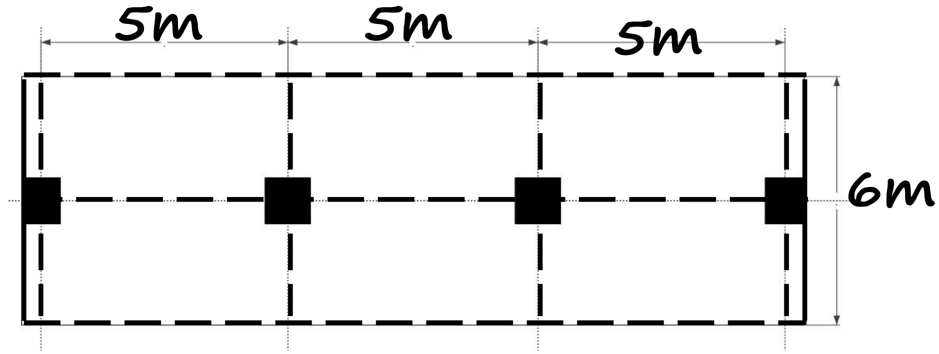
Example (6)

For the interior strip shown, find the factored moments in exterior and interior columns in the direction of analysis. q_{Lu}=5 kN/m², t=150mm, Cols.=(400x400)mm, ℓ_{col}=3000mm



Sol.:

Interior Columns:

$$M_{sc} = 0.07(0.5 q_{Lu} l_2 l_n^2)$$

$$M_{sc} = 0.07(0.5 \times 5 \times 6 \times 4.6^2) = 22.218 kN.m$$

The size and length of columns above and below the slab are the same.

$$\therefore M_{col} = \frac{22.218}{2} = 11.11 kN.m$$

The moment is combined with factored axial load for each story to design the interior column.

Exterior Columns:

$$W_u = 1.2 D.L + 1.6 L.L$$

$$W_u = 1.2 \times (0.15 \times 24) + 5 = 9.32 kN/m^2$$

$$l_n = 5 - 0.4 = 4.6 > 0.65 l_1 = 0.65 \times 5 = 3.25 m$$

$$M_o = \frac{Wu \times l_2 \times l_n^2}{8} = \frac{9.32 \times 6 \times (4.6)^2}{8} = 148 kN.m$$

$$M_{ext.}^{-} = 0.26 M_o = 0.26 \times 148 = 38.48 kN.m$$ **Case(3)**

$$M_{col} = 38.48/2 = 19.24 kN.m$$

(Interior in the other direction)

Shear in Two Way Slab System

If the thickness shown for deflection is not adequate to carry the shear, use one or more of the following:

(a) Increase the column dimension.

(b) Increase concrete strength.

(c) Increase slab thickness.

(d) Use special shear reinforcement.

(e) Use drop panels or column capitals to improve shear strength.

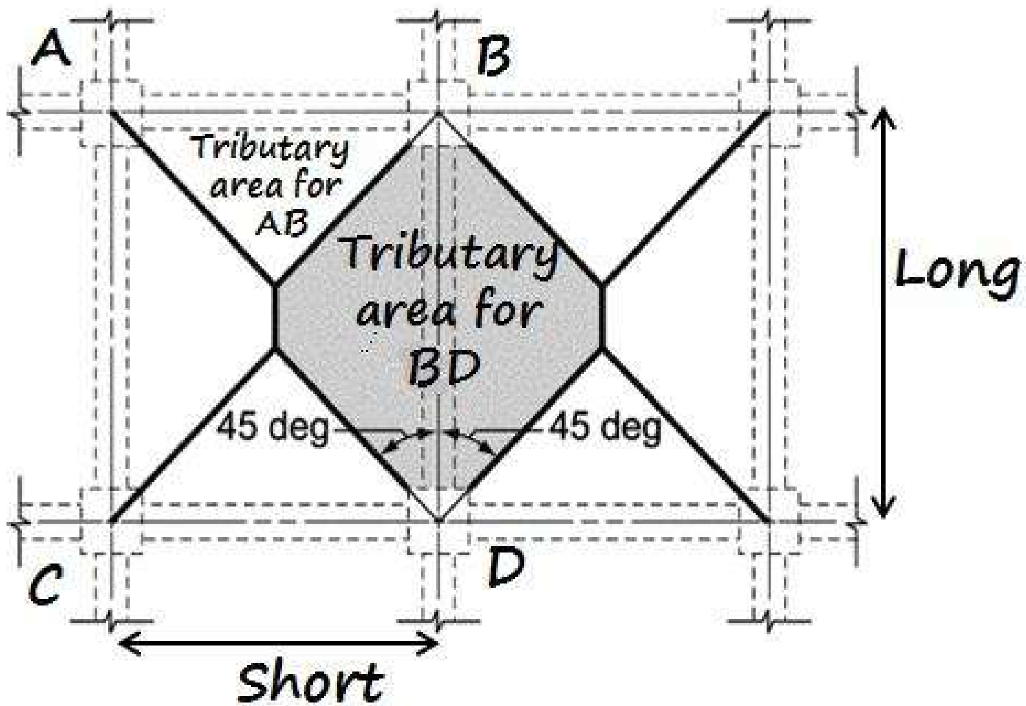
A- Shear in Two Way System with Beams ACI (8.10.8)

1- loads on Beams

Beams with $(\alpha_{f1} l_2 / l_1) \geq 1$ shall be proportioned to resist shear caused by factored loads on tributary areas which are bounded by 45-degree lines drawn from the corners of the panels and the centerlines of the adjacent panels parallel to the long sides.

For value of $0 < (\alpha_{f1} l_2 / l_1) < 1$, the proportion of load carried by beam is found by linear interpolation, assuming beams carry no load at $\alpha f1 = 0$.

In addition to shears calculated based on loads transferred from the supporting slab, beams shall be proportioned to resist shears caused by factored loads applied directly on beams.



2- Shear Force in the Supported Slab

the shear per unit width of slab along the beam is highest at the ends of slab strip perpendicular to the long beams (AC and BD). Considering the increased shear at the exterior face of first interior supports, the shear in the slab can be approximately equal to:

$$Vu = 1.15\left(\frac{Wu.S}{2}\right)$$

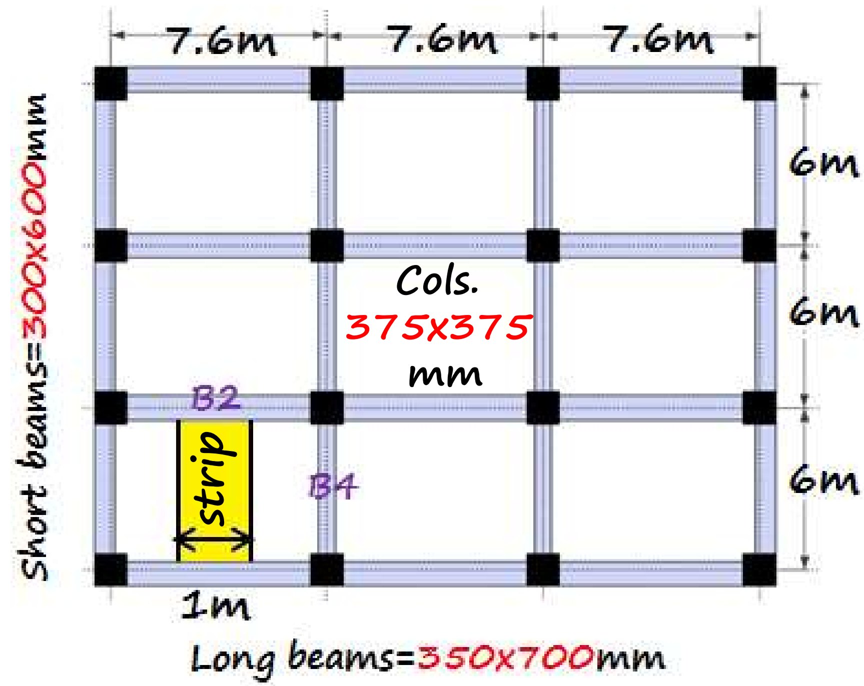
where S: is clear span (face to face of beams)

Example (1)

check the adequacy of slab for shear and compute the design shear force in the long and short beams. Wu=13.95 kN/m² , f'c=30 MPa, t=165mm

Sol:

Max. shear force in the slab can be computed as the reaction of the strip of (1m).



$$Vu_{slab} = 1.15\left(\frac{Wu \times S}{2}\right)$$

$$= 1.15 \times \frac{13.95 \times 5.662}{2} = 45.42 kN/m$$

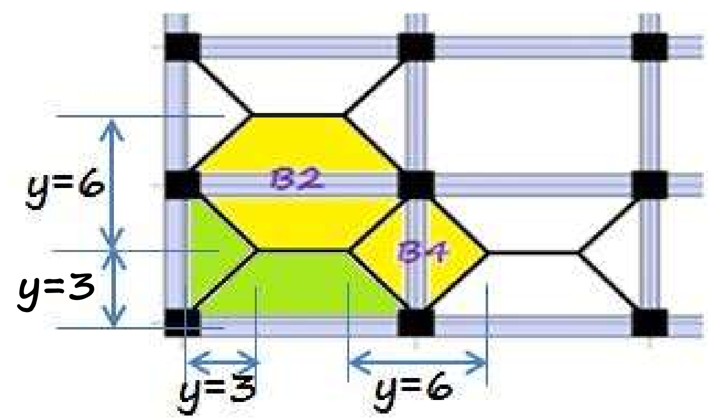
Slab design shear strength can be taken as:

b= 1000mm, d=165-20-12=133mm

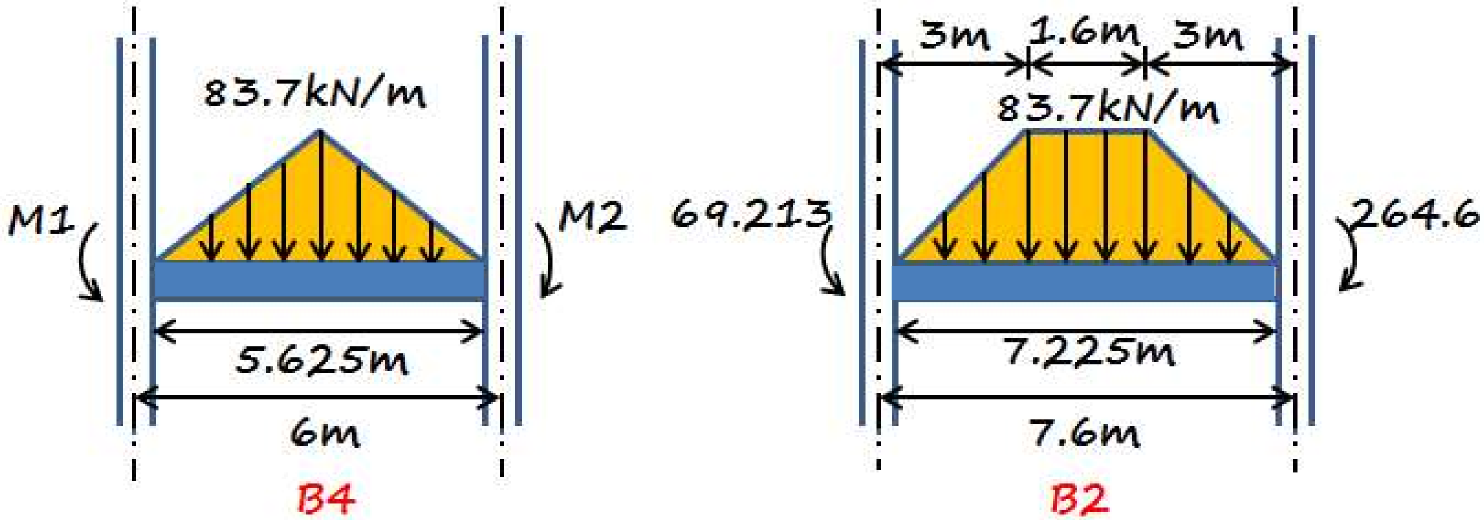
$$\phi Vc = 0.75 \times \frac{\sqrt{f'c}}{6} bd = 0.75 \times 0.17\sqrt{30} \times 1000 \times 133 \times 10^{-3} = 92.88 kN/m$$

$$\phi Vc = 92.88 kN > Vu = 45.32 kN/m \quad \text{o.k}$$

Beam shear force: $(\alpha_{f1} l_2 / l_1) > 1$



The forces that acting on the critical beams in the short and long directions and that transferred from the supported slab only can be summarized and shown in figures below:



$$Wu \times y = 13.95 \times 6 = 83.7 kN/m$$

The moments (69.213 and 264.6) for B2 are beam moments from column strip in the longitudinal direction (page 63). Also find the moments (M1 and M2) for B4 in the transverse direction.

The total load transmitted to B2 from the supported slab:

$$(Wu_{B2})_{slab} = \frac{1.6 + 7.225}{2} \times 83.7 = 369.32 kN$$

Self weight of stem of B2=1.2(0.7-0.165) x 0.35 x 24= 5.4 kN/m

$$Vu_{B2}=1.15\times\frac{5.4\times7.225}{2}+\frac{369.32}{2}+\frac{264.6-69.213}{7.225}=234.13kN$$

The total load transmitted to B4 from the supported slab:

$$(Wu_{B4})_{slab}=\frac{5.625\times83.7}{2}=235.4kN$$

Self weight of stem of B4=1.2(0.6-0.165) x 0.3 x 24= 3.75 kN/m

$$Vu_{B4}=1.15\times\frac{3.75\times5.625}{2}+\frac{235.4}{2}+\frac{M_1-M_2}{5.625}=?kN$$

For B2: d= 700-40-10-10=640mm

$$\phi Vc=\phi\times0.17\sqrt{f'c}bd=0.75\times0.17\sqrt{30}\times350\times640\times10^{-3}=156.43<Vu$$

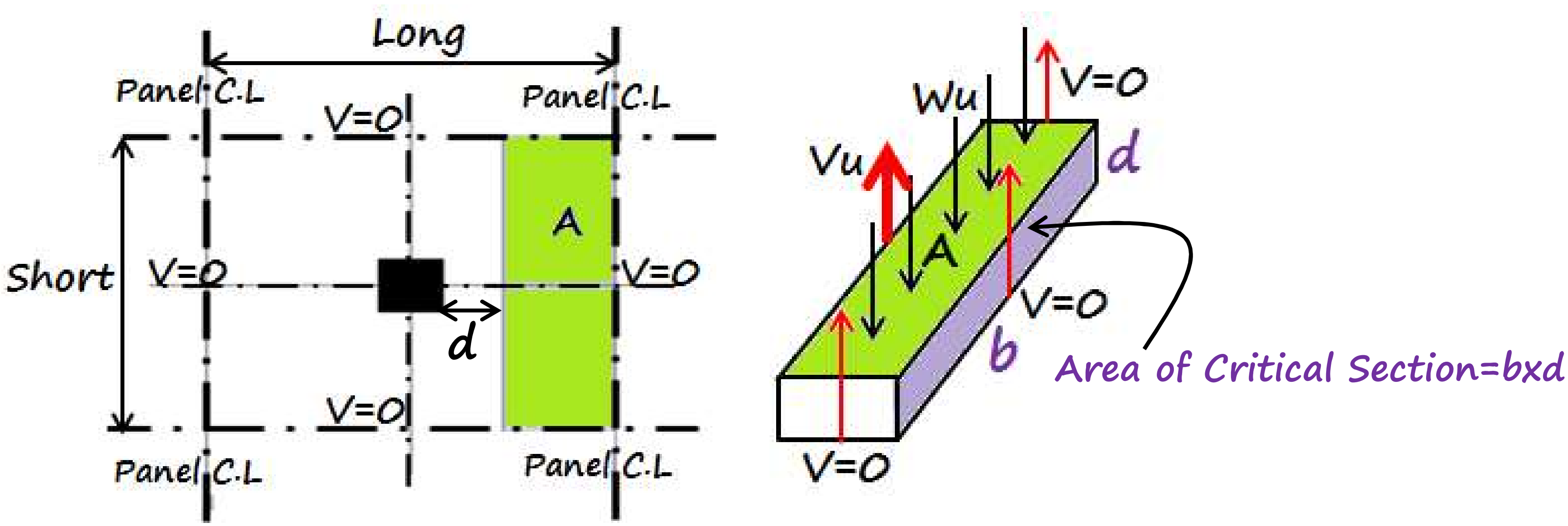
Then ; design the shear reinforcement.

B- Shear in Flat Plate and Flat Slab

1-One Way Shear Action: (Beam Action)

The critical section at distance (d) from face of support (column or column capital) or drop panel.

Circular or regular polygon shaped supports shall be treated as square support with the same area.



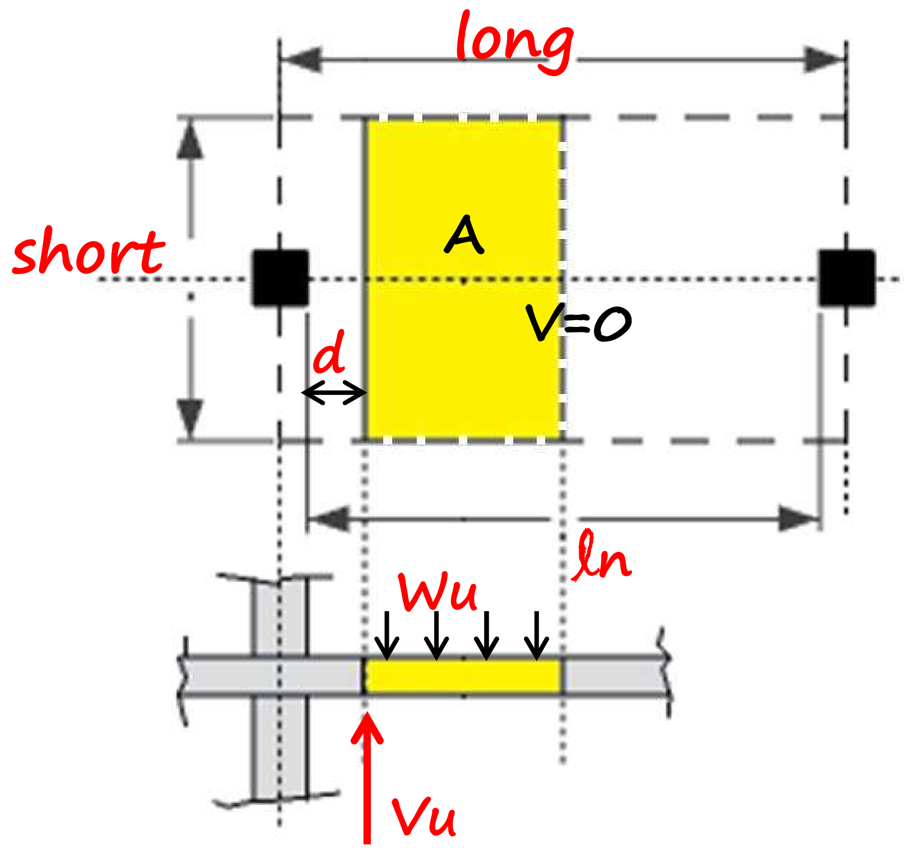
1-Flat Plate

The critical section for checking the shear capacity is at a distance effective depth (d) from the face of the column, across the entire width of the frame.

$$Vu = Wu \times A$$

$$\phi Vc = \phi(0.17\sqrt{f'c} \times b \times d)$$

Where b= short length

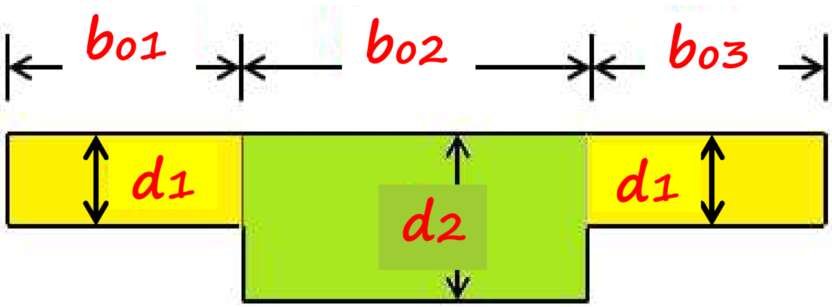


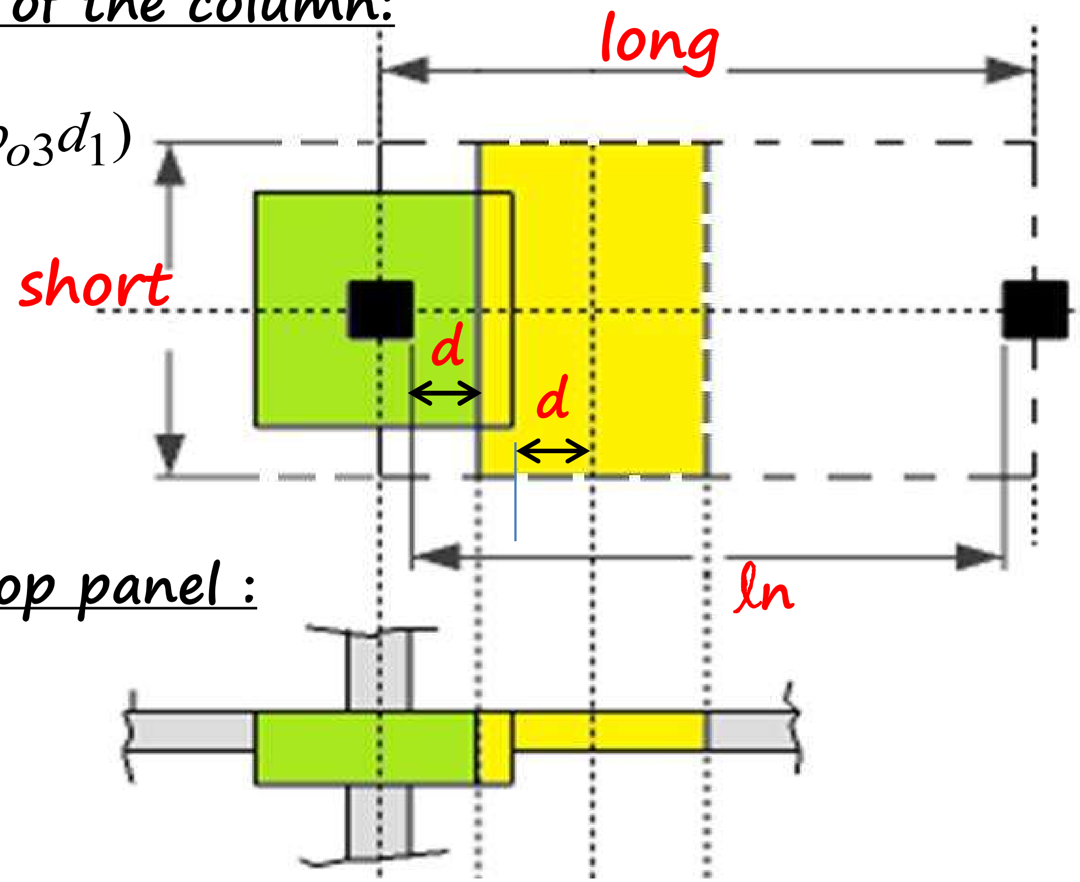
2-Flat Slab with Drop Panel

Two critical sections need to be checked. The first section is at a distance d from the face of the column or column capital, and the second section is at a distance d from the face of the drop panel <u>where d is the effective depth of the slab.</u>

<u>Critical Section at d from the face of the column:</u>

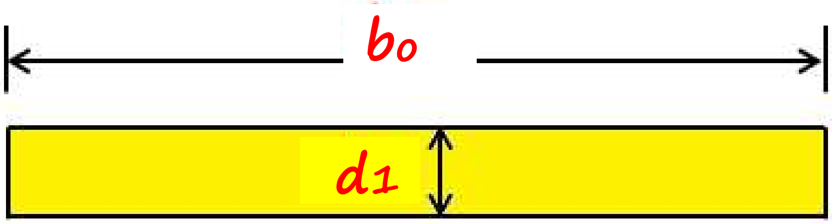
$$\phi Vc = \phi(0.17\sqrt{f'c}) \times (b_{o1}d_1 + b_{o2}d_2 + b_{o3}d_1)$$





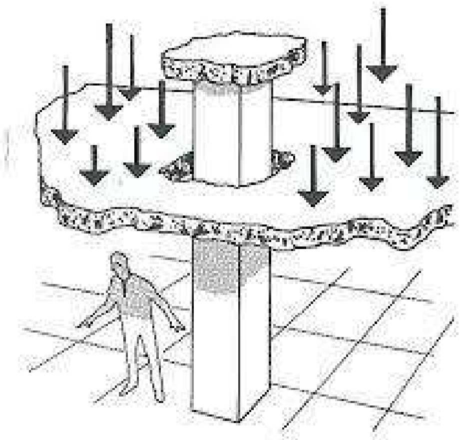
<u>Critical Section at d face of the drop panel :</u>

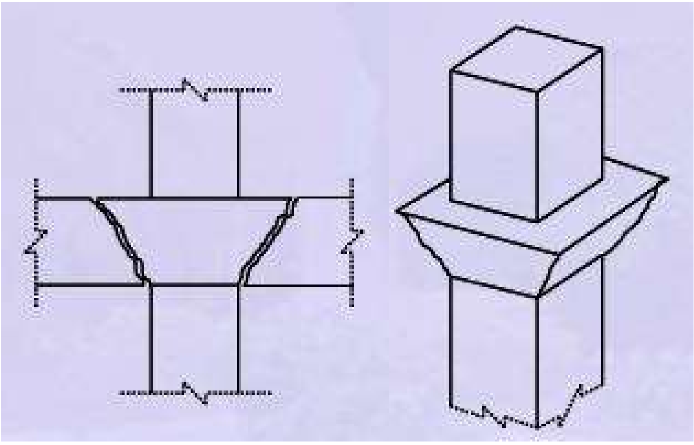
$$\phi Vc = \phi(0.17\sqrt{f'c}) \times (b_o \times d_1)$$



2-Two Way Shear Action: (Punching Shear) ACI 22.6

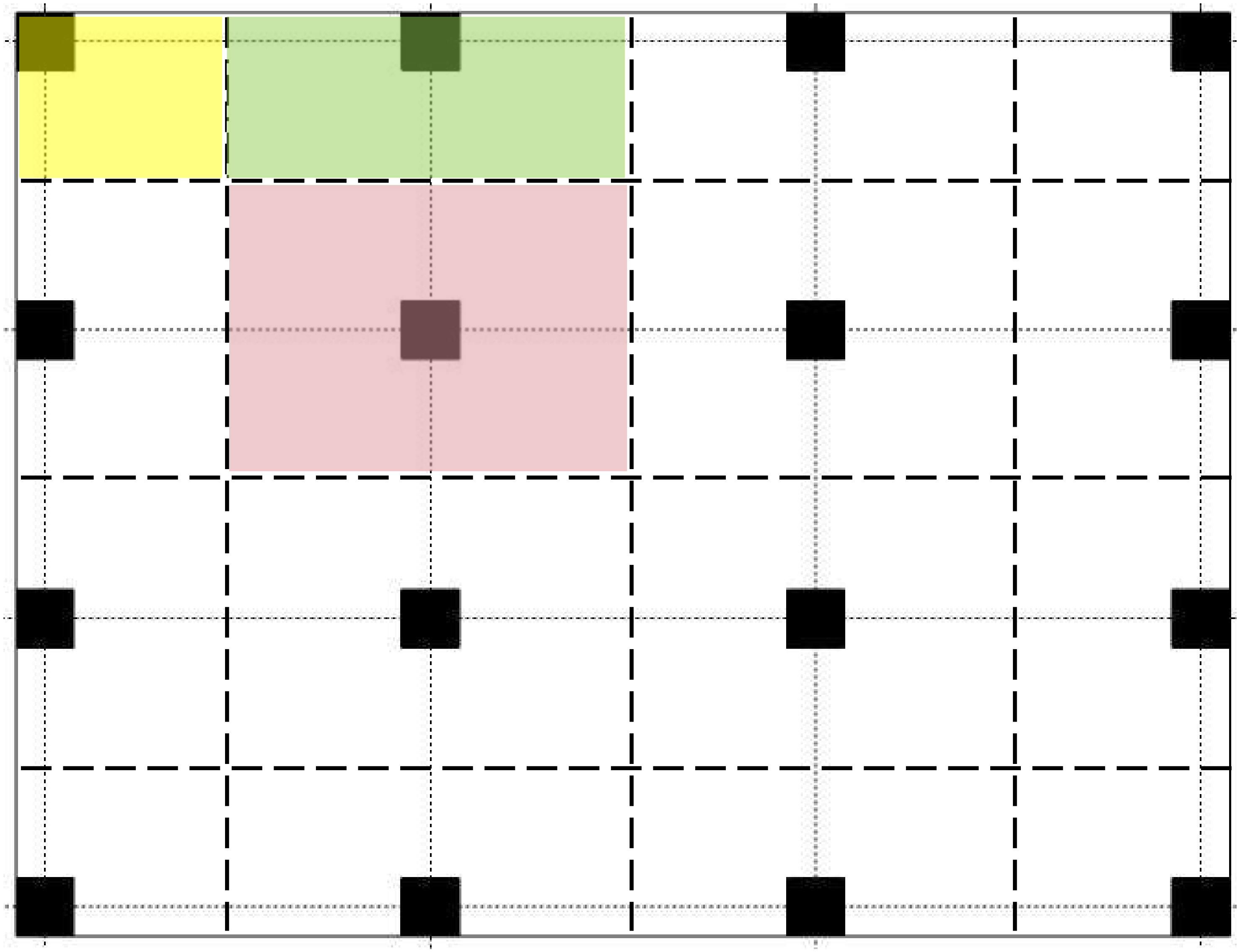
The two-way shear is specific to two-way slabs. Critical shear occurs around columns in the absence of beams (in flat plate) and around column capital or drop panel (in flat slab). If the capacity is inadequate, the slab may fail due to punching around a column. The following figure illustrates the punching shear failure.





The critical section for checking the shear capacity is geometrically similar to the column perimeter and is at a distance (d / 2) from the face of the column. The depth of the critical section is equal to the average of the effective depths of the slab in the two directions.

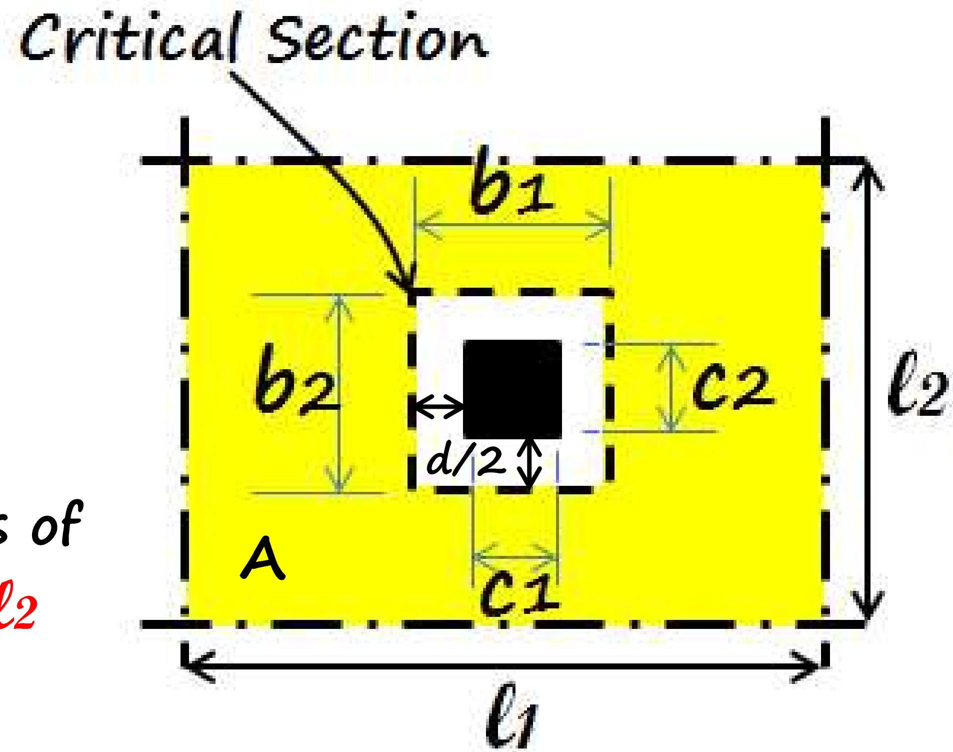
Loading Area on Column Causing Punching Shear



1-Flat Plate

$b_1 = C_1 + d$

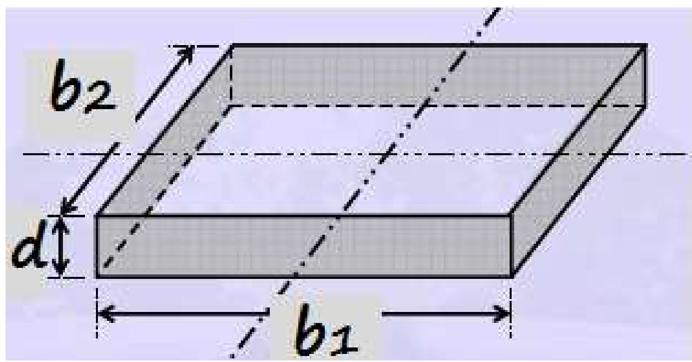
$b_2 = C_2 + d$



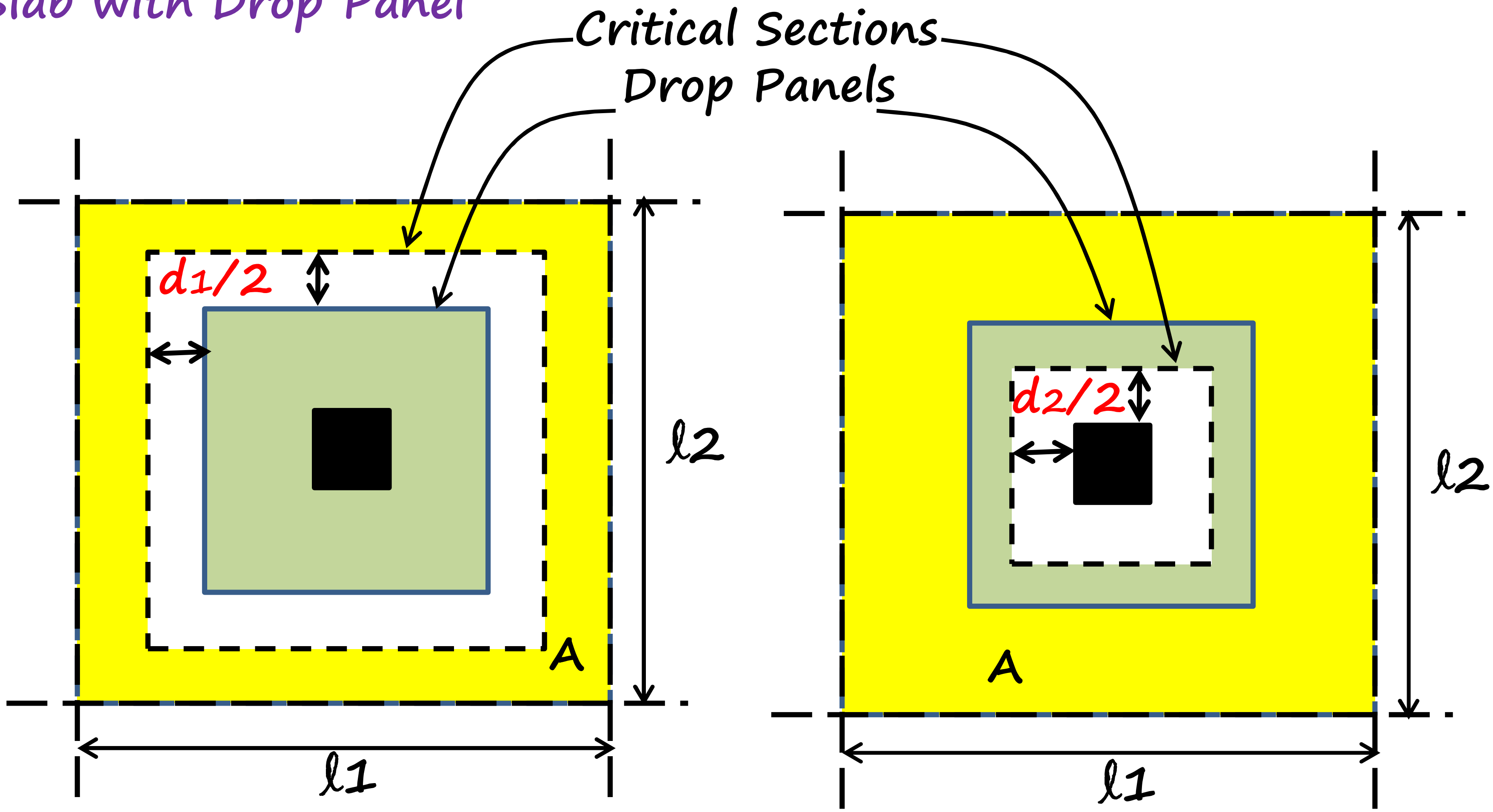
The lengths of the sides of the critical section (b_1 and b_2) and the dimensions of column (C_1 and C_2) are along ℓ_1 and ℓ_2 respectively.

The area of critical section = $2(b_1+b_2) \times d$

Vu=Wu x A , Vu: applied shear force



2-Flat Slab with Drop Panel

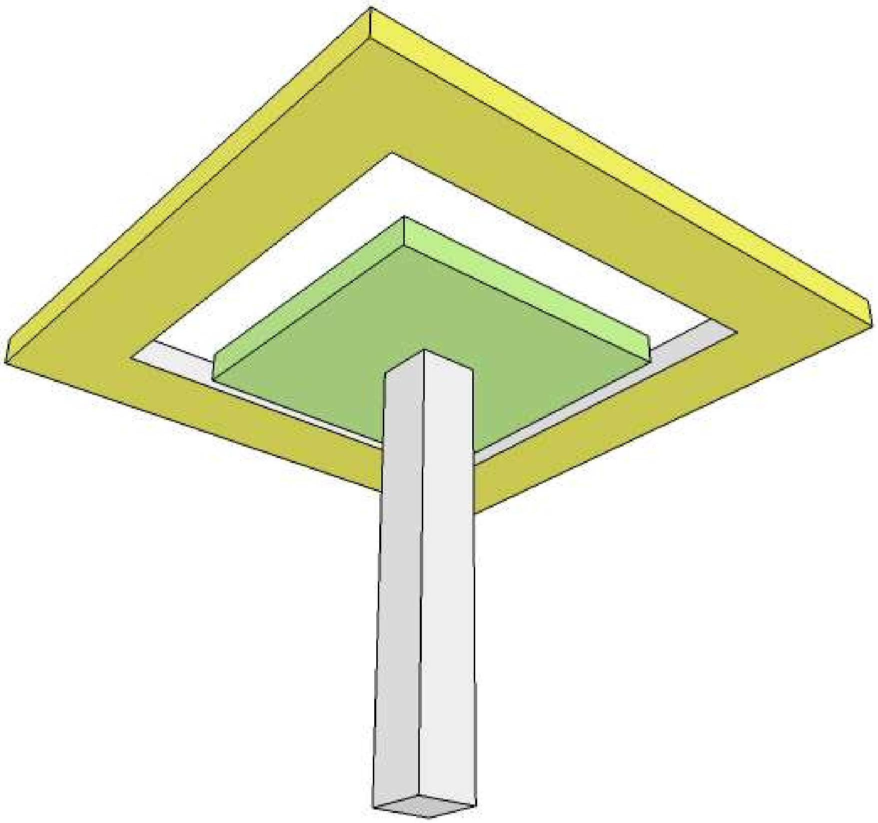


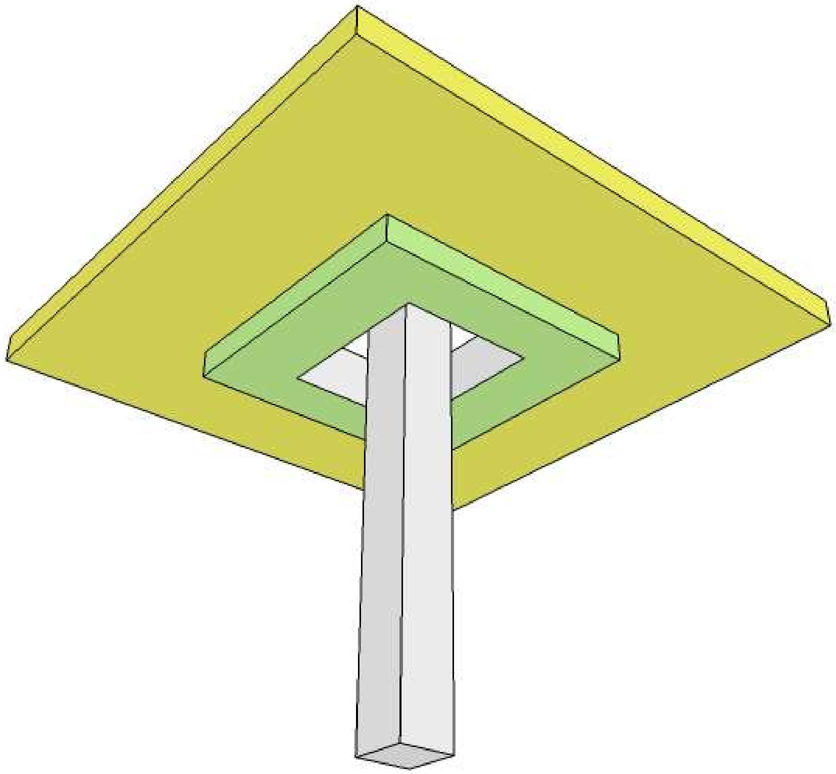
Critical Section at $d_1/2$ from drop panel

Critical Section at $d_2/2$ from column

where d_1 is the effective depth of slab and d_2 is the effective depth of drop panel .

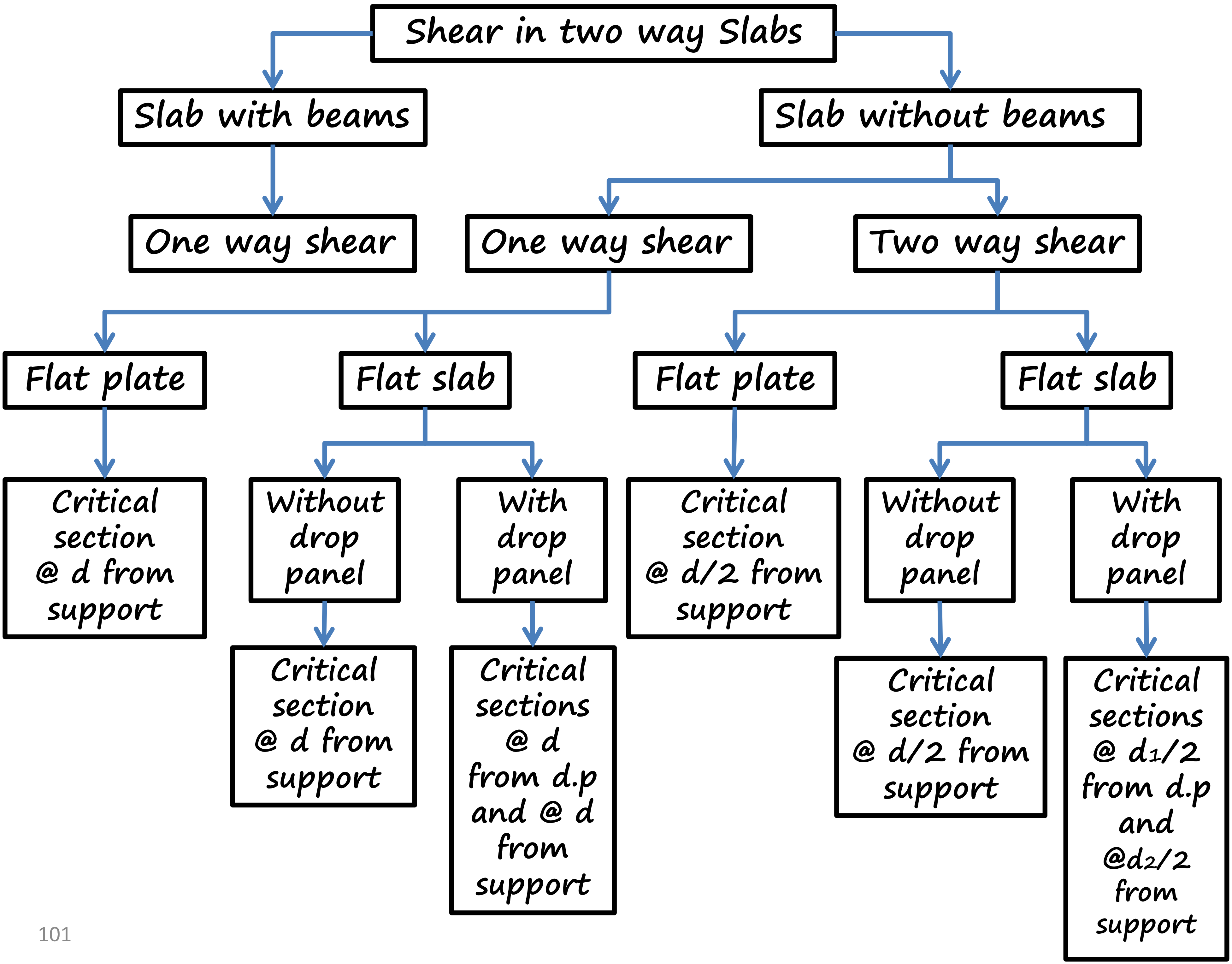
Area of critical section= perimeter of critical section x effective depth of section.





Critical Section at $d_1/2$ from drop panel

Critical Section at $d_2/2$ from column


Shear in two way Slabs
Slab with beams
Slab without beams
One way shear
One way shear
Two way shear
Flat plate
Flat slab
Flat plate
Flat slab
Critical section @ d from support
Without drop panel
With drop panel
Critical section @ d/2 from support
Without drop panel
With drop panel
Critical section @ d from support
Critical sections @ d from d.p and @ d from support
Critical section @ d/2 from support
Critical sections @ d1/2 from d.p and @d2/2 from support

The nominal shear strength of slab taken as the smallest of the following equations:

$$Vc = 0.33\lambda_s\lambda\sqrt{f'c} \times b_o d$$

$$Vc = (0.17 + \frac{0.33}{\beta})\lambda_s\lambda\sqrt{f'c} \times b_o d$$

$$Vc = (0.17 + \frac{0.083\alpha_s d}{b_o})\lambda_s\lambda\sqrt{f'c} \times b_o d$$

β= (Long side/ Short side) of column, concentrated load, reaction area, column capital or drop panel

Where: $\lambda_s = \sqrt{\frac{2}{1+0.004d}} \le 1$

factor used to modify shear strength based on the effects of member depth, commonly referred to as the size effect factor.

$\lambda = 1.0$

modification factor to reflect the reduced mechanical properties of lightweight concrete relative to normal-weight concrete of the same compressive strength.

bo : perimeter of critical section

αs= 40 for interior column (critical sides).

αs= 30 for edge column (critical sides).

αs= 20 for corner column (critical sides).

If $Vu \le \phi Vc$ no shear reinforcement is required

If $\phi Vc < Vu \le \frac{\phi}{2}\sqrt{f'c} \times b_o d$ provide shear reinforcement

Reinforcement of Shear

A- Stirrups:

$$\phi Vs = Vu - \phi 0.17\sqrt{f'c} \times b_o d$$

$$S = \frac{\phi A_v f_y d}{\phi V_s} < S_{\max} = \frac{d}{2}$$

B- Bent Bar Reinforcement:

$$\phi Vs = Vu - \phi Vc$$

$$\max .\phi Vs = \frac{\phi}{4}\sqrt{f'c} \times b_o d$$

$$\phi Vc = \phi \times 0.17\sqrt{f'c} \times b_o d$$

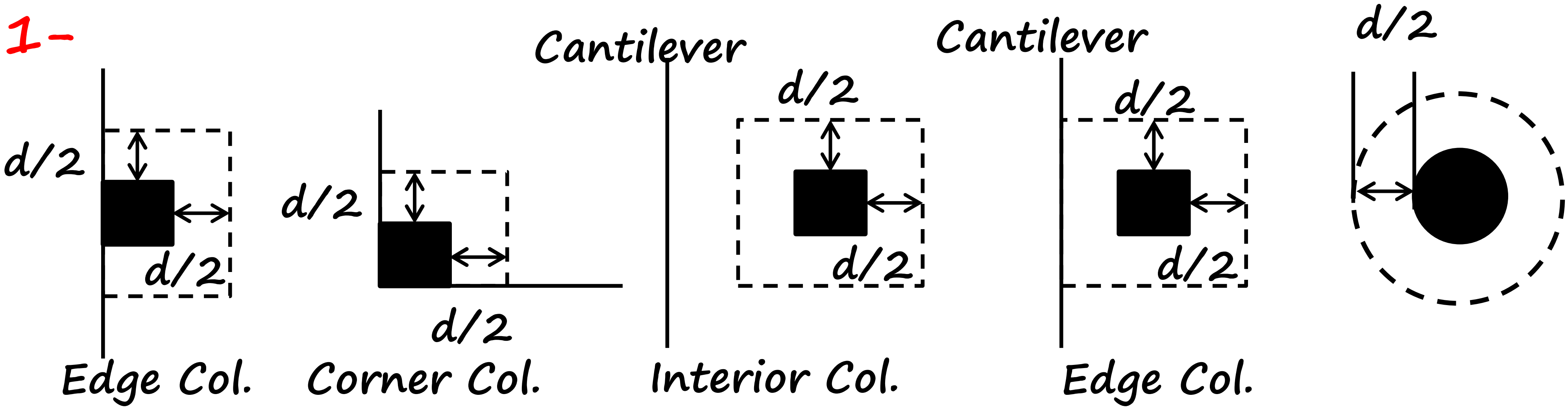
$$A_v = \frac{\phi Vs}{\phi f_y \sin\alpha} \qquad no. = \frac{Av}{2 \times \frac{\pi}{4} D^2}$$

Where α is angle of bent bar

If $Vu > \frac{\phi}{2}\sqrt{f'c} \times b_o d$ increase slab thickness

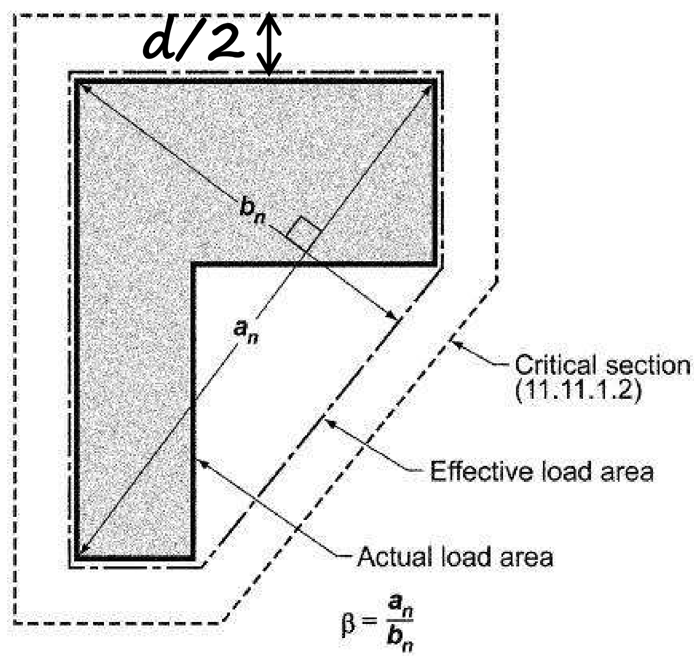
Notes:

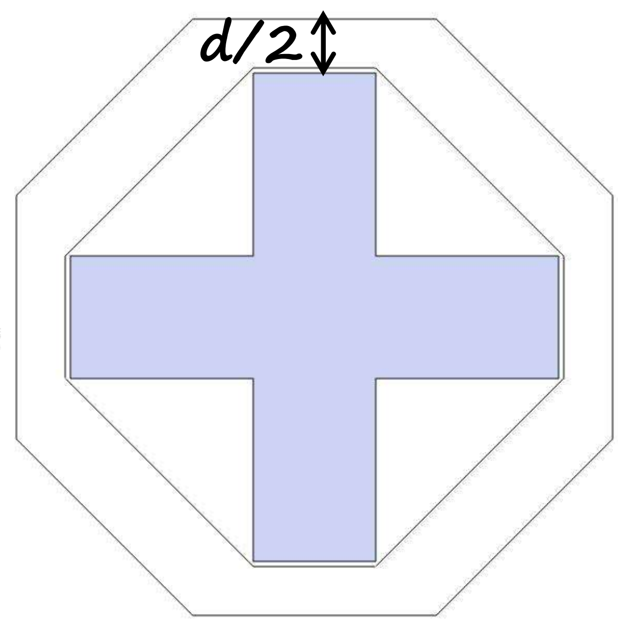
1-



The effective loaded area is that area totally enclosing the actual loaded area for which the perimeter is a minimum.

$$\beta = a/b$$





2-

At corner column:

$V_u = W_u \times A$

$A = ab - x_1 x_2$

$b_o = x_1 + x_2$

Perimeter of critical section= $x_1 + x_2$

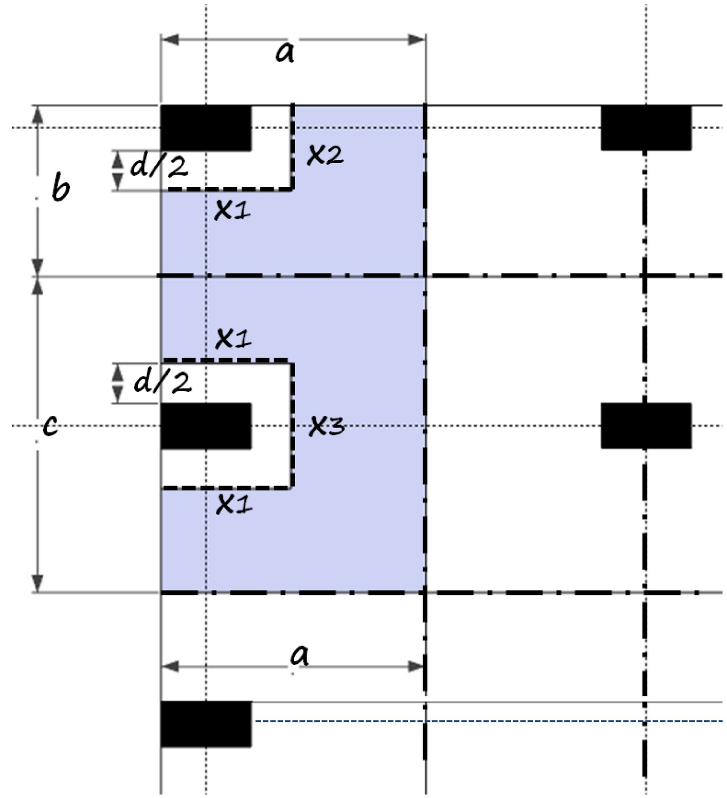
At edge column:

$V_u = W_u \times A$

$A = ac - x_1 x_3$

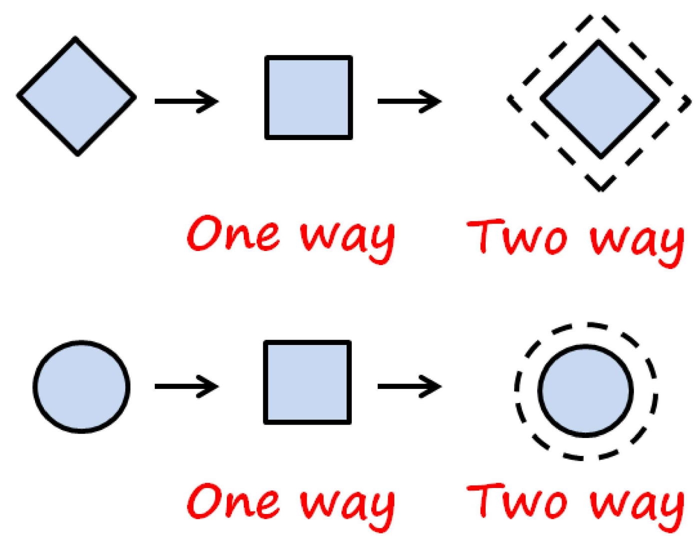
$b_o = 2x_1 + x_3$

Perimeter of critical section= $2x_1 + x_3$



3-

Critical sections for arbitrary shapes



4-

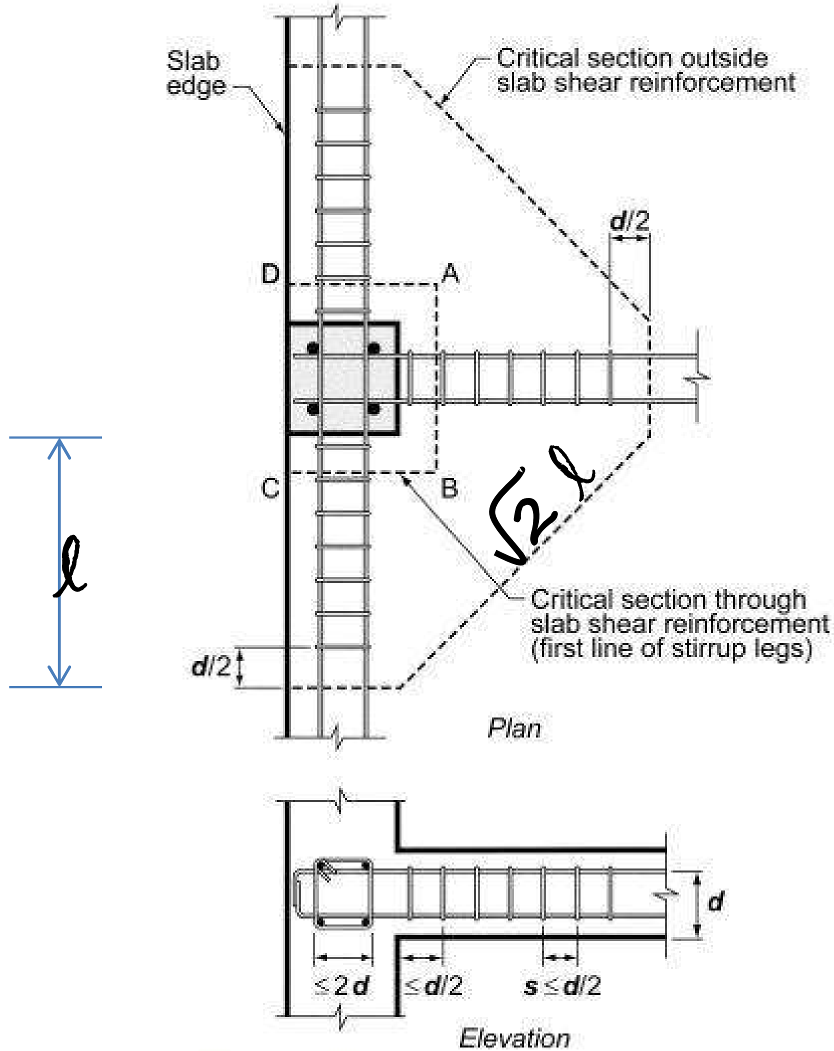


Fig. R8.7.6e—Arrangement of stirrup shear reinforcement, edge column.

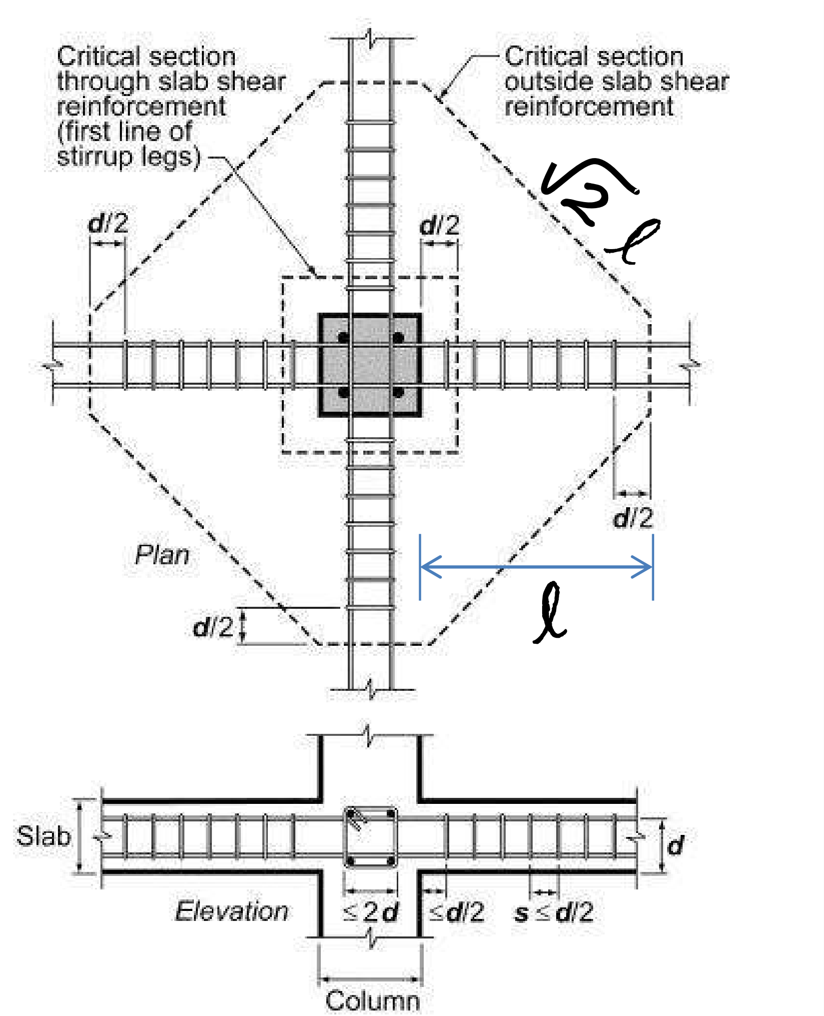


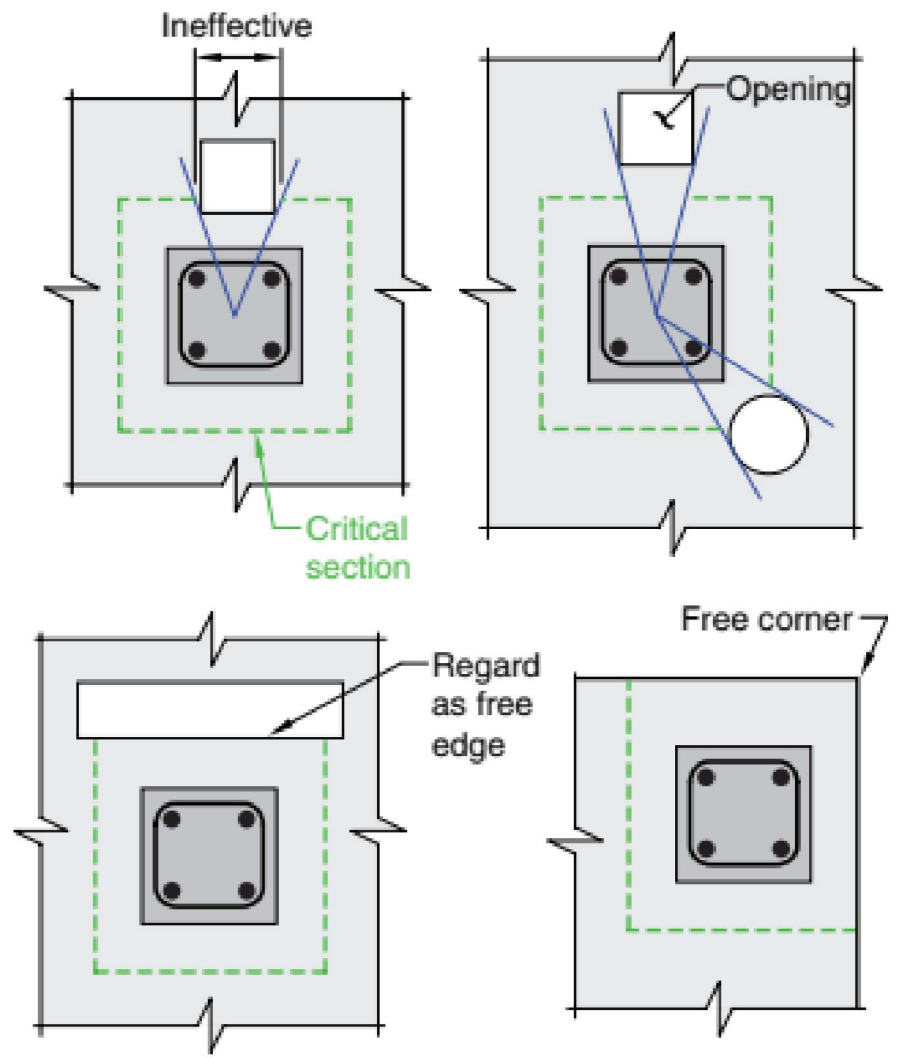
Fig.R8.7.6d—Arrangement of stirrup shear reinforcement, interior column.

To find the value of ℓ in the critical section:

$$b_o = 4(\sqrt{2}\times l + Col.) \rightarrow Vu = \frac{\phi}{6}\sqrt{f'c}\times b_o d\times 10^{-3}, no._{stirrups} = \frac{l - d/2}{S}$$

Effect of Openings on Shear Strength (ACI 22.6.4.3):

If an opening is located closer than 4h from the periphery of a column, concentrated load, or reaction area, the portion of bo enclosed by straight lines projecting from the centroid of the column, concentrated load or reaction area and tangent to the boundaries of the opening shall be considered ineffective. the critical section for shear have a reduced perimeter (bo-x)



Note: Openings shown are located within 4*h* of the column periphery.

Fig. R22.6.4.3—Effect of openings and free edges (effective perimeter shown with dashed lines).

Example (1): For the flat plate system shown, $W_u=14\ kN/m^2$, $f_y=400\ MPa$, $f'_c=20\ MPa$, $t=240\ mm$, $d=t-40$, Col.300x300 mm.
Check the shear strength of the slab at the column support and if not adequate increase shear strength by:

1- Using drop panel.

2- Using shear reinforcement (stirrup Ø 10 mm).

Sol.

A- One way shear action:

Critical section at (d) from face of column

d=240-40=200mm=0.2m

X=(6.6/2)-(0.3/2)-0.2= 2.95 m

Vu=Wu . A=14 x 6.6 x 2.95=273 kN

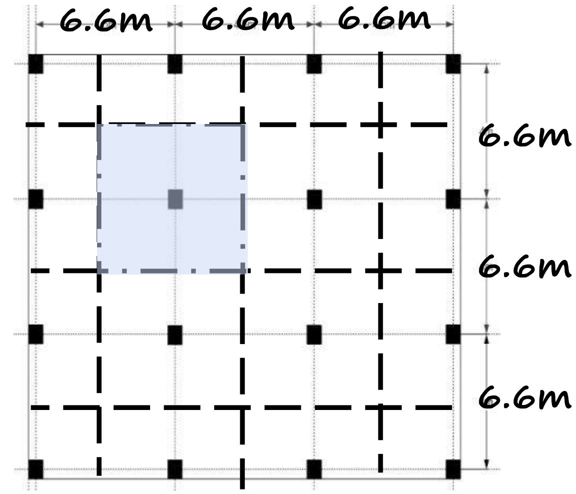
$$\phi Vc=\phi(0.17\sqrt{f'c}\times b_o d)=0.75\times 0.17\sqrt{20}\times 6600\times 200\times 10^{-3}$$

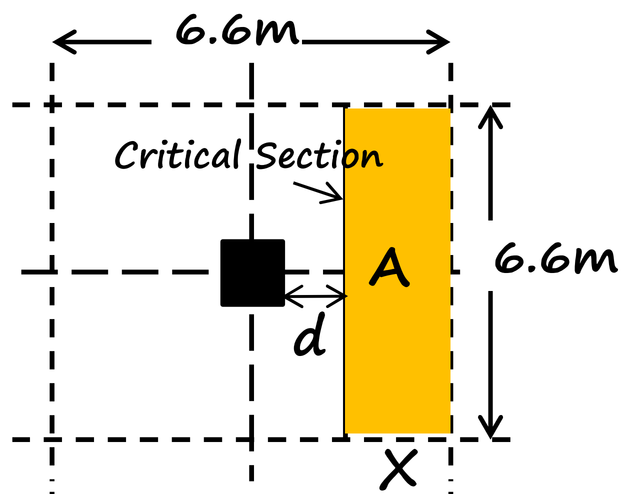
$$\phi Vc=735kN>Vu=273kN$$ One way shear (o.k)

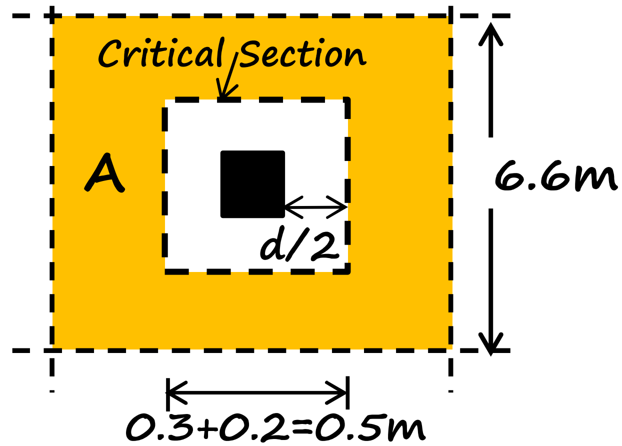
B- Two way shear (punching): d/2= 200/2=100mm

$$Vu=Wu\times A=14\times(6.6^2-0.5^2)=606kN$$

$$\beta=C_1/C_2=300/300=1 \quad , \quad \alpha_s=40 \quad , \quad b_o=4\times 0.5=2m$$







The nominal shear strength of slab taken as the smallest of the following equations:

$$\lambda_s = \sqrt{\frac{2}{1+0.004d}} = \sqrt{\frac{2}{1+0.004\times 200}} = 1.05 > 1 \qquad \therefore \lambda_s = 1$$

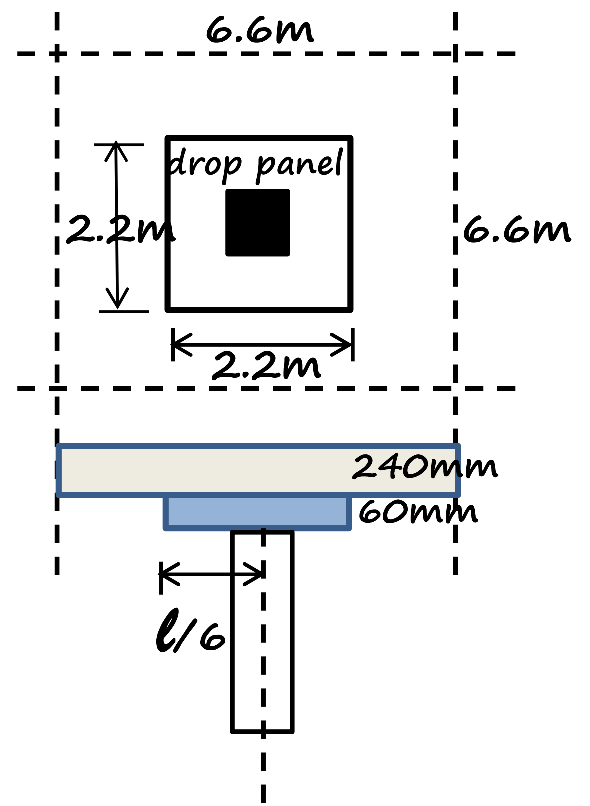
$$Vc = 0.33\lambda_s\lambda\sqrt{f'c}\times b_o d = 0.33\sqrt{f'c}\times b_o d$$ **Smallest** Eq. 11-31

$$Vc = (0.17+\frac{0.33}{\beta})\lambda_s\lambda\sqrt{f'c}\times b_o d = 0.51\sqrt{f'c}\times b_o d$$ Eq. 11-32

$$Vc = (0.17+\frac{0.083\alpha_s d}{b_o})\lambda_s\lambda\sqrt{f'c}\times b_o d = 0.498\sqrt{f'c}\times b_o d$$ Eq. 11-33

$$\phi Vc = 0.75\times 0.33\sqrt{20}\times 2000\times 200\times 10^{-3} = 443kN < Vu = 606kN$$

1- Using drop panel



According to ACI 8.2.4, the min. required dimensions of drop panel are:

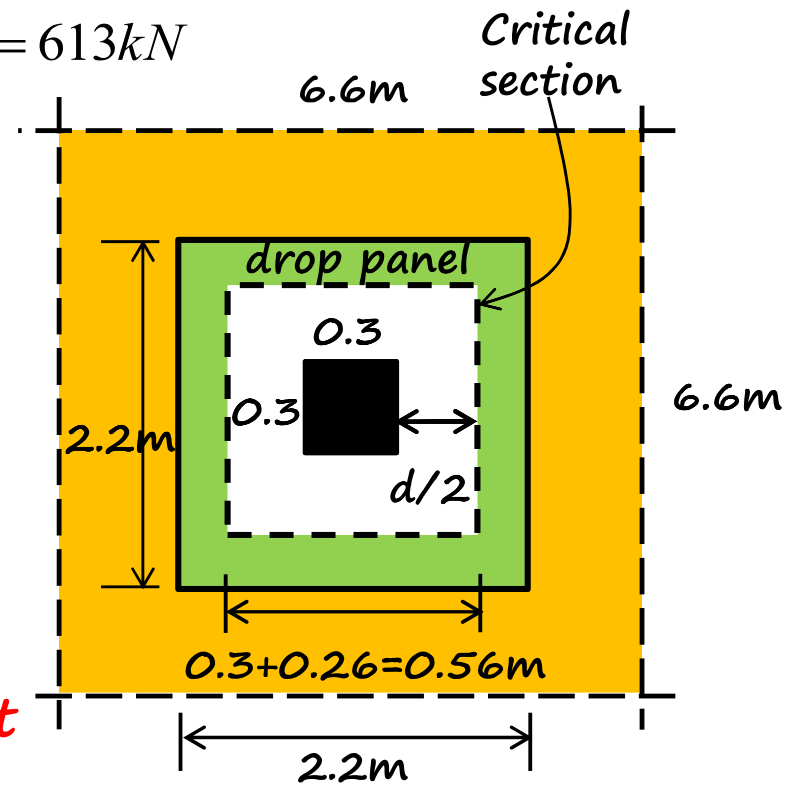
$$2\times\frac{l}{6} = 2\times\frac{6.6}{6} = 2.2m \qquad t_{drop} = \frac{t_{slab}}{4} = \frac{240}{4} = 60mm$$

a) check the shear at d/2 from face of column

$d_{drop}=d_2=$ t-40=300-40=260mm

$$Vu = 14\times(6.6^2-0.56^2)+1.2\times(2.2^2-0.56^2)\times0.06\times24 = 613kN$$

$$b_o = 4\times0.56 = 2.24m$$



To check drop panel thickness for max. strength:

$$Vc_{\max} = \frac{\phi}{2}\sqrt{f'c}\times b_o d = \frac{0.75}{2}\sqrt{20}\times2240\times260\times10^{-3}$$

$$Vc_{\max} = 977kN > 613kN$$ If not ; increase D.P dimensions

$$\beta = \frac{0.3}{0.3} = 1 \qquad \lambda_s = \sqrt{\frac{2}{1+0.004\times260}} = 0.990$$

$$Vc = 0.33\lambda_s\lambda\sqrt{f'c}\times b_o d = 0.33\lambda_s\sqrt{f'c}\times b_o d$$ **Smallest**

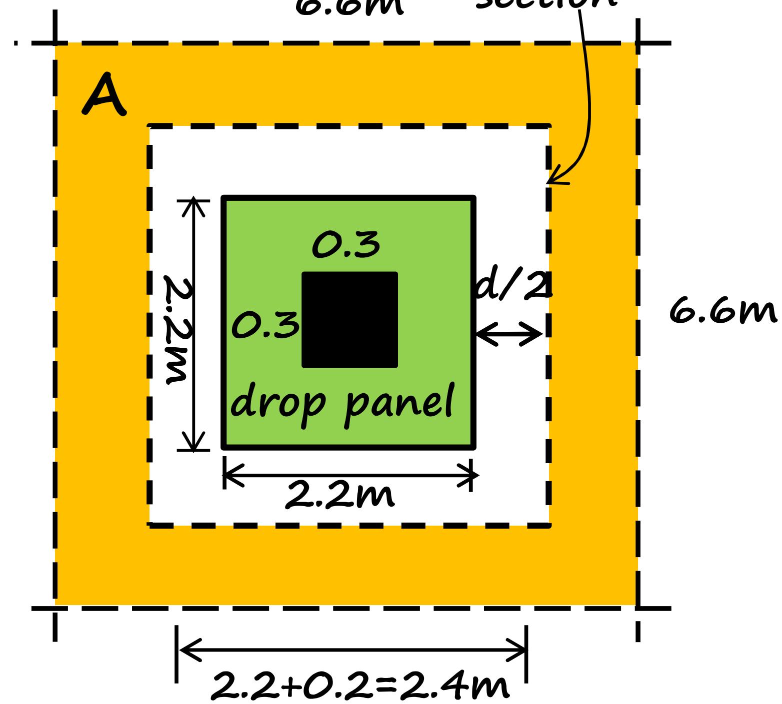
$$Vc = (0.17+\frac{0.33}{1})\lambda_s\lambda\sqrt{f'c}\times b_o d = 0.51\lambda_s\sqrt{f'c}\times b_o d$$

$$Vc = (0.17+\frac{0.083\times40\times260}{2240})\lambda_s\lambda\sqrt{f'c}\times b_o d = 0.51\lambda_s\sqrt{f'c}\times b_o d$$

$$\phi Vc = 0.75\times0.33\times0.99\times\sqrt{20}\times2240\times260\times10^{-3} = 638.55kN > Vu = 613kN$$ o.K

Drop panel thickness is o.k

b) check the shear at d/2 from face of drop panel:



$d_{slab}=d_1=$ 200 mm

$$Vu = 14\times(6.6^2 - 2.4^2) = 529.2kN$$

$$b_o = 4\times 2.4 = 9.6m$$

$$Vc_{\max} = \frac{\phi}{2}\sqrt{f'c}\times b_o d = \frac{0.75}{2}\sqrt{20}\times 9600\times 200\times 10^{-3}$$

$$Vc_{\max} = 3213kN > 529.2kN$$

$$\beta = \frac{2.2}{2.2} = 1 \qquad \lambda_s = \sqrt{\frac{2}{1+0.004\times 200}} = 1.05 > 1 \qquad \therefore \lambda_s = 1$$

$$Vc = 0.33\lambda_s\lambda\sqrt{f'c}\times b_o d = 0.33\lambda_s\sqrt{f'c}\times b_o d$$

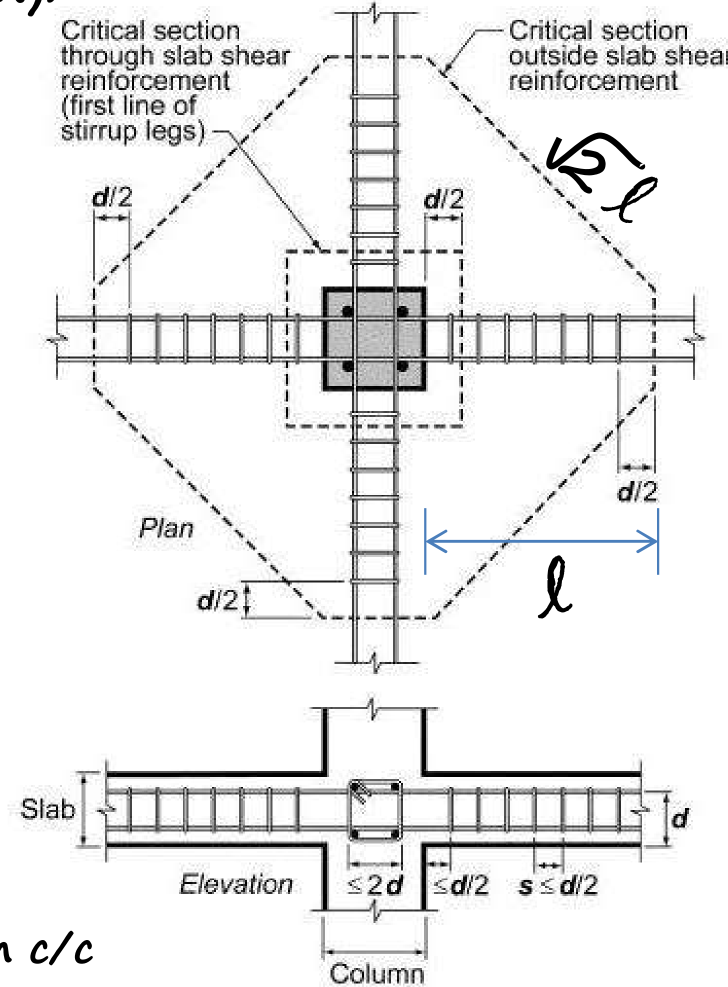
$$Vc = (0.17+\frac{0.33}{1})\lambda_s\lambda\sqrt{f'c}\times b_o d = 0.51\lambda_s\sqrt{f'c}\times b_o d$$

$$Vc = (0.17+\frac{0.083\times 40\times 200}{9600})\lambda_s\lambda\sqrt{f'c}\times b_o d = 0.235\lambda_s\sqrt{f'c}\times b_o d$$ Smallest

$$\phi Vc = 0.75\times 0.235\sqrt{20}\times 9600\times 200\times 10^{-3} = 1514.4kN > Vu = 529.2kN$$

Drop panel dimensions are o.k

2- Using shear reinforcement (stirrup Ø 10 mm).
According to ACI 8.7.6 :
Stirrups permitted to use in shear reinforcement in slab with



$$d > 16d_b \qquad 16d_b = 16\times 10 = 160mm$$

and $d \geq 150mm \qquad d = 200mm$ o.k

$$Vu = 606kN$$

$$\phi Vs = Vu - \phi 0.17\sqrt{f'c}\times b_o d$$

$$\phi Vs = 606 - 0.75\times 0.17\sqrt{20}\times 2000\times 200\times 10^{-3}$$

$$\phi Vs = 382kN$$

$$S = \frac{\phi A_v f_y d}{\phi V_s} = \frac{0.75\times 8\times 78.5\times 400\times 200}{382} = 99mm$$

$$S < S_{\max} = \frac{200}{2} = 100mm$$ Use Ø 10mm stirrup @90mm c/c

$$b_o = 4(\sqrt{2}\times l + Col.)$$

$$Vu = \frac{\phi}{6}\sqrt{f'c}\times b_o d\times 10^{-3} \rightarrow 606 = 0.75\times\frac{1}{6}\sqrt{20}\times 4(\sqrt{2}l + 300)\times 200\times 10^{-3}$$

$$l = 746mm \rightarrow use: l = 750mm \quad no._{stirrups} = \frac{l - d/2}{S} = \frac{750-100}{90} = 7.2$$

Use 8-Ø 10 mm stirrup along each col. line

Example (2):

Based on two way shear action, find the minimum dimensions of interior column by using maximum shear reinforcement. Wu=15.2 kN/m², fy=400 MPa, f'c=20 MPa, t=200 mm.

Sol:

d=200-20-10=170mm

$$Vu = 15.2 \times \left[7 \times 5 - (C + 0.17)^2\right]$$

Maximum reinforcement used for maximum shear strength:

$$b_o = 4 \times (C + 0.17)$$

$$Vc_{max} = \frac{\phi}{2}\sqrt{f'c} \times b_o d = \frac{0.75}{2}\sqrt{20} \times 4 \times (C + 0.17) \times 170 \times 10^{-3}$$

$$Vc_{max} = 1140.4 \times (C + 0.17)$$

$$Vu = Vc_{max}$$

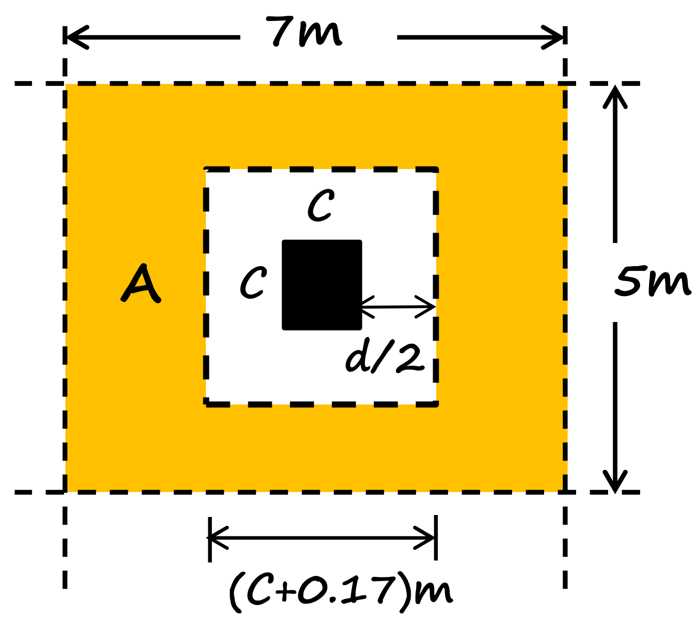
$$15.2 \times (35 - C^2 - 0.34C - 0.0289) = 1140.4 \times (C + 0.17)$$

$$532 - 0.44 - 15.2C^2 - 5.168C = 1140.4C + 193.86$$

$$15.2C^2 + 1145.56C - 337.7 = 0$$

$$C^2 + 75.36C - 22.21 = 0$$

$$\therefore C = \frac{-75.36 \pm \sqrt{(75.36)^2 - 4 \times (-22.21)}}{2} = 0.29m$$

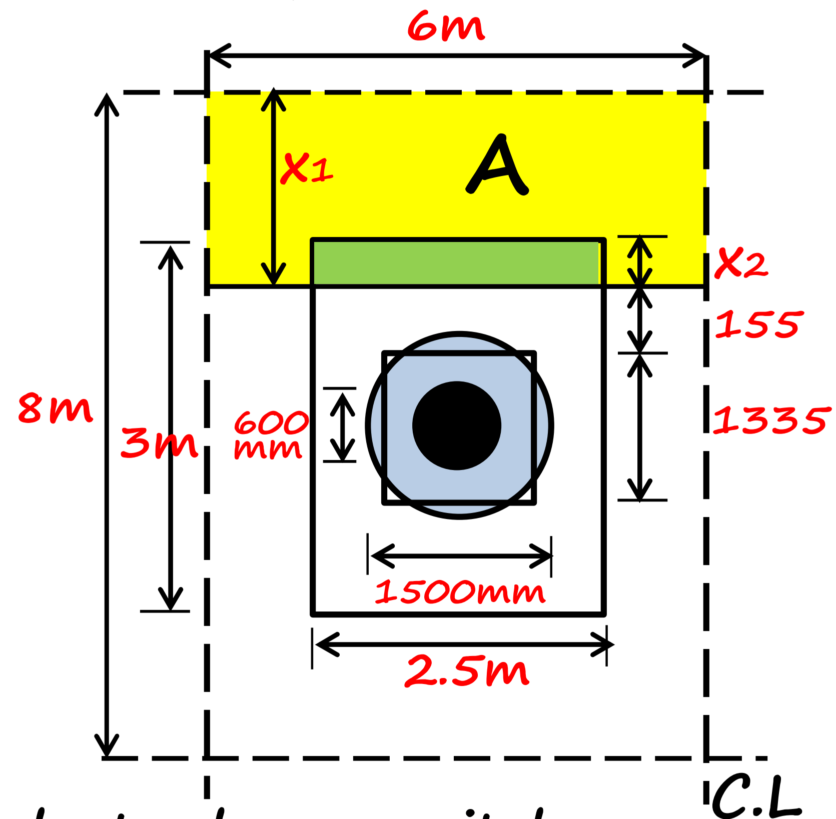


Use Col.= 325 x 325 mm

Example (3):

For the interior panel of flat slab which thickness= 185mm, drop panel thickness= 50mm, D.L= slab weight + (2 kN/m² flooring), L.L=4.5 kN/m², f'c=25 MPa, Circular column= 600 mm dia., Column Capital=1500mm dia., d=(t-30)mm.

Check the one and two way shear strength of the slab.



Sol.

Self weight= 0.185 x 24= 4.44 kN/m²

D.L= 4.44 + 2= 6.44 kN/m²

Wu= 1.2 x 6.44 +1.6 x 4.5= 15 kN/m²

A- One way shear action:

a) Critical section at (d) from face of equivalent column capital:

0.89 x 1500 =1335 mm

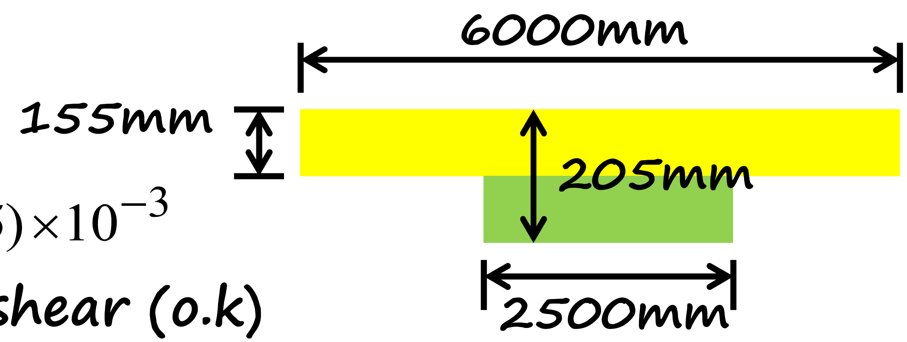
d_{slab}=185-30=155 mm effective depth of slab

d_{drop}=185+50-30=205 mm effective depth of drop panel

X_1=(8/2)-(1.335/2)-0.155= 3.1775 m

X_2=(3/2)-(1.335/2)-0.155= 0.6775 m

Vu=Wu . A=15 x 6 x 3.1775 + 1.2 (2.5 x 0.6775 x 0.05) x 24
Vu= 288.41 kN

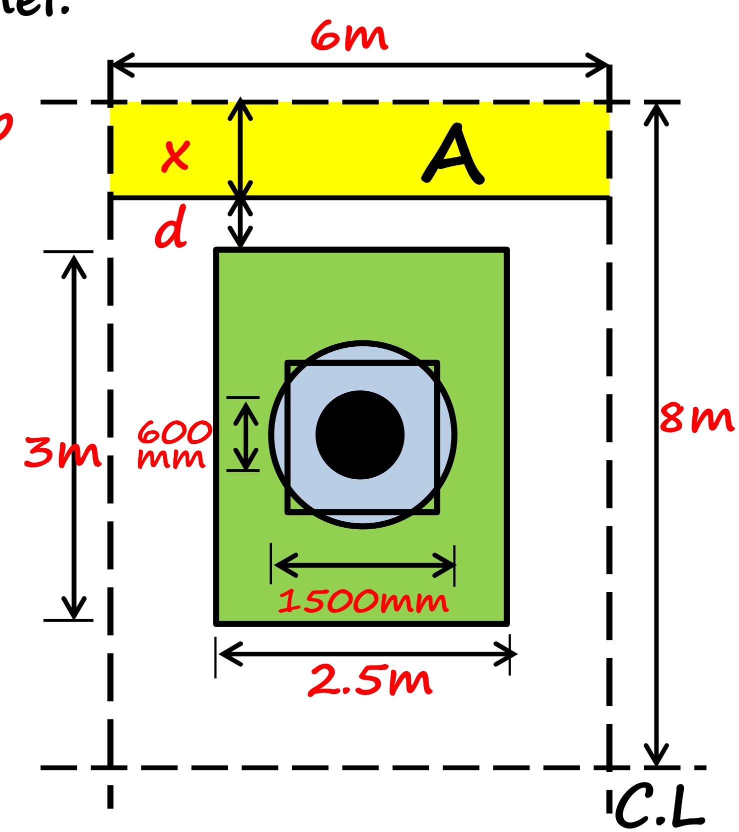


$$\phi Vc = \phi(0.17\sqrt{f'c} \times b_o d)$$

$$\phi Vc = 0.75 \times 0.17\sqrt{25} \times (3500 \times 155 + 2500 \times 205) \times 10^{-3}$$

$$\phi Vc = 672.5kN > Vu = 288.41kN$$ One way shear (o.k)

b) Critical section at (d) from face of drop panel:



dslab=185-30=155 mm effective depth of slab

X=(8/2)-(3/2)-0.155= 2.345 m

Vu=Wu . A=15 x 6 x 2.345= 211 kN

$$\phi Vc = \phi(0.17\sqrt{f'c} \times b_o d)$$

$$\phi Vc = 0.75 \times 0.17\sqrt{25} \times 6000 \times 155 \times 10^{-3}$$

$$\phi Vc = 593kN > Vu = 211kN$$

One way shear (o.k)

B- two way shear (punching) H.W

B- Two way shear (punching):

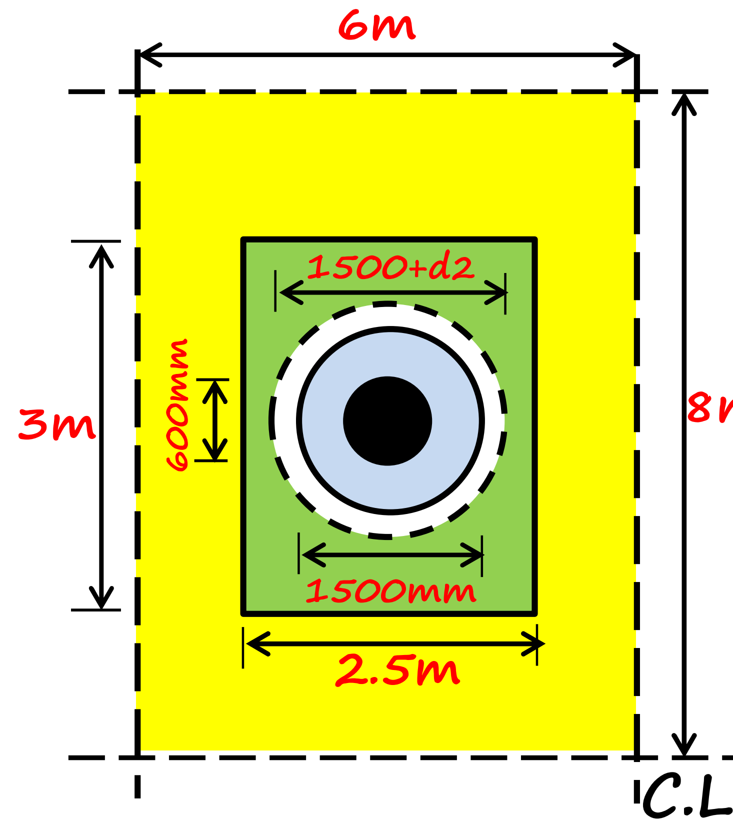
a) Critical section at d/2 from support (column capital):

Diameter of critical section= 1500 + d2 = 1500+205=1705mm

$$Vu = Wu \times A = 15 \times \left[(6 \times 8) - \frac{\pi}{4}(1.705)^2 \right] + 1.2 \times \left[2.5 \times 3 - \frac{\pi}{4}(1.705)^2 \right] \times 0.05 \times 24 = 693.26kN$$

$b_o = 2\pi R = 1705\pi = 5358.6mm$, $\alpha_s = 40$, $\beta = 1$ Circular section

Complete the Solution



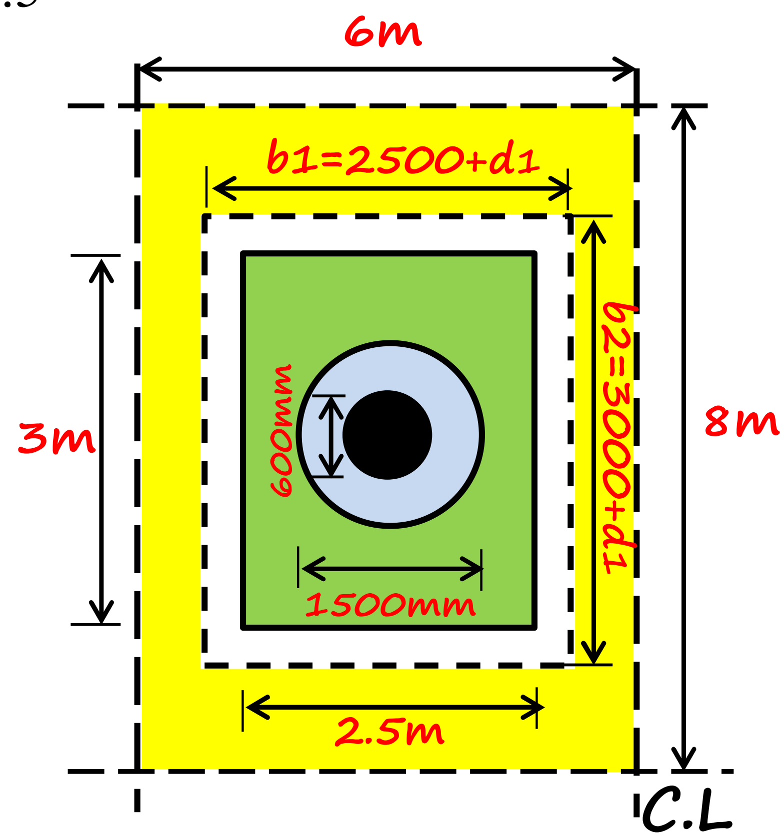
b) Critical section at d/2 from drop panel:

d1= 155mm

b1=2500+d1=2655mm , b2=3000+d1=3155mm

$$Vu = Wu \times A = 15 \times (6 \times 8 - 2.655 \times 3.155) = 594.35kN$$

$$b_o = 2(2655 + 3155) = 11620mm, \quad \alpha_s = 40 \quad , \beta = \frac{3}{2.5} = 1.2$$



Complete the Solution