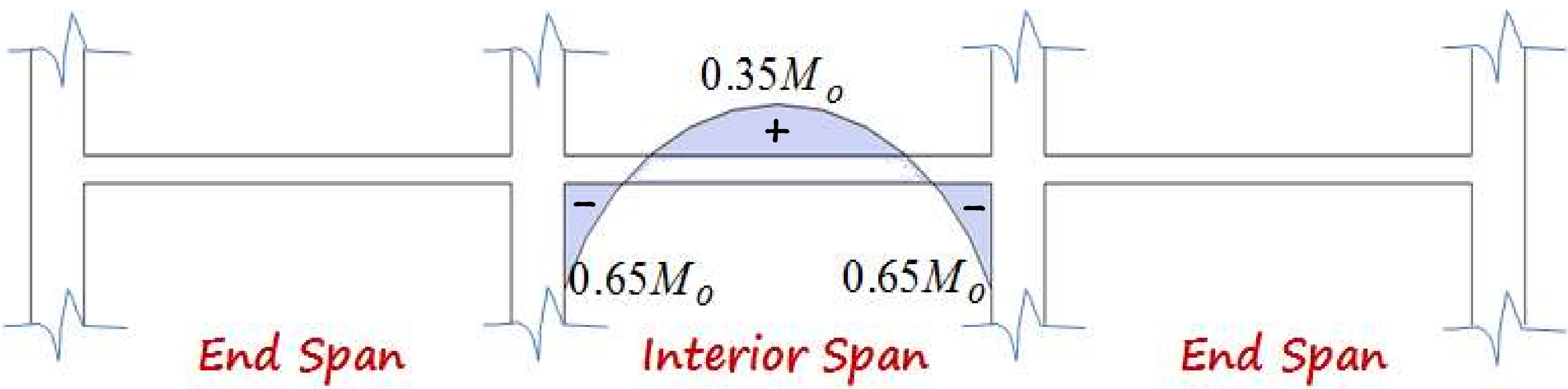
(b) Exterior Supports with Brackets or Corbels

4- Distribute the total moment (M_o) into –ve and +ve moments in the longitudinal direction for each of the interior span and end span ACI (8.10.4).

Note: all the moments are at the faces of supports

a) Interior Span ACI (8.10.4.1)

In an interior span, total static moment, Mo , shall be distributed as follows:

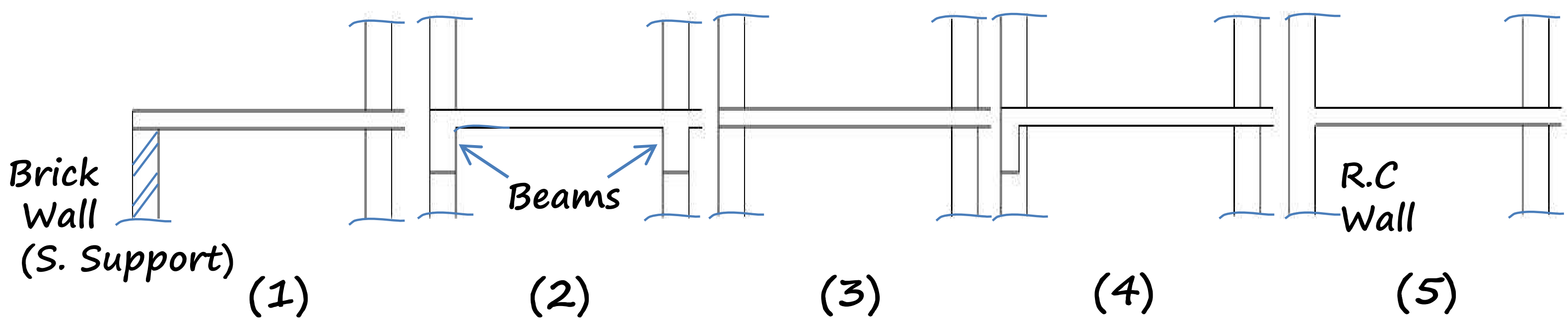


b) End Span: For different end span conditions, total static moment (M_o) shall be distributed as shown in Table (8.10.4.2).

Table 8.10.4.2—Distribution coefficients for end spans

	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports		Exterior edge fully restrained
			Without edge beam	With edge beam	
Interior negative	0.75	0.70	0.70	0.70	0.65
Positive	0.63	0.57	0.52	0.50	0.35
Exterior negative	0	0.16	0.26	0.30	0.65

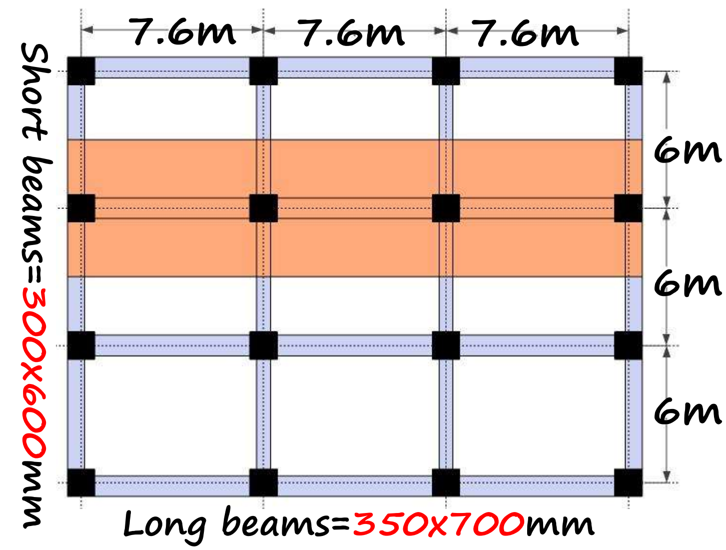
	(1)	(2)	(3)	(4)	(5)
	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports: Without edge beam	Slab without beams between interior supports: With edge beam	Exterior edge fully restrained
Interior negative factored moment	0.75	0.70	0.70	0.70	0.65
Positive factored moment	0.63	0.57	0.52	0.50	0.35
Exterior negative factored moment	0	0.16	0.26	0.30	0.65



Example (1)

Two way slab with beams, L.L=5.75 kN/m² , t=165mm , cols.= 375x375 mm, find the negative and positive moments in each span in the longitudinal interior frame using D.D.M

Sol.



$$W_u = 1.2D.L + 1.6L.L$$

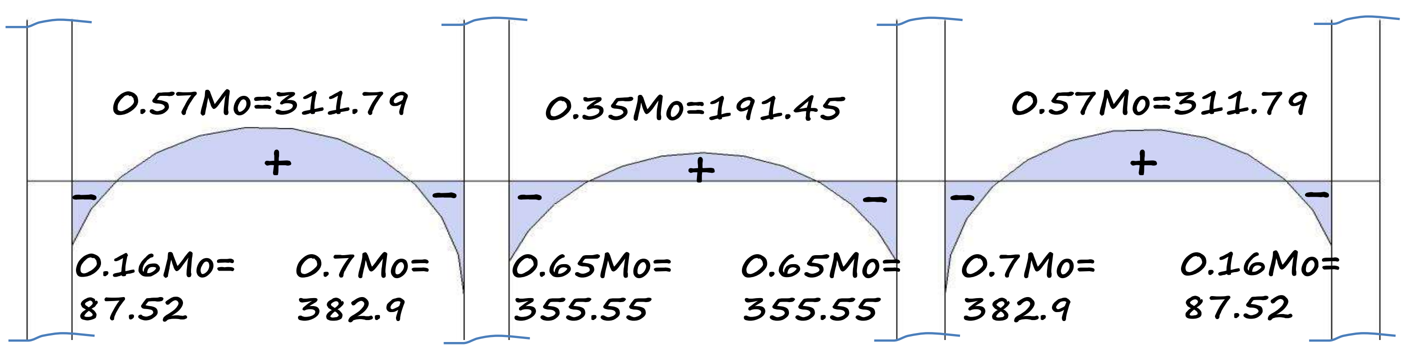
$$W_u = 1.2\times(24\times0.165)+1.6\times5.75 = 13.95 \text{ kN/m}^2$$

$$l_n = 7.6-0.375 = 7.23 > 0.65 l_1 = 0.65\times7.6 = 4.94m$$

$$l_2 = \left(\frac{6}{2}+\frac{6}{2}\right) = 6$$

$$M_o = \frac{Wu\times l_2\times l_n^2}{8} = \frac{13.95\times6\times(7.23)^2}{8} = 547 \text{ kN.m}$$

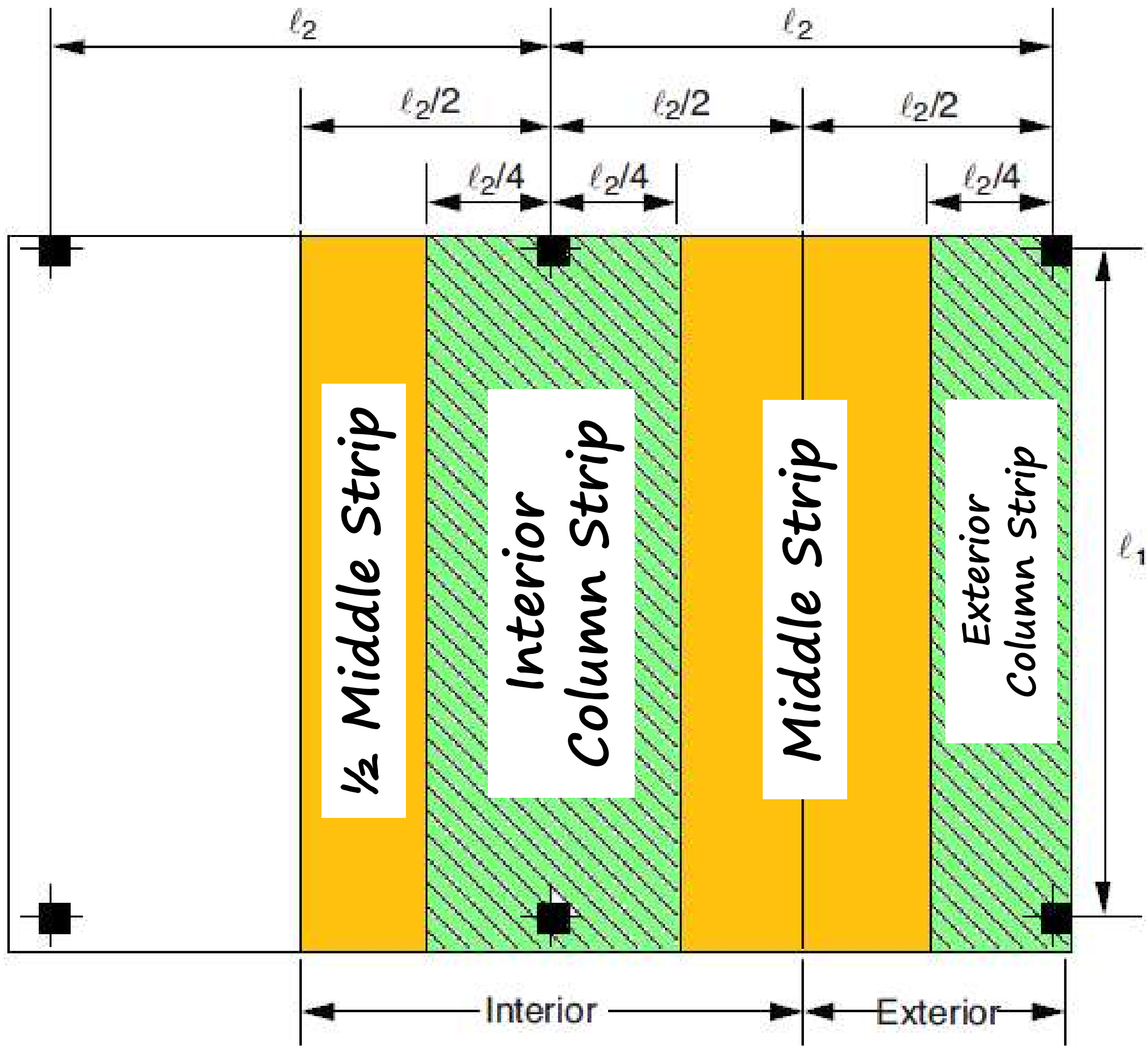
Equal spans give equal moments. Table (13.6.3.3)



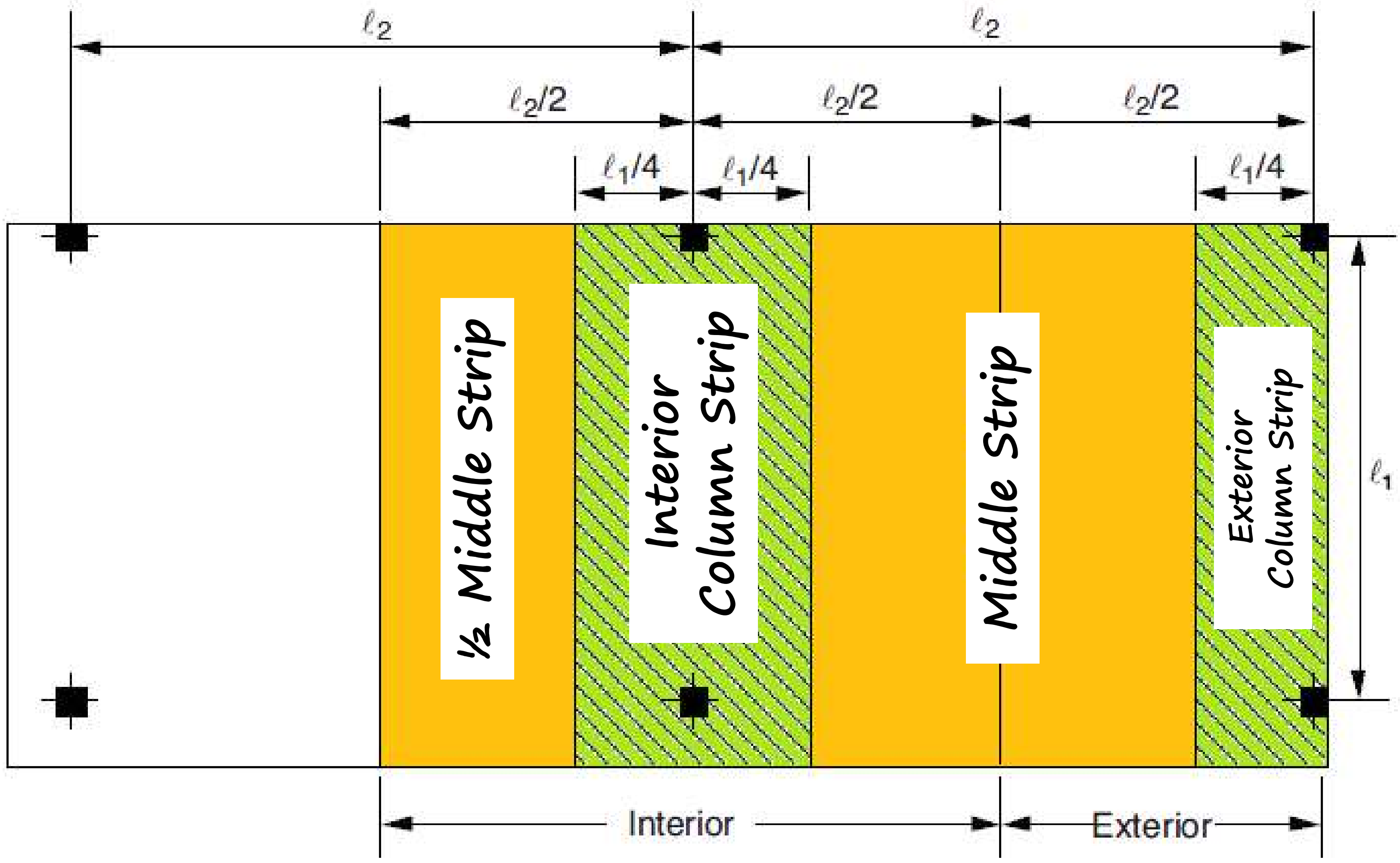
Note: For design use the larger negative factored moment determined on faces of a column.

5- Distribute each of the negative and positive moments in the transverse direction into the column strip and middle strip for each design frame.

1) Defining column and middle strips:



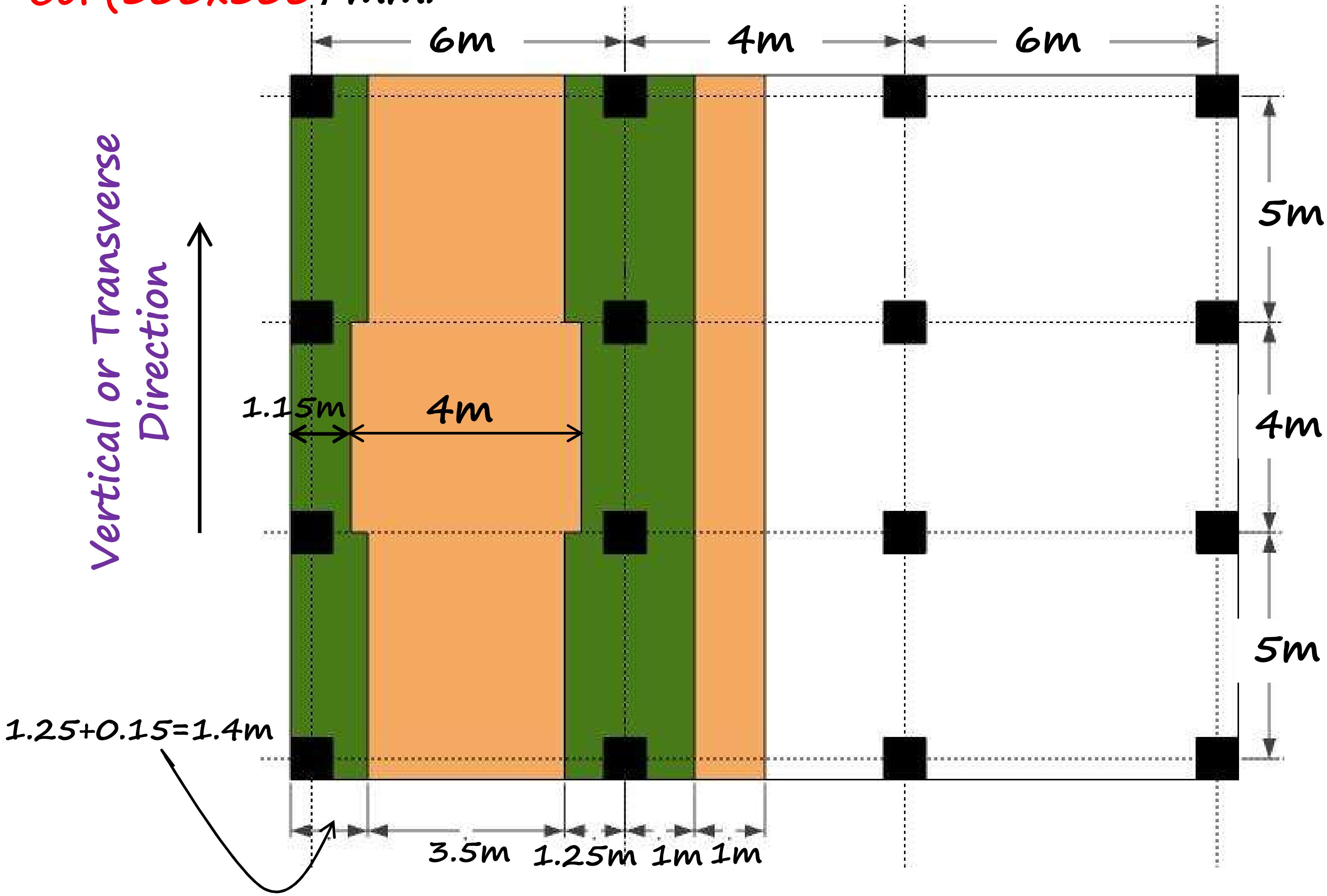
Column Strip for $l_1 > l_2$



Column Strip for $l_2 > l_1$

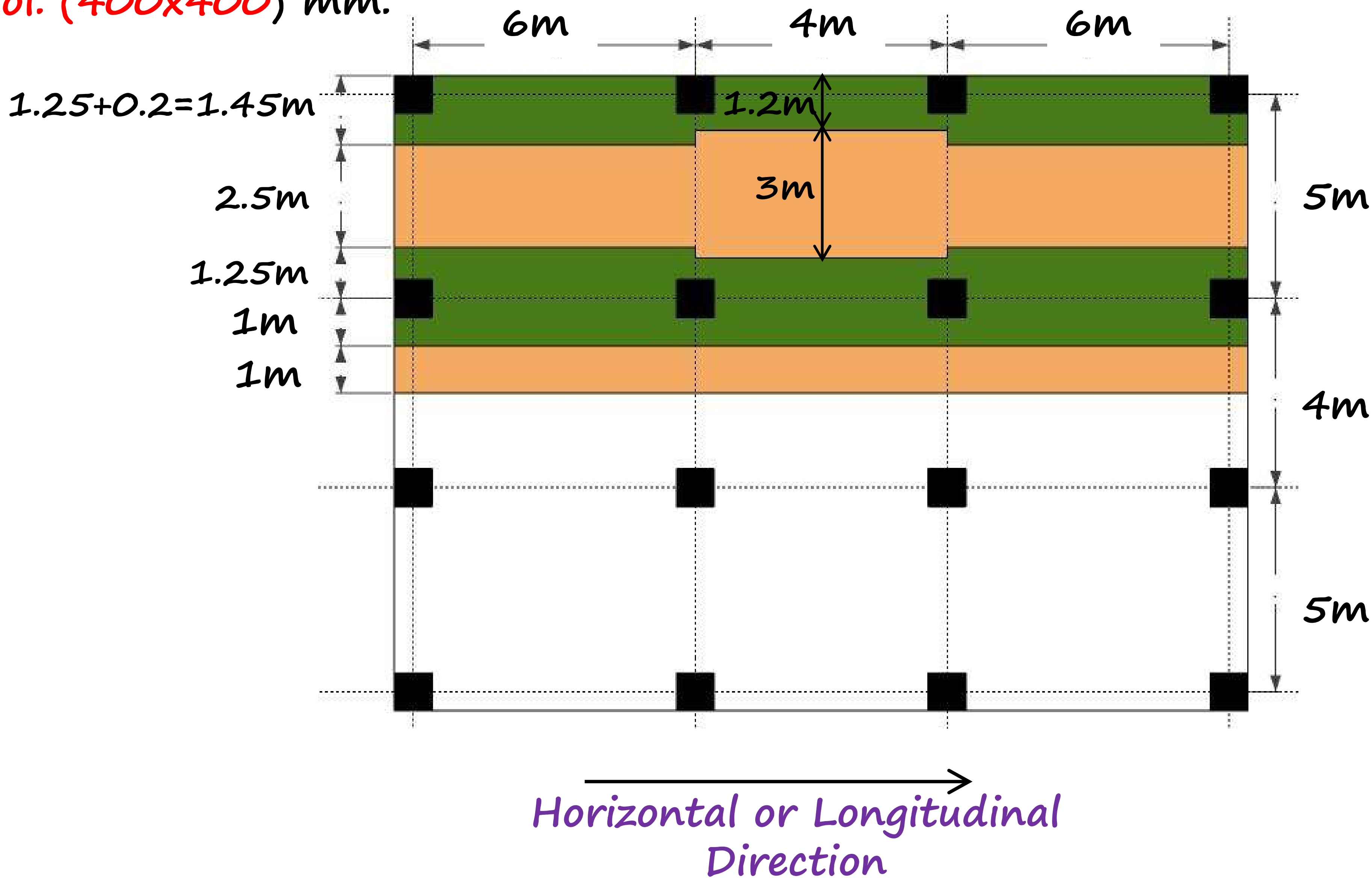
Exercise (1)

Determine the width of column and middle strips in the transverse direction of the exterior and interior frame of the system shown, Col (300x300) mm.



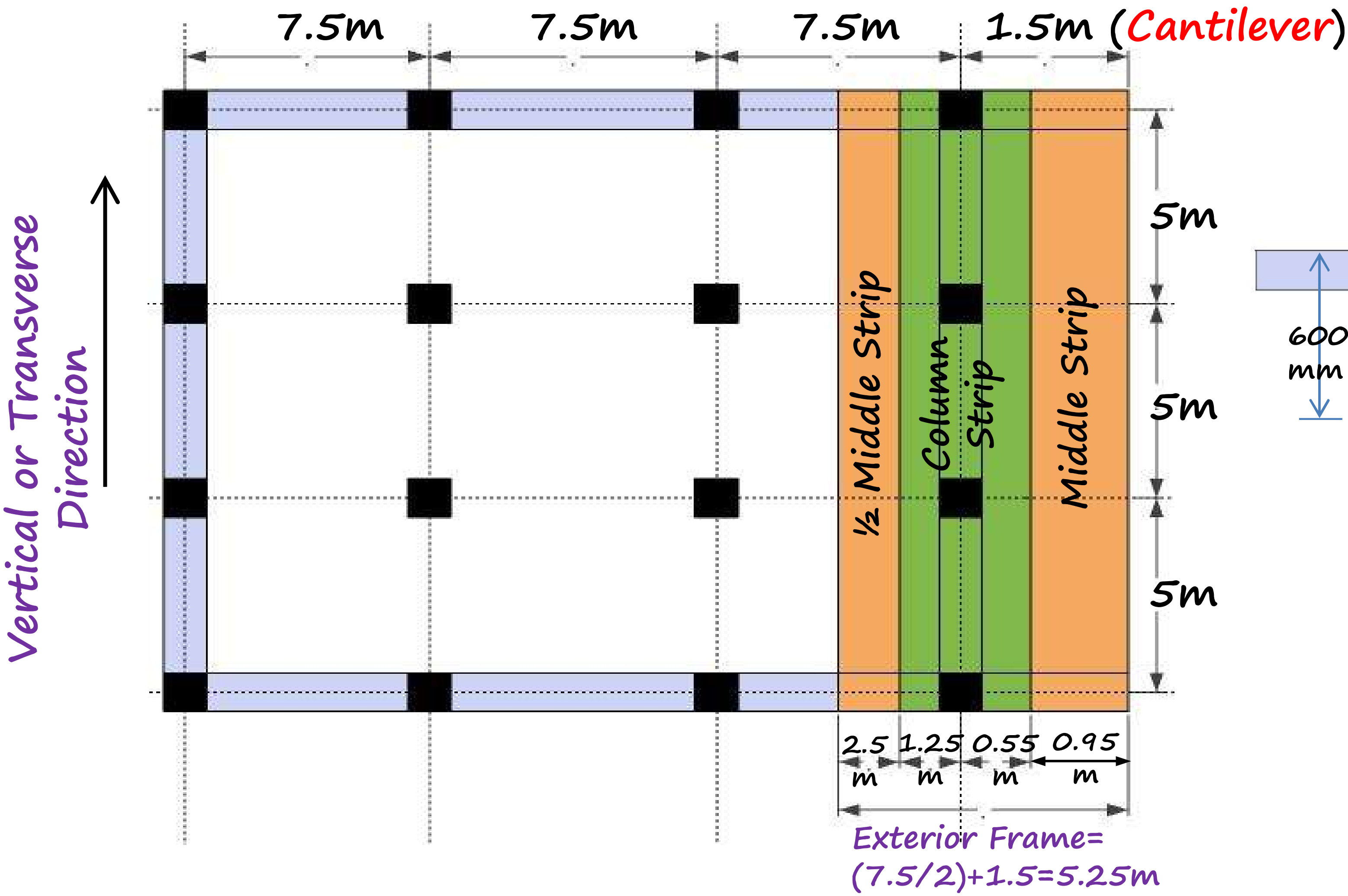
Exercise (2)

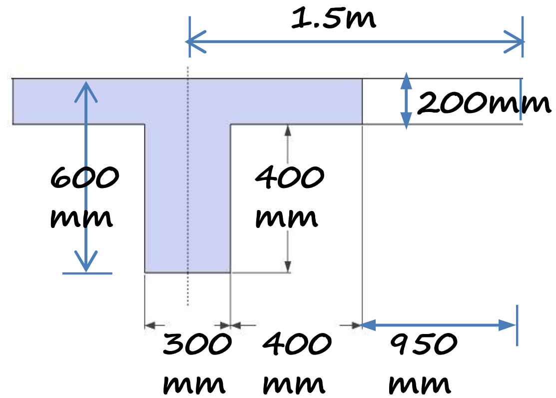
Determine the width of column and middle strips in the longitudinal direction of the exterior and interior frame of the system shown, Col. (400x400) mm.



Exercise (3)

Determine the width of column and middle strips in the transverse direction of the exterior frame of the system shown, Col (300x300) mm, beams (300x600) mm, t=200 mm .





2) Factored moments in the column and middle strips ACI (8.10.5)

The amounts of negative and positive factored moments to be resisted by a column strip depends on the relative beam-to-slab stiffness ratio and the panel width-to-length ratio in the direction of analysis. The percentage of total negative and positive factored moments to be resisted by a column strip may be determined from the tables.

A) Interior negative factored moments in the column strips: ACI (8.10.5.1)

Column strips shall be proportioned to resist the following percentages of interior negative factored moments:

Table 8.10.5.1—Portion of interior negative M_u in column strip

$\alpha_{f1}\ell_2/\ell_1$	ℓ_2/ℓ_1		
	0.5	1.0	2.0
0	0.75	0.75	0.75
≥1.0	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

Or from the following expression: (for linear interpolation)

$$Percentage(C.S) = 75 + 30(\alpha_{f1}\frac{l_2}{l_1})(1-\frac{l_2}{l_1}),$$ **if** $(\alpha_{f1}\frac{l_2}{l_1}) > 1$ **then use** $(\alpha_{f1}\frac{l_2}{l_1}) = 1$

where: $\frac{l_2}{l_1}$ measured c/c of supports and α_{f1} in the direction of l_1

The interior negative factored moments are:

$$M^-_{c.s}(\text{int.}) = (Percentage) \times M^-(\text{int.})$$

$$M^-_{m.s}(\text{int.}) = (1 - Percentage) \times M^-(\text{int.})$$

B) Exterior negative factored moments in the column strips: ACI (8.10.5.2)

Column strips shall be proportioned to resist the following percentages of exterior negative factored moments:

Table 8.10.5.2—Portion of exterior negative M_u in column strip

$\alpha_{f1}\ell_2/\ell_1$	β_t	ℓ_2/ℓ_1		
		0.5	1.0	2.0
0	0	1.0	1.0	1.0
	≥2.5	0.75	0.75	0.75
≥1.0	0	1.0	1.0	1.0
	≥2.5	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown. β_t is calculated using Eq. (8.10.5.2a), where C is calculated using Eq. (8.10.5.2b).

Or from the following expression: (for linear interpolation)

$$Percentage(C.S) = 100 - 10\beta_t + 12\beta_t(\alpha_{f1}\frac{l_2}{l_1})(1 - \frac{l_2}{l_1})$$

if $(\alpha_{f1}\frac{l_2}{l_1}) > 1$ **use** $(\alpha_{f1}\frac{l_2}{l_1}) = 1$

If $\beta_t > 2.5$ **use** $\beta_t = 2.5$

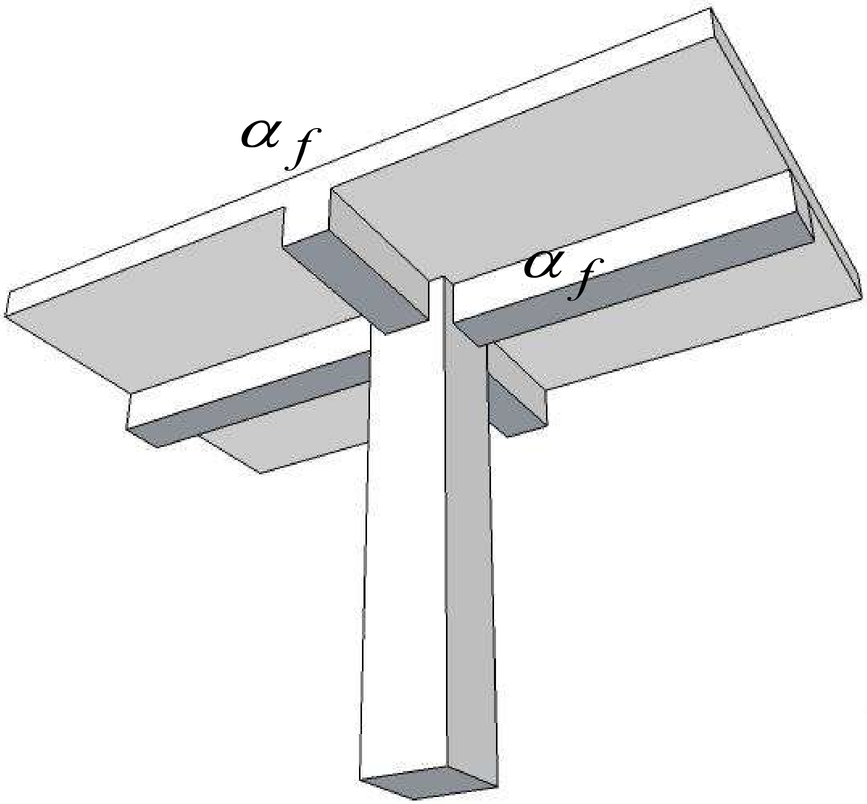
The exterior negative factored moments are:

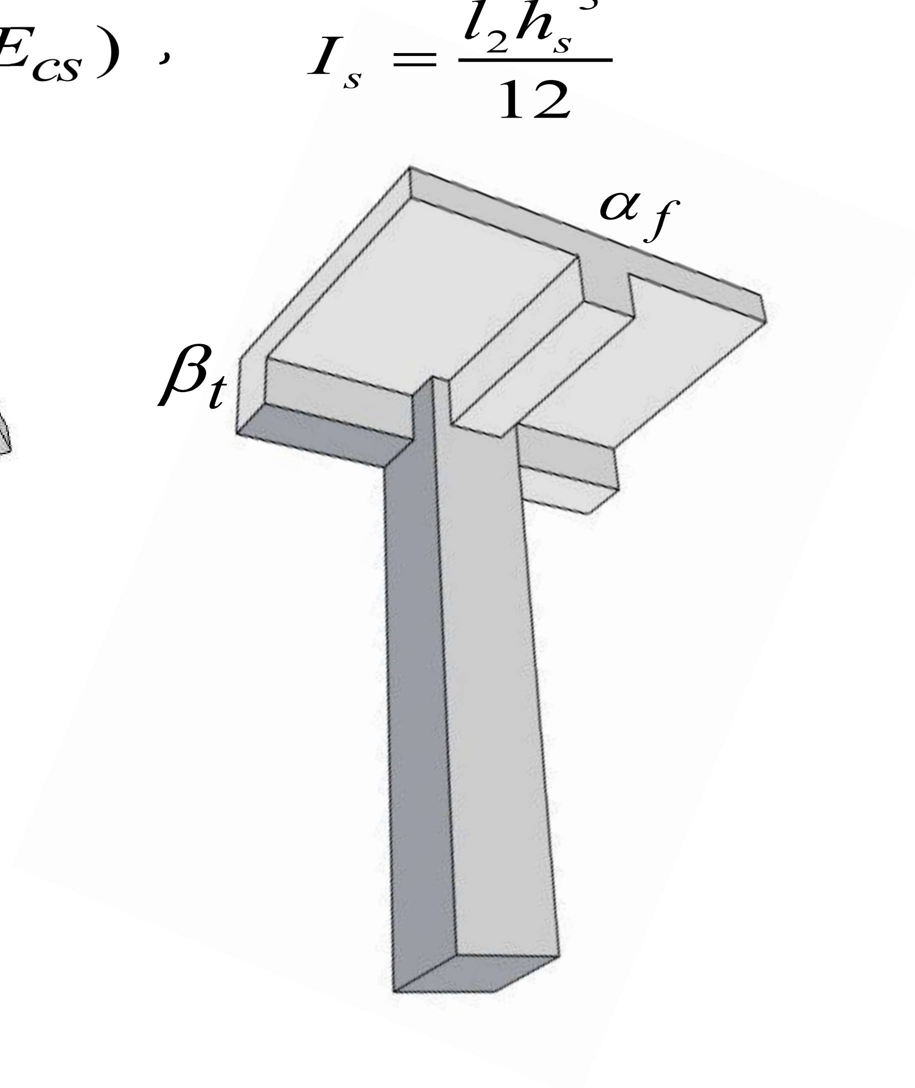
$$M^{-}_{c.s}(ext.) = (Percentage) \times M^{-}(ext.)$$

$$M^{-}_{m.s}(ext.) = (1 - Percentage) \times M^{-}(ext.)$$

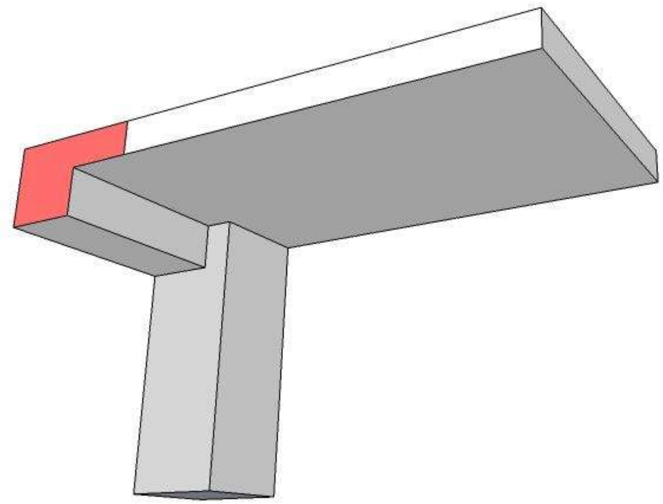
β_t : The ratio of torsional stiffness of edge beam section to flexural stiffness of a width of slab equals to the span length of the torsional edge beam c/c of supports. (l_2 is the average length of torsional member)

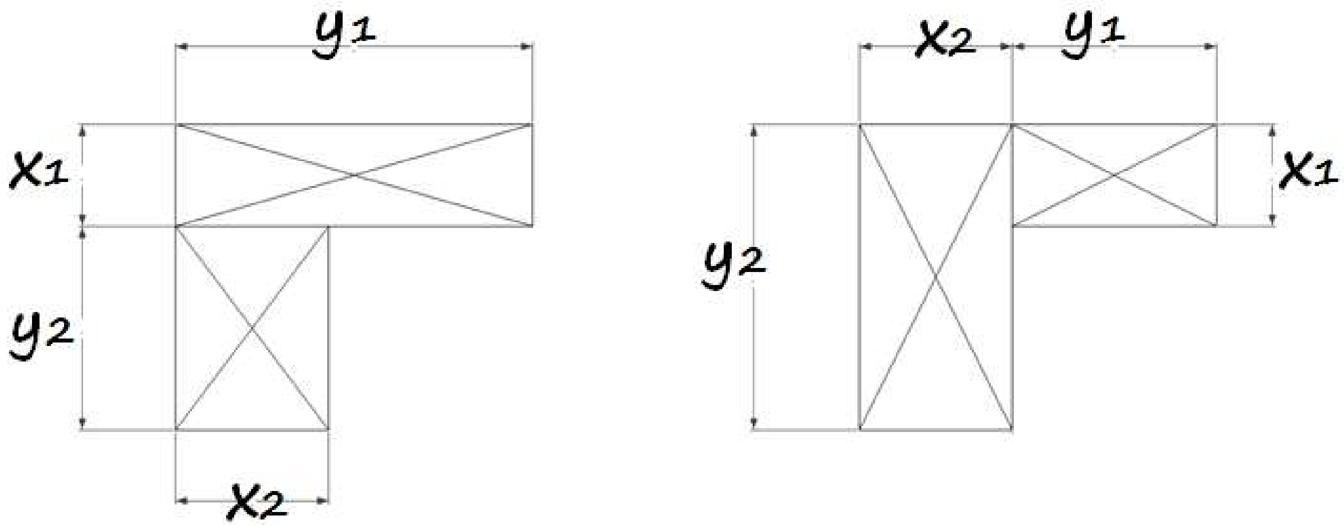
$$\beta_t = \frac{E_{cb} C}{2 E_{cs} I_s} = \frac{C}{2 I_s}, if\ (E_{cb} = E_{cs}), \qquad I_s = \frac{l_2 h_s^3}{12}$$





$$C = \sum (1 - 0.63\frac{x}{y})\frac{x^3 y}{3}$$





x: smaller dimension
y: larger dimension
Use the greatest value of (C) for the two cases

C) Positive factored moments in the column strips (for all spans): (8.10.5.5)

Column strips shall be proportioned to resist the following percentages of positive factored moments:

Table 8.10.5.5—Portion of positive M_u in column strip

$\alpha_{f1}\ell_2/\ell_1$	ℓ_2/ℓ_1		
	0.5	1.0	2.0
0	0.60	0.60	0.60
≥1.0	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

Or from the following expression:

$$Percentage(C.S) = 60 + 30(\alpha_{f1}\frac{l_2}{l_1})(1.5 - \frac{l_2}{l_1}) \quad , \quad \text{if } (\alpha_{f1}\frac{l_2}{l_1}) > 1 \quad \text{use } (\alpha_{f1}\frac{l_2}{l_1}) = 1$$

The positive factored moments are:

$$M^+_{c.s} = (Percentage) \times M^+$$

$$M^+_{m.s} = (1 - Percentage) \times M^+$$

6-Distribute the moment in column strip between the slab and the beam portions of the column strip. ACI (8.10.5.7.1)

a) Beams between supports shall be proportioned to resist (85%) of column strip moments if: $(\alpha_{f1}\frac{l_2}{l_1}) > 1$

b) For values of $0 < (\alpha_{f1}\frac{l_2}{l_1}) < 1$, proportion of column strip moments resisted by beams shall be obtained by linear interpolation between (85%) and (0%). Or from the following expression:

$$M_{beam} = 0.85(\alpha_{f1}\frac{l_2}{l_1}) \times M_{c.s} \leq 0.85 M_{c.s}$$

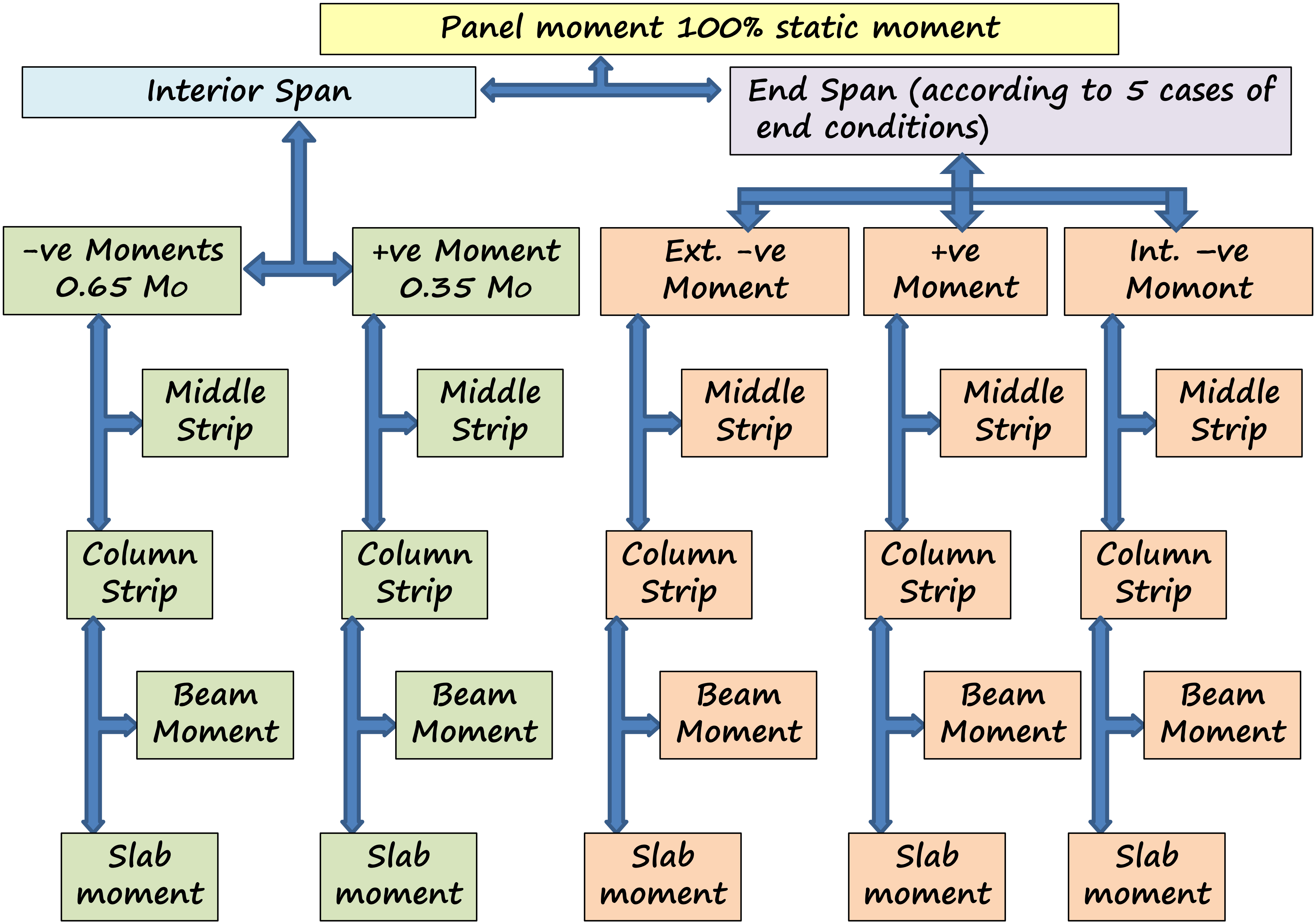
Note:

In addition to moments calculated for uniform loads according to (a) and (b), beams shall be proportioned to resist all moments caused by concentrated or linear loads applied directly to beams, including weight of projecting beam stem above or below the slab.

Table 8.10.5.7.1—Portion of column strip M_u in beams

$\alpha_{f1}\ell_2/\ell_1$	Distribution coefficient
0	0
≥1.0	0.85

Note: Linear interpolation shall be made between values shown.

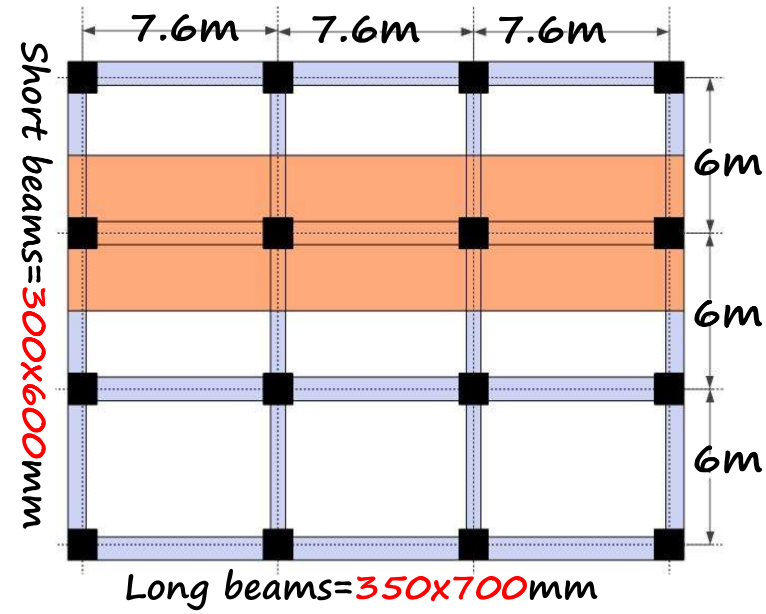

Panel moment 100% static moment
Interior Span
End Span (according to 5 cases of end conditions)
-ve Moments 0.65 Mo
+ve Moment 0.35 Mo
Ext. -ve Moment
+ve Moment
Int. -ve Momont
Middle Strip
Middle Strip
Middle Strip
Middle Strip
Middle Strip
Column Strip
Column Strip
Column Strip
Column Strip
Column Strip
Beam Moment
Beam Moment
Beam Moment
Beam Moment
Beam Moment
Slab moment
Slab moment
Slab moment
Slab moment
Slab moment

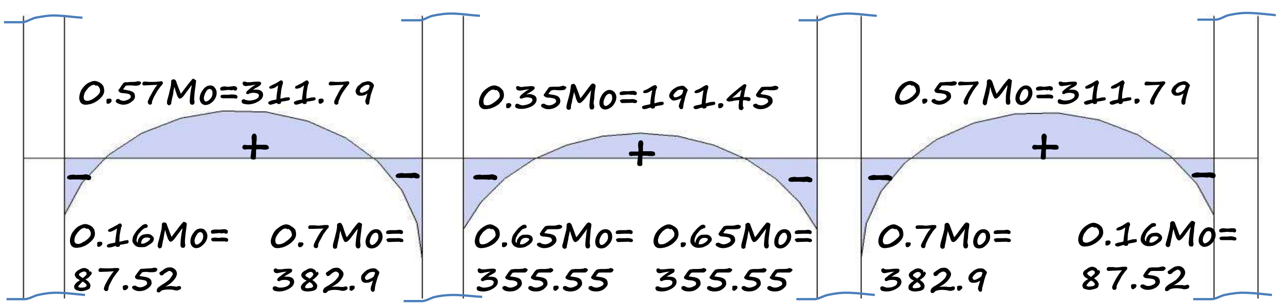
Example (2)

Two way slab with beams, L.L=5.75 kN/m² , t=165mm , cols.= 375x375 mm, distribute the negative and positive moments in each span into column and middle strips in the longitudinal interior frame. Find the proportion of column strip moments resisted by beam.

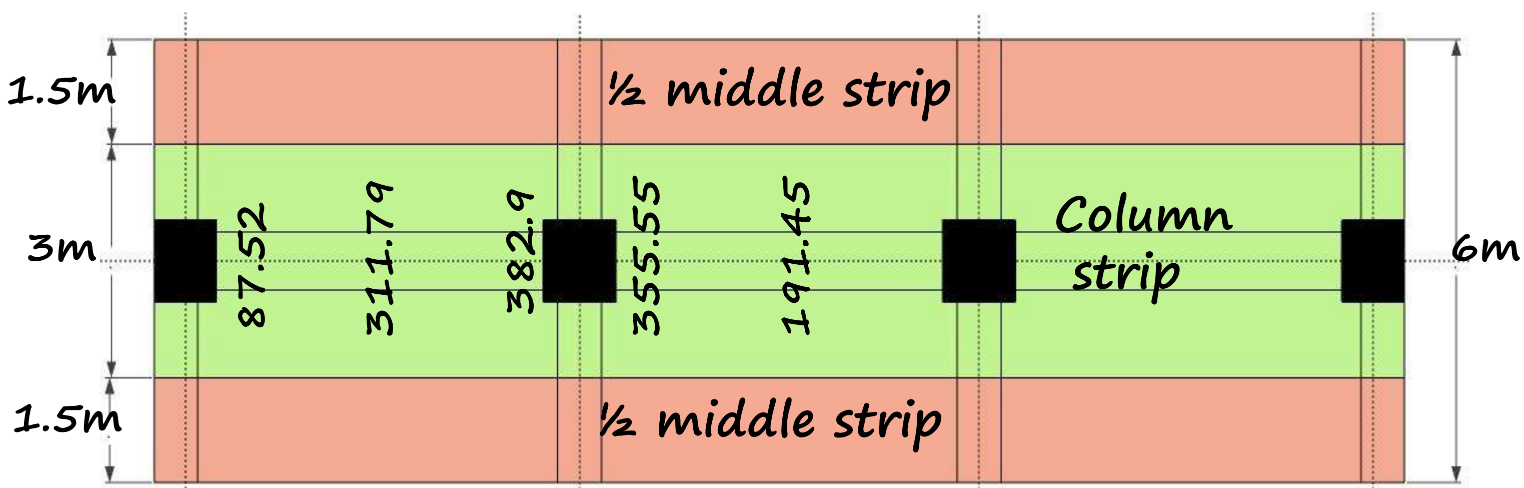
Sol.

The moments in each span had been calculated in example (1).





Width of column and middle strips:



1)Exterior Span

A)Percent of interior –ve moment to column strip: ACI (8.10.5.1)

$$\frac{l_2}{l_1}=\frac{6}{7.6}=0.79 \quad , \quad \alpha_{f1}\frac{l_2}{l_1}=8.9\times0.79=7.03>1 \quad , \quad \alpha_{f1}=8.9 \quad (\alpha_{f1} \text{ for } B_2)$$

Use linear interpolation:

$$Percentage(C.S)=75+30(\alpha_{f1}\frac{l_2}{l_1})(1-\frac{l_2}{l_1})=75+30\times1\times(1-0.79)=81.3\%$$

$$Percentage(m.s)=100\%-81.3\%=18.7\%$$

$$M^-_{c.s}(\text{int.})=(Percentage)\times M^-(\text{int.})=81.3\%\times382.9=311.297kN.m$$

$$M^-_{m.s}(\text{int.})=(1-Percentage)\times M^-(\text{int.})=18.7\%\times382.9=71.603kN.m$$

B)Percent of interior +ve moment to column strip: ACI (8.10.5.5)

Use linear interpolation:

$$Percentage(C.S)=60+30(\alpha_{f1}\frac{l_2}{l_1})(1.5-\frac{l_2}{l_1})=60+30\times1\times(1.5-0.79)=81.3\%$$

$$Percentage(m.s)=100\%-81.3\%=18.7\%$$

$$M^+_{c.s}=(Percentage)\times M^+=81.3\%\times311.79=253.48kN.m$$

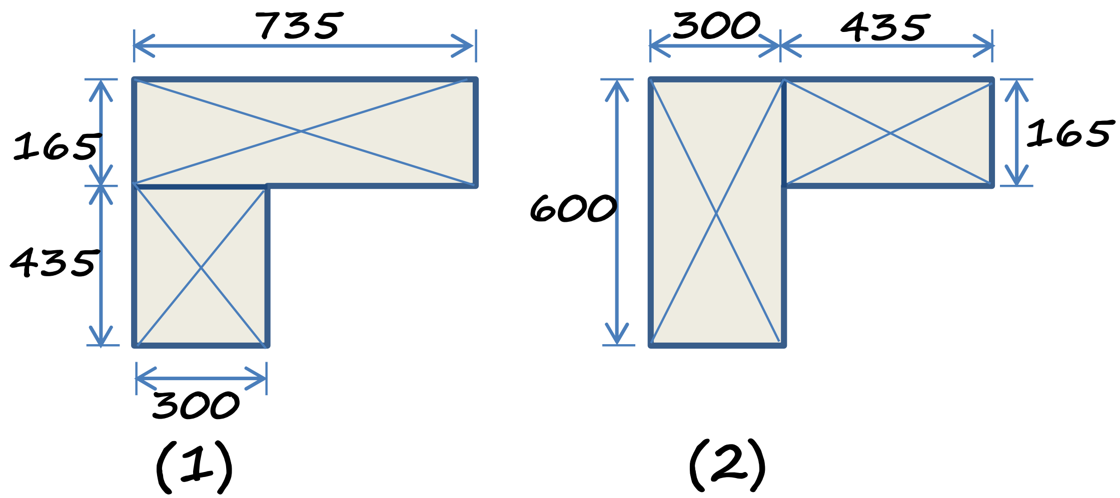
$$M^+_{m.s}=(1-Percentage)\times M^+=18.7\%\times311.79=58.31kN.m$$

C)Percent of exterior -ve moment to column strip: ACI (8.10.5.2)

Use linear interpolation:

$$\beta_t = \frac{C}{2I_s} \quad , \quad I_s = \frac{l_2 h_s^3}{12} = \frac{6000(165)^3}{12} = 2.24\times10^9\, mm^4 \quad \text{(where } l_2 \text{ c/c)}$$

$$C = \sum (1-0.63\frac{x}{y})\frac{x^3 y}{3}$$



$$C_1 = (1-0.63\times\frac{300}{435})\frac{300^3\times435}{3} + (1-0.63\times\frac{165}{735})\frac{165^3\times735}{3} = 3.159\times10^9\, mm^4$$

$$C_2 = (1-0.63\times\frac{300}{600})\frac{300^3\times600}{3} + (1-0.63\times\frac{165}{435})\frac{165^3\times435}{3} = 4.195\times10^9\, mm^4$$

Use the greater value of C

$$\beta_t = \frac{C}{2I_s} = \frac{4.195\times10^9}{2\times2.24\times10^9} = 0.93$$

Use linear interpolation: (double interpolation)

$$Percentage(C.S) = 100 - 10\beta_t + 12\beta_t(\alpha_{f1}\frac{l_2}{l_1})(1 - \frac{l_2}{l_1}) = 100 - 10\times0.93 + 12\times0.93\times1\times(1-0.79) = 93.04\%$$

$$Percentage(m.s) = 100\% - 93.04\% = 6.95\%$$

$$M^-_{c.s}(ext.) = (Percentage)\times M^-(ext.) = 93.04\%\times87.52 = 81.42kN.m$$

$$M^-_{m.s}(ext.) = (1 - Percentage)\times M^-(ext.) = 6.95\%\times87.52 = 6.08kN.m$$

2)Interior Span

A)Percent of interior –ve moment to column strip:

$$\frac{l_2}{l_1} = \frac{6}{7.6} = 0.79 \quad , \alpha_{f1}\frac{l_2}{l_1} = 8.9\times0.79 = 7.03 > 1 \quad , \alpha_{f1} = 8.9$$ (α_{f1} for B_2)

$$M^-_{c.s}(\text{int.}) = (Percentage)\times M^-(\text{int.}) = 81.3\%\times355.55 = 289.06kN.m$$

$$M^-_{m.s}(\text{int.}) = (1 - Percentage)\times M^-(\text{int.}) = 18.7\%\times355.55 = 66.14kN.m$$

B)Percent of interior +ve moment to column strip:

$$M^+_{c.s} = (Percentage)\times M^+ = 81.3\%\times191.45 = 155.648kN.m$$

$$M^+_{m.s} = (1 - Percentage)\times M^+ = 18.7\%\times191.45 = 35.802kN.m$$

Proportion of column strip moments resisted by beam:

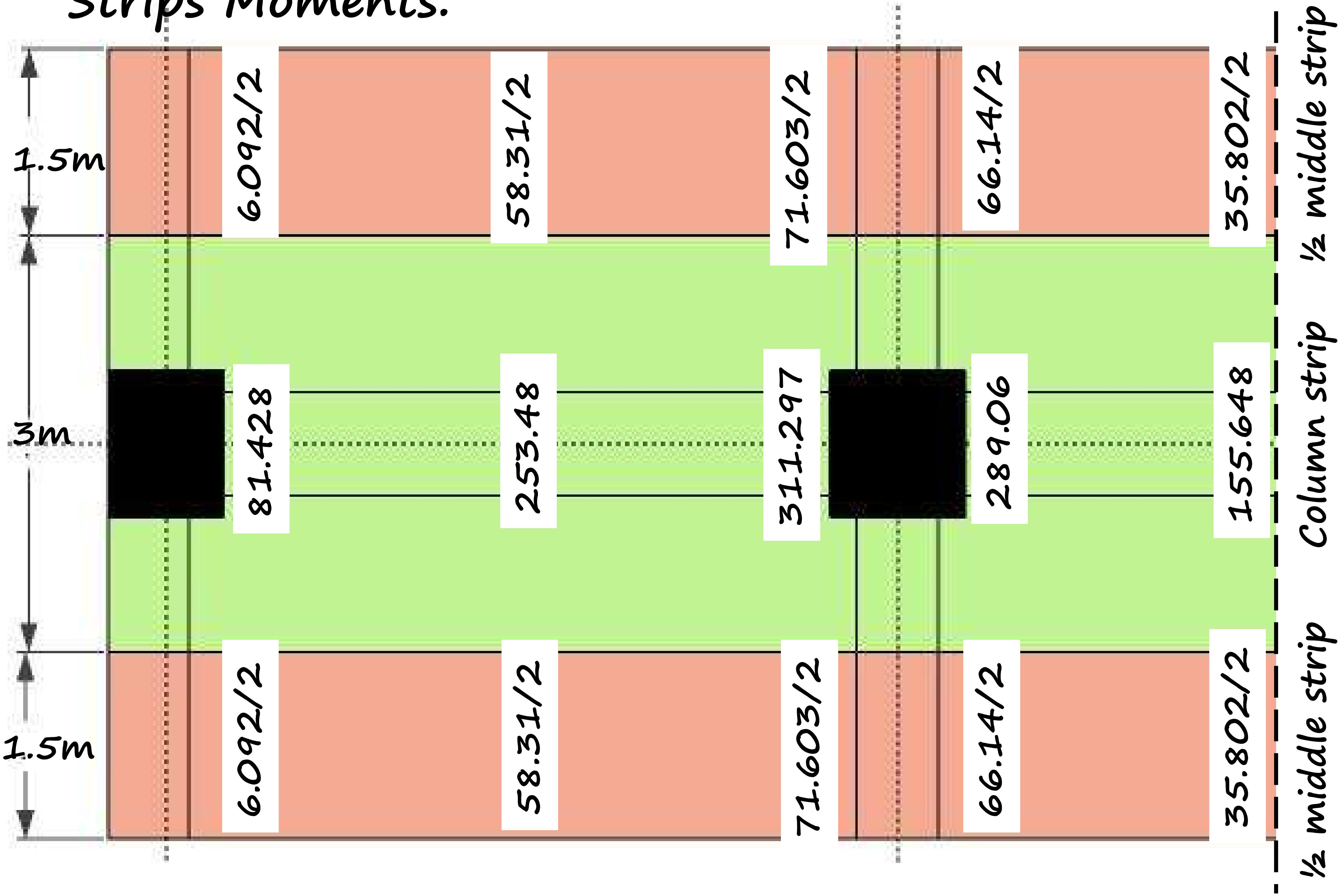
$$\alpha_{f1}\frac{l_2}{l_1} = 7.03 > 1 \quad , \quad M_{beam} = 85\% M_{c.s} \quad , \quad (M_{c.s})_{net} = 15\% M_{c.s}$$

Distribution of Moments:

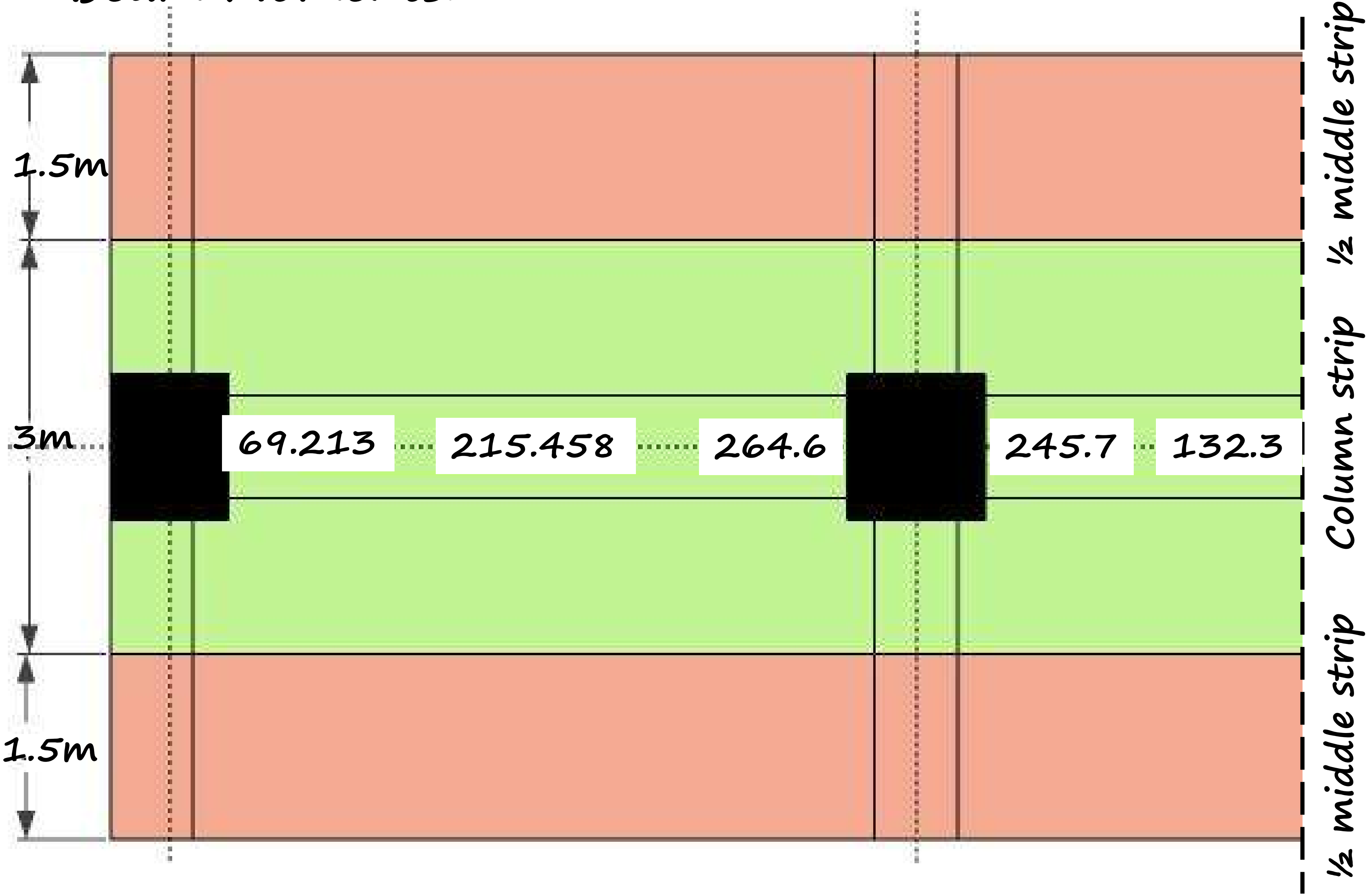
span	position	Span Moment (kN.m)	Column strip				Middle strip
			Percentage %	Mc.s	Mbeam	Mslab	
Exterior Span	-ve M(ext.)	87.52	93.04	81.428	69.21	12.214	6.092
	+ve M	311.79	81.3	253.48	215.45	38.02	58.31
	-ve M(int.)*	382.9	81.3	311.297	264.6	46.69	71.603
Interior Span	-ve M(int.)*	355.55	81.3	289.06	245.7	43.36	66.14
	+ve M	191.45	81.3	155.648	132.3	23.347	35.802

*Use the larger moment on two faces of column for design.

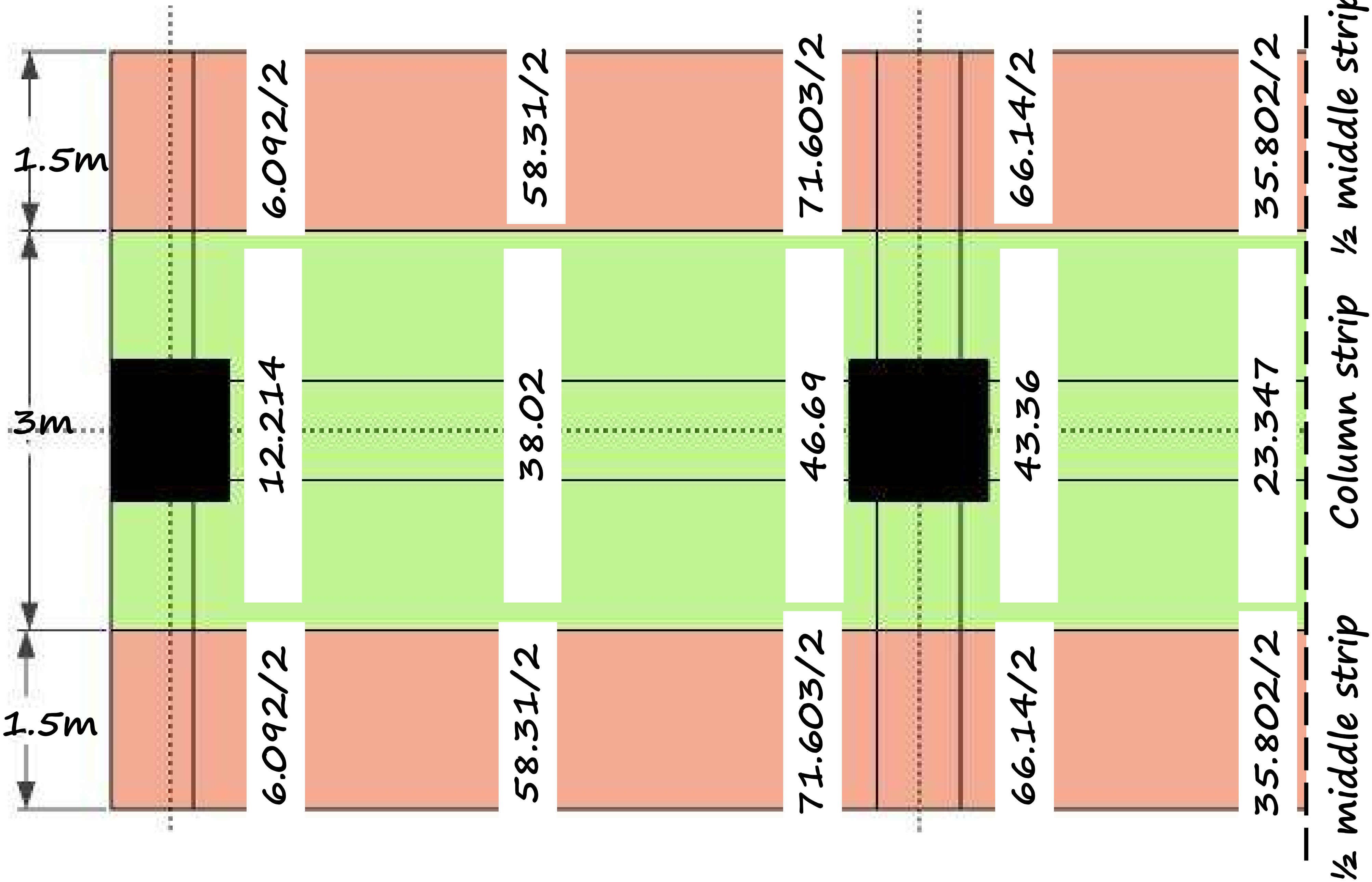
Strips Moments:



Beam Moments:



Slab Moments:



7-Find the area and number of flexural bars in each of the column and middle strips:

A) Slab Reinforcement: ACI (24.4.3.1)

$$\rho_{required} = \frac{1}{m}\left(1-\sqrt{1-\frac{2mR}{f_y}}\right) \quad \text{where} \quad m=\frac{f_y}{0.85 f'_c} \quad \text{and} \quad R=\frac{10^6 M}{0.9bd^2}$$

Or $$\rho_{required} = \frac{1-\sqrt{1-(2.622M_u / f'_c bd^2}}{1.18 f_y / f'_c}$$

$$A_{s\ required} = \rho_{required} \times b \times d \quad \text{and} \quad A_{s\ required} \geq A_{s\ min} = \rho_{min} bh$$

$\rho_{min} = 0.002$ $\quad f_y = (280) or (350) MPa$

$\rho_{min} = 0.0018$ $\quad f_y = 420 MPa$

$\rho_{min} = 0.0018\frac{420}{f_y}$ $\quad f_y > 420 MPa$

$\rho_{required} < \rho_{max}$, $\rho_{max} = 0.75 \times \left(0.85\beta\frac{f'_c}{f_y}\frac{600}{600+f_y}\right)$

$\beta = 0.85 - 0.05 \times (f'_c - 30)$ where $(f'_c \geq 30)$

$\beta = 0.85$ where $(f'_c < 30)$

Table 24.4.3.2—Minimum ratios of deformed shrinkage and temperature reinforcement area to gross concrete area

Reinforcement type	f_y, MPa	Minimum reinforcement ratio	
Deformed bars	< 420	0.0020	
Deformed bars or welded wire reinforcement	≥ 420	Greater of:	$\frac{0.0018 \times 420}{f_y}$
			0.0014

B) Required Spacing:

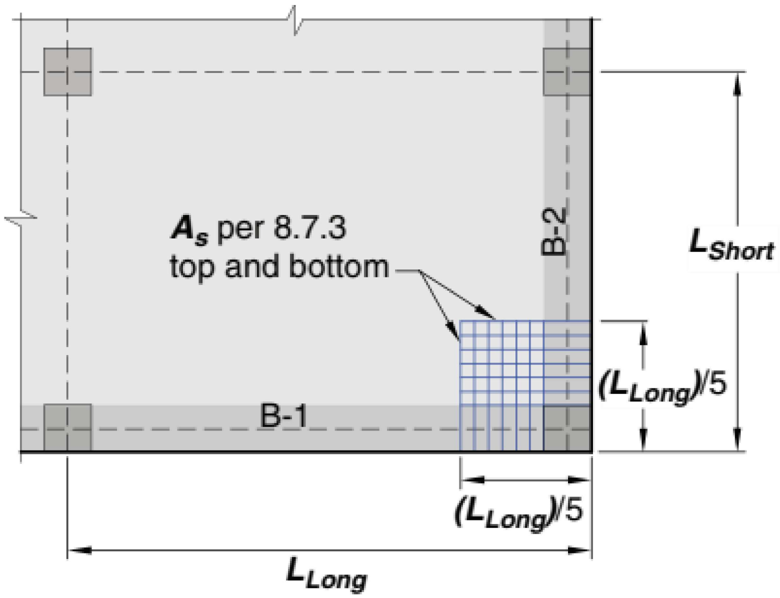
$$S_{required} = \frac{A_{bar}}{A_s} \times b(width) \quad \text{and} \quad S_{required} \leq 2h \quad \text{(h: slab thickness)}$$

Corner Reinforcement: ACI (8.7.3)

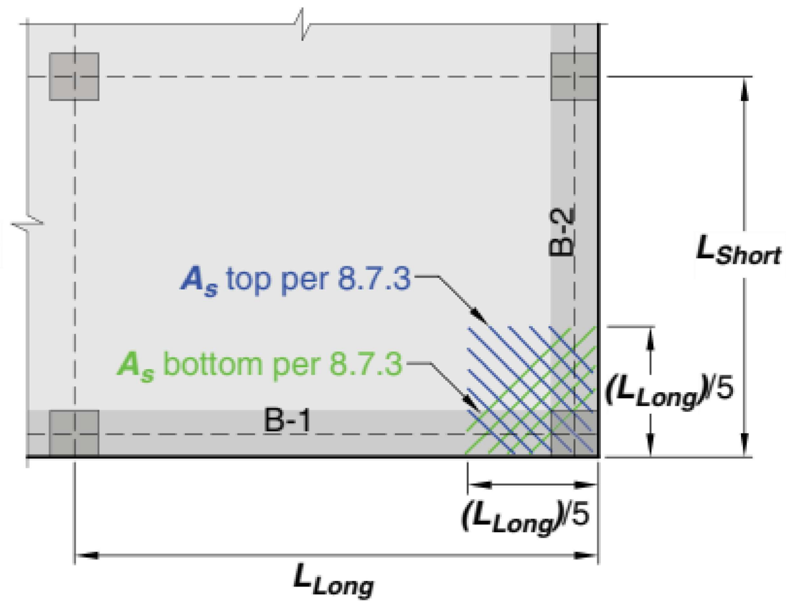
Unrestrained corners of two-way slabs tend to lift when loaded. If this lifting tendency is restrained by edge walls or beams, bending moments result in the slab. This section provides steel to resist these moments and control cracking. Reinforcement provided for flexure in the primary directions may be used to satisfy this requirement.

Corner reinforcement shall be placed parallel to the diagonal in the top of the slab and perpendicular to the diagonal in the bottom of the slab. Alternatively, reinforcement shall be placed in two layers parallel to the sides of the slab in both the top and bottom of the slab.

At exterior corners of slabs supported by edge walls or where one or more edge beams have a value of ($\alpha_f \geq 1$), top and bottom slab reinforcement shall be provided at exterior corners.



OPTION 2



OPTION 1

Notes:

1. Applies where B-1 or B-2 has $\alpha_f > 1.0$
2. Max. bar spacing 2h, where h = slab thickness

Example (3)

HOME WORK

Find the reinforcement area for the column and middle strips in each span in example (2)

Sol.

$d_{average}$= h-cover-db =165-20-12=133mm

Width of C.S=3m, Width of M.S=3m, f'c=30 MPa, fy=420 MPa

Location		Strip	M	R	ρreq.	As req. =ρbd mm²/m	Reinforcement	As prov. mm²/m
Exterior Span	M^- int.	C.S	12.214	0.255	0.00061	297	Ø12 @300 mm	377
		M.S	6.092					
	M^+	C.S	38.02					
		M.S	58.31					
	M^- ext.	C.S	46.69					
		M.S	71.603					
Interior Span	M^- int.	C.S	43.36					
		M.S	66.14					
	M^+	C.S	23.347					
		M.S	35.802					

Sample of Calculation:

$$R = \frac{10^6 M}{0.9bd^2} = \frac{10^6 \times 12.214}{0.9 \times 3000 \times 133^2} = 0.255$$

$$m = \frac{f_y}{0.85 f_c'} = \frac{420}{0.85 \times 30} = 16.47$$

$$\rho_{required} = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{f_y}}\right) = \frac{1}{16.47}\left(1 - \sqrt{1 - \frac{2 \times 16.47 \times 0.255}{420}}\right) = 0.00061$$

$$A_{s\,\min} = \rho_{\min} bh = 0.0018 \times 1000 \times 165 = 297 mm^2 / m$$

$$A_{s\,required} = \rho_{required} \times b \times d = 0.00061 \times 1000 \times 133 = 81.13 mm^2 / m < A_{s\min}$$

$$S_{required} = \frac{A_{bar}}{A_s} \times b(width) = \frac{113}{297} \times 1000 = 380 mm$$

$$S_{\max} = 2h = 2 \times 165 = 330 mm$$

Use $S = 300 mm$

$$A_{s\,provided} = \frac{A_{bar}}{S} \times 1000 = \frac{113}{300} \times 1000 = 377 mm^2 / m$$

Notes:

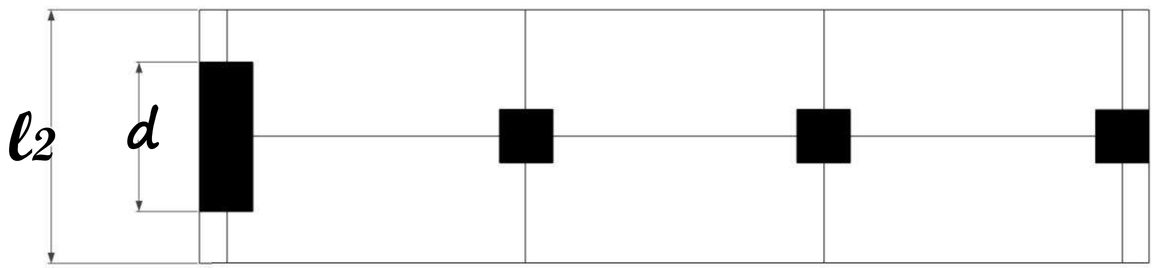
1- $\beta t=0$ (flat plate or flat slab without edge beams, brick wall)
$\beta t > 2.5$ (Reinforced concrete wall)

2-where supports consist of columns or wall extending for a distance ≥ 0.75 ℓ_2 (used to compute Mo which is equal to width of frame), negative moment shall be considered to be uniformly distributed across ℓ_2. ACI (8.10.5.4)

$$M^- = \frac{M^-}{l_2}$$

$$M^-_{c.s} = \frac{M^-}{l_2} \times width(c.s)$$

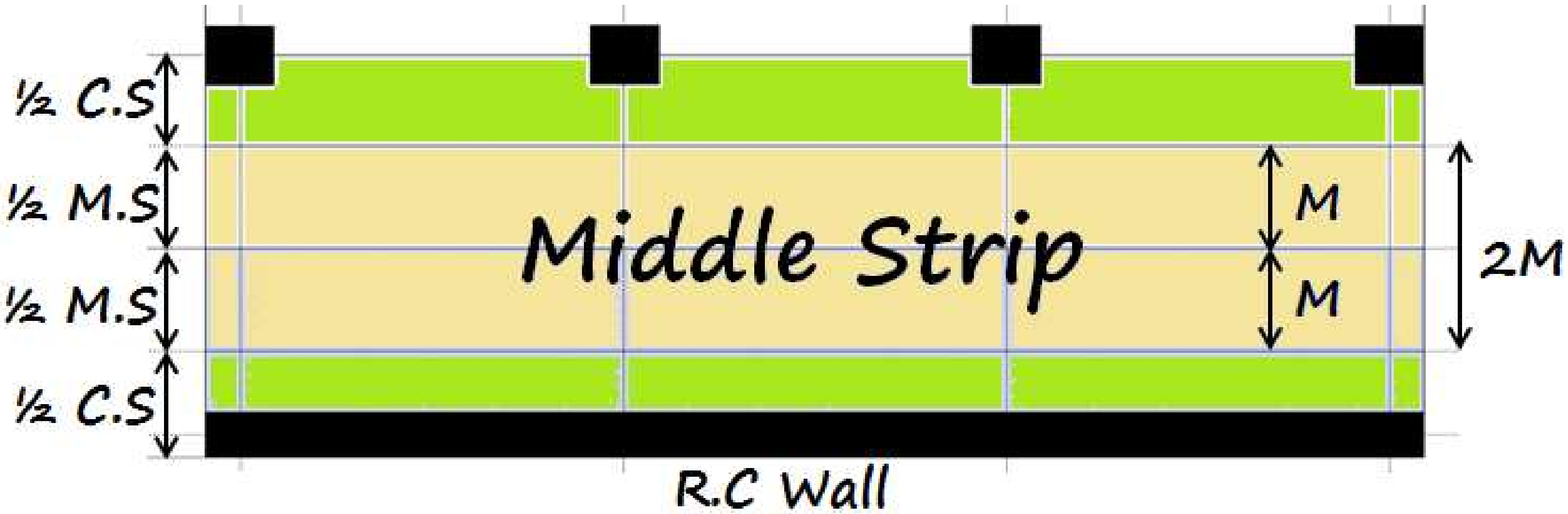
$$M^-_{m.s} = \frac{M^-}{l_2} \times width(m.s)$$



3- If walls are used as support along columns lines, they can regarded as very stiff beam with $(\alpha_{f1}\frac{l_2}{l_1}) > 1$.

4-Each middle strip shall be proportioned to resist the sum moments assigned to its two half middle strip ACI (8.10.6.2).

5- A middle strip adjacent to and parallel to a wall supported edge shall be proportioned to resist twice the moment assigned to the half middle strip corresponding to the first row of interior supports. ACI (8.10.6.3)



6- For the purpose of establishing moments in the half column strip adjacent to an edge supported by a wall, ℓ_n may be assumed equals to ℓ_n of the parallel adjacent column to column span, and a wall may be considered as a beam having a moment of inertia equal to infinity. (R 8.10.5)

Example(4)

Flat plate, L.L=2.5 kN/m², D.L=7 kN/m² (including slab weight), Col.=(600x300)mm, Beams=(600x300)mm, t=200mm, fy=400 MPa.

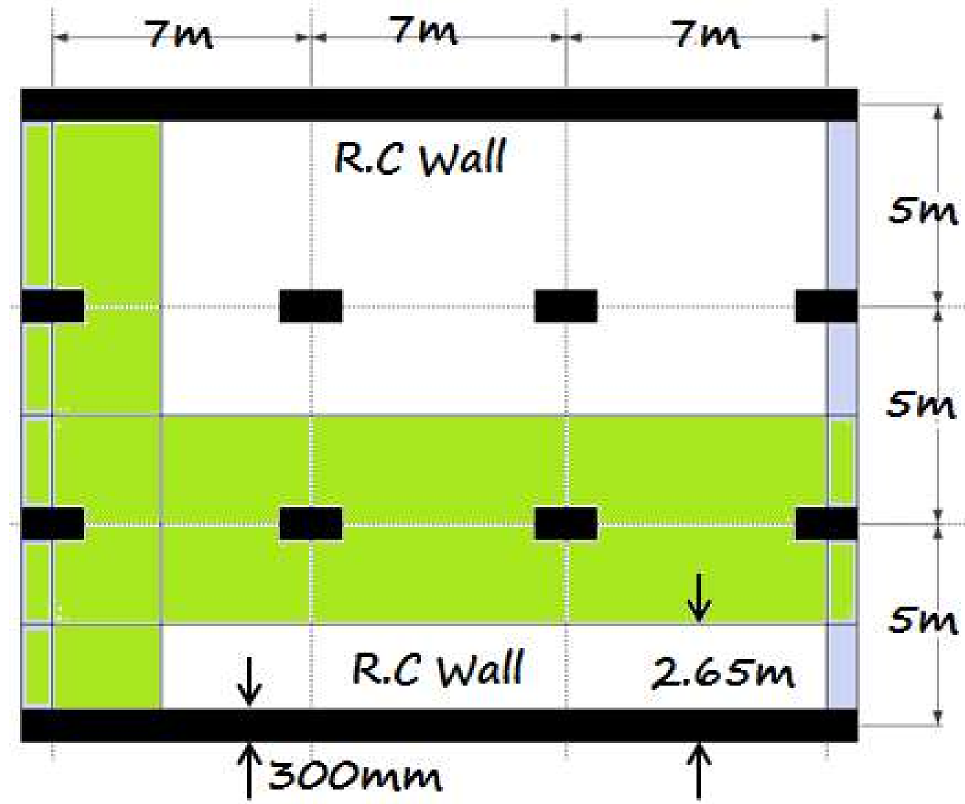
1-Find the exterior negative bending moment in beam

2-Find the positive moment per meter in the middle strip of the end span of the exterior frame in horizontal direction.

Sol.

1-take exterior frame in y-direction:

End Span



$$l_n = 5 - 0.3 = 4.7 > 0.65 l_1 = 0.65 \times 5 = 3.25 m$$

$$l_2 = \frac{7}{2} + 0.3 = 3.8 m$$

$$W_u = 1.2 D.L + 1.6 L.L$$

$$W_u = 1.2 \times 7 + 1.6 \times 2.5 = 12.4 kN/m^2$$

Total moment in the end span:

$$M_o = \frac{Wu \times l_2 \times l_n^2}{8} = \frac{12.4 \times 3.8 \times (4.7)^2}{8} = 130.11 kN.m$$

Table 8.10.4.2

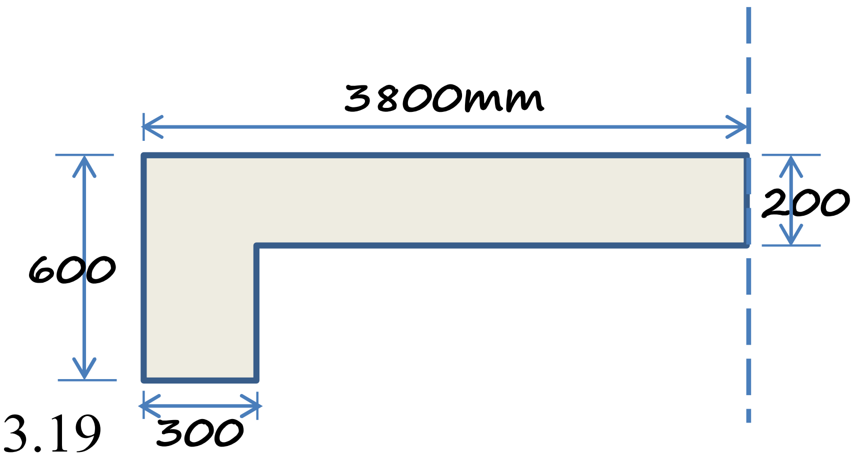
$$M^-_{ext.} = 0.65 M_o = 0.65 \times 130.11 = 84.57 kN.m$$ Case (5)

Note (2): (d> 0.75 ℓ2) therefore the moment distributed uniformly.

$M^{-}_{ext.}$ (Per meter) $= \frac{84.6}{3.8} = 22.26 kN.m/m$

Width of C.S=(5/4)+0.3=1.55m

Mc.s=1.55x22.26=34.5 kN.m



$$M_{beam} = 0.85(\alpha_{f1}\frac{l_2}{l_1}) \times M_{c.s} \leq 0.85 M_{c.s}$$

$$\alpha_{f1} = \frac{I_b}{I_{slab}} = \frac{300 \times 600^3 \times 1.5/12}{3800 \times 200^3/12} = \frac{8.1 \times 10^9}{2.53 \times 10^9} = 3.19$$

$$(\alpha_{f1}\frac{l_2}{l_1}) = 3.19 \times \frac{7}{5} = 4.47 > 1$$

$$M_{beam} = 0.85 M_{c.s} = 0.85 \times 34.5 = 29.33 kN.m$$

Adding the weight of stem:

Self weight of stem=1.2(0.3x0.4x24)=3.46 kN/m

Moment due to self weight= $\frac{1}{16} W_u l_n^2 = \frac{1}{16} \times 3.46 \times 4.7^2 = 4.77 kN.m$ T (6.5.2)

$$M_{beam} = 29.33 + 4.77 = 34.1 kN.m$$

Table 6.5.2—Approximate moments for nonprestressed continuous beams and one-way slabs

Moment	Location	Condition	M_u
Positive	End span	Discontinuous end integral with support	$w_u \ell_n^2/14$
		Discontinuous end unrestrained	$w_u \ell_n^2/11$
	Interior spans	All	$w_u \ell_n^2/16$
Negative[1]	Interior face of exterior support	Member built integrally with supporting spandrel beam	$w_u \ell_n^2/24$
		Member built integrally with supporting column	$w_u \ell_n^2/16$
	Exterior face of first interior support	Two spans	$w_u \ell_n^2/9$
		More than two spans	$w_u \ell_n^2/10$
	Face of other supports	All	$w_u \ell_n^2/11$
	Face of all supports satisfying (a) or (b)	(a) slabs with spans not exceeding 3 m (b) beams where ratio of sum of column stiffnesses to beam stiffness exceeds 8 at each end of span	$w_u \ell_n^2/12$

[1]To calculate negative moments, ℓ_n shall be the average of the adjacent clear span lengths.

2-Take interior frame in x-direction: (end span)

Note (5):

$$l_n = 7 - 0.6 = 6.4m > 0.65 l_1 = 0.65 \times 7 = 4.55m \quad , \quad l_2 = 5m$$

$$M_o = \frac{Wu \times l_2 \times l_n^2}{8} = \frac{12.4 \times 5 \times (6.4)^2}{8} = 317.44 kN.m$$

Table 8.10.4.2

$$M^+ = 0.5 M_o = 0.5 \times 317.44 = 158.73 kN.m$$ **Case (4)**

Table 8.10.5.5

$$(\alpha_{f1} \frac{l_2}{l_1}) = 0 \quad , \quad \frac{l_2}{l_1} = \frac{5}{7} = 0.7$$

$$M_{c.s}^+ = 0.6 M^+ = 0.6 \times 158.72 = 95.23 kN.m$$

$$M_{m.s} = 158.72 - 95.23 = 63.5 kN.m$$

$M_{m.s}$ adjacent to and parallel to the wall=63.5 kN.m

Moment in ½ middle strip in the exterior frame= $\frac{63.5}{2} = 31.75 kN.m$

Width of exterior frame=(5/2)+(0.3/2)=2.65m

Width of column strip=(5/4)+(0.15)=1.4m

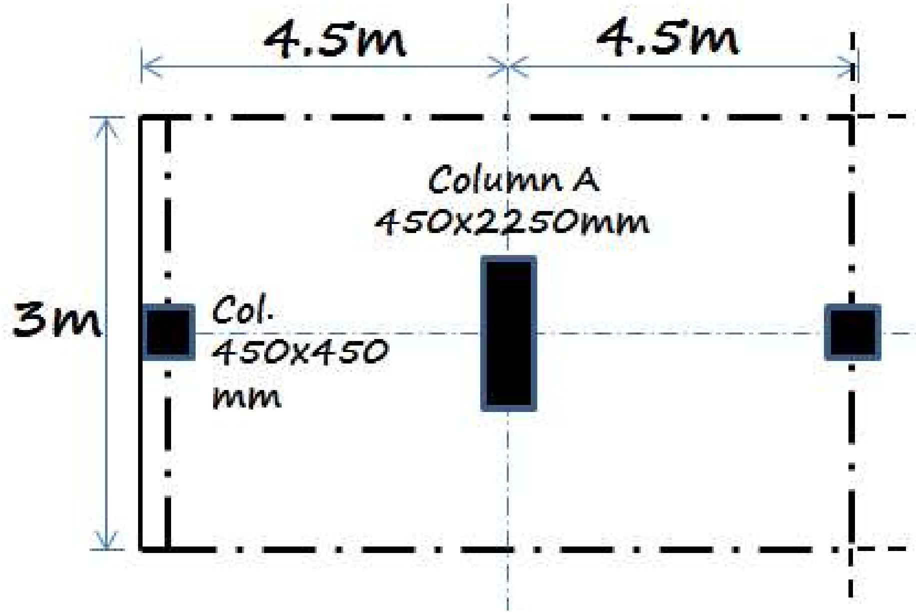
Width of ½ middle strip=2.65-1.4=1.25m

Positive moment (per meter) in ½ middle strip=31.75/1.25=25.4kN.m/m

Example (5)

For the interior design strip shown, find the spacing of reinforcement (Ø10mm bar) at column A if: L.L=4 kN/m² , D.L=6 kN/m² (including slab weight), t=175mm, fy=420 MPa, f'c=25 MPa.

Sol.



$$W_u = 1.2D.L + 1.6L.L$$

$$W_u = 1.2\times 6 + 1.6\times 4 = 13.6kN/m^2$$

$$l_n = 4.5 - 0.45 = 4.05m > 0.65l_1 = 0.65\times 4.5 = 2.92m$$

$$l_2 = 3m$$

$$M_o = \frac{Wu\times l_2\times l_n^2}{8} = \frac{13.6\times 3\times (4.05)^2}{8} = 83.65kN.m$$

End span:

Table (8.10.4.2)

$$M^- \text{int.} = 0.7M_o = 0.7\times 83.65 = 58.55kN.m > 0.65M_o$$ **Case (3)**

$$d_{col.} = 2.25 \ge 0.75l_2 = 0.75\times 3 = 2.25m$$ **Note (2)**

The moment distributed uniformly

Spacing of reinforcement:

b=3 m, d=175-20-10=145mm

$$R = \frac{10^6 M}{0.9bd^2} = \frac{10^6 \times 58.55}{0.9 \times 3000 \times 145^2} = 1.03 \quad , \quad m = \frac{f_y}{0.85 f'_c} = \frac{420}{0.85 \times 25} = 19.76$$

$$\rho_{required} = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{f_y}}\right) = \frac{1}{19.76}\left(1 - \sqrt{1 - \frac{2 \times 19.76 \times 1.03}{420}}\right) = 0.0025$$

$$\beta = 0.85 \quad , \quad (f'_c < 30) \quad , \quad \rho_{\max} = 0.75 \times \left(0.85\beta \frac{f'_c}{f_y} \frac{600}{600 + f_y}\right) = 0.0189 > 0.0025$$

$$A_{s\ required} = \rho_{required} \times b \times d = 0.0025 \times 1000 \times 145 = 363 mm^2 / m$$

$$\rho_{\min} = 0.0018 \quad , \quad f_y = 420 MPa$$

$$A_{s\ \min} = \rho_{\min} bh = 0.0018 \times 1000 \times 175 = 315 mm^2 / m < As_{req.}$$

$$\therefore A_s = 363 mm^2 / m \quad , \quad S_{required} = \frac{A_{bar}}{A_s} \times b(width) = \frac{78}{363} \times 1000 = 214 mm$$

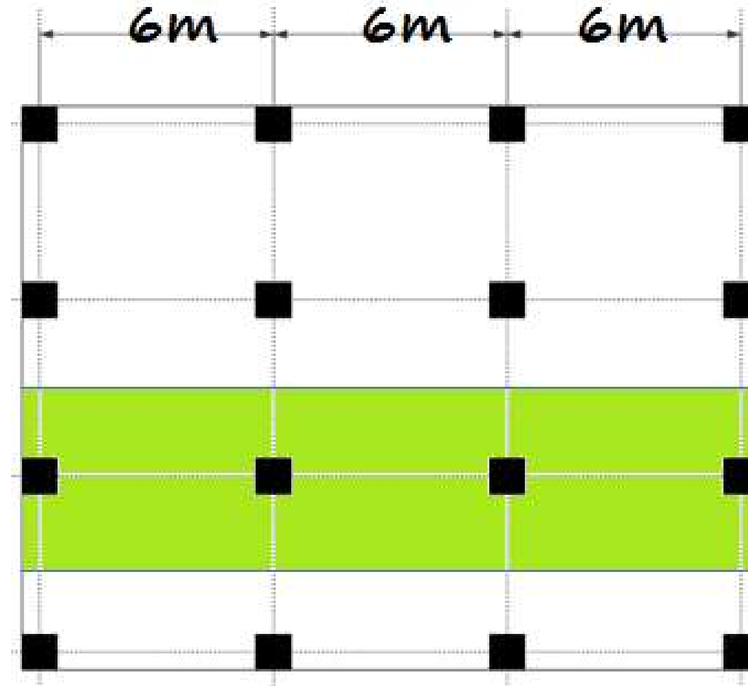
$S_{\max} = 2h = 2 \times 175 = 350 mm > S_{req.}$ **Use** $S = 200 mm$

Use Ø 10 @ 200mm c/c

Example (6)

Flat plate without edge beams. L.L=4 kN/m², t=200mm, fy=420 MPa, f'c=25 MPa, Cols. (300x300)mm.

Design the size and spacing of bar at the exterior edge of an interior frame in the long direction.



Sol.

$$W_u = 1.2D.L + 1.6L.L$$

$$W_u = 1.2\times(24\times0.2)+1.6\times4 = 12.16kN/m^2$$

$$l_n = 6-0.3 = 5.7m > 0.65l_1 = 0.65\times6 = 3.9m$$

$$l_2 = 5.5m$$

$$M_o = \frac{Wu\times l_2\times l_n^2}{8} = \frac{12.16\times5.5\times(5.7)^2}{8} = 271.6kN.m$$

Table (8.10.4.2) Case (3)

$$M^{-}ext. = 0.26M_o = 0.26\times271.6 = 70.62kN.m$$

Width of column strip=(5.5/4)x2=2.75m

Table (8.10.5.2): $\beta t=0$, $\alpha_{f1}=0$

Mc.s=100% x 70.62=70.62 kN.m

d=200-20-10=170mm

$$R = \frac{10^6 M}{0.9bd^2} = \frac{10^6 \times 70.62}{0.9 \times 2750 \times 170^2} = 0.987 \quad , \quad m = \frac{f_y}{0.85 f_c'} = \frac{420}{0.85 \times 25} = 19.76$$

$$\rho_{required} = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{f_y}}\right) = \frac{1}{19.76}\left(1 - \sqrt{1 - \frac{2 \times 19.76 \times 0.987}{420}}\right) = 0.00241$$

$$\beta = 0.85 \quad , \quad (f_c' < 30) \quad , \quad \rho_{\max} = 0.75 \times \left(0.85\beta \frac{f_c'}{f_y} \frac{600}{600 + f_y}\right) = 0.0189 > 0.00241$$

$$A_{s\ required} = \rho_{required} \times b \times d = 0.00241 \times 1000 \times 170 = 410 mm^2 / m$$

$$\rho_{\min} = 0.0018 \quad , \qquad f_y = 420 MPa$$

$$A_{s\ \min} = \rho_{\min} bh = 0.0018 \times 1000 \times 200 = 360 mm^2 / m < As_{req.}$$

$$\therefore A_s = 410 mm^2 / m \quad , \qquad S_{required} = \frac{A_{bar}}{A_s} \times b(width) = \frac{78}{410} \times 1000 = 190 mm$$

$$S_{\max} = 2h = 2 \times 200 = 400 mm > S_{req.}$$

Use Ø 10 @ 175mm c/c

Middle Strip:

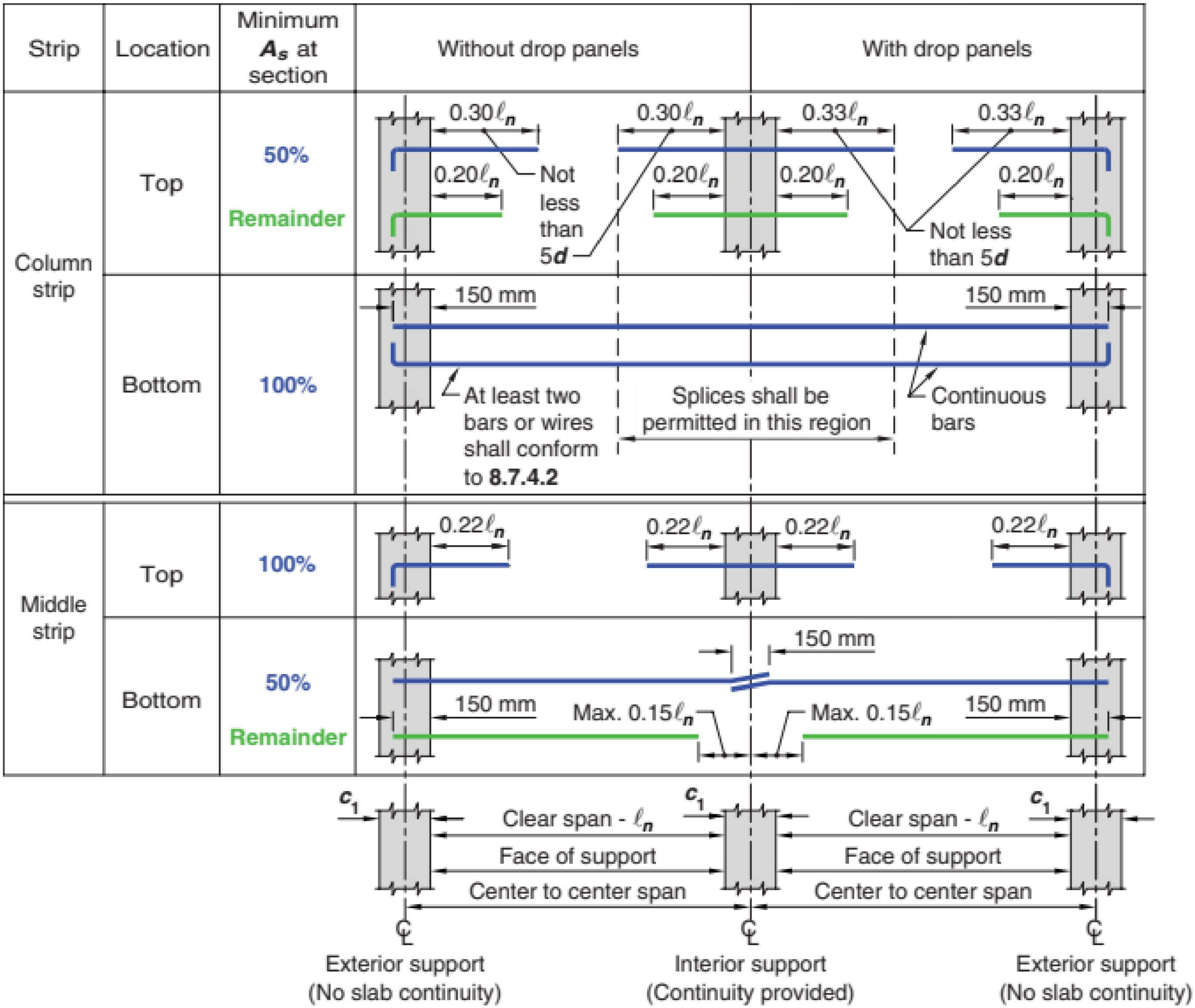
Mm.s=0 (100% of moment to column strip)

Use $A_{s_{\min}}$ $360mm^2/m$

$$S_{required} = \frac{A_{bar}}{A_s} \times b(width) = \frac{78}{360} \times 1000 = 216mm < 2h = 400mm$$

Use Ø 10 @200 mm

Minimum extension for reinforcement in slabs without beams



Factored Moments in Columns ACI 318-2014 (8.10.7)

Columns and walls built integrally with a slab system shall resist moments caused by factored loads on the slab system.

1-At an interior support, supporting elements above and below the slab shall resist the factored moment specified by the following equation :

$$M_{sc} = 0.07\left[(q_{Du} + 0.5q_{Lu})l_2 l_n^2 - q'_{Du} l'_2 (l'_n)^2\right]$$ Eq. (8.10.7.2)

$$M_{sc} = 0.07(0.5q_{Lu} l_2 l_n^2)$$ (Equal adjacent spans)

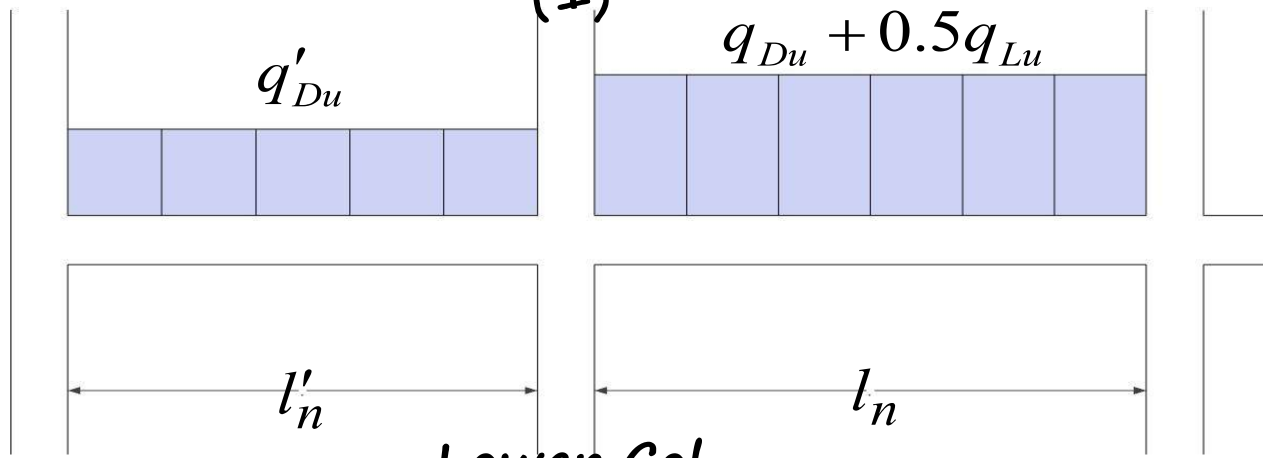
Where:

M_{sc} Factored slab moment that is resisted by the column at a joint.

q'_{Du}, l'_2, l'_n Refers to shorter span and

q_{Du} : Factored D.L and q_{Lu} : Factored L.L

The moment is distributed to the upper and lower column in proportion of the relative stiffness.



$$M_{col(1)}=\frac{K_{col(1)}}{K_{col(1)}+K_{col(2)}}\times Mu$$ and $$M_{col(2)}=\frac{K_{col(2)}}{K_{col(1)}+K_{col(2)}}\times Mu$$

Where $K_{col}=(\frac{4EI}{l})_{col}$

If the same size and length of upper and lower columns, then: $M_{col}=\frac{Mu}{2}$

2- At an exterior support, the total exterior negative factored moment from the slab system ACI (8.10.4.2) is transferred directly to the supporting member.