Flow net under Hydraulic Structures

Laplace Equation

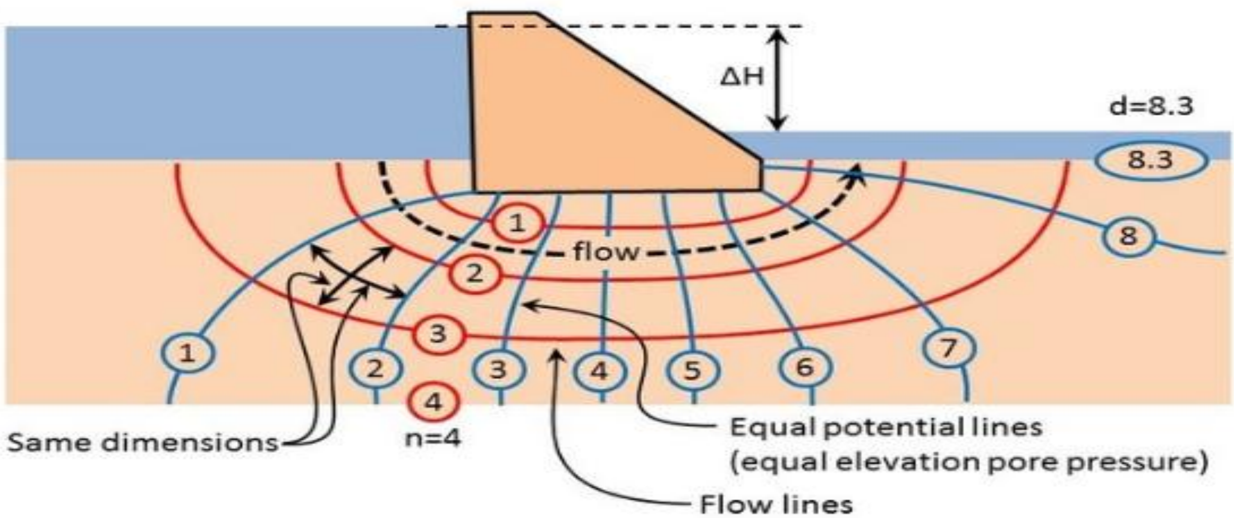
$$K_x \frac{\partial^2 h}{\partial x^2} + K_y \frac{\partial^2 h}{\partial y^2} = 0$$

Assume homogenous and Isotropic; $K_x = K_y$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

Which is Laplace equation of two dimensions steady state condition.

1- *Graphical Solution_ Flow net*



$h_{tu} = h_u + h_e$= total head at upstream head

$h_{td} = h_u + h_e$ = total head at downstream head

h_u is the water pressure and h_e is the elevation head (Z)

n_f = number of flow channel

n_d = total number of drop head

$(\Delta H \text{ or } H) = h_{tu} - h_{td}$ = total difference head between the upstream and downstream head

Δh = Losses head between any two equipotential lines, $\Delta h = \frac{\Delta H}{n_d} = \frac{H}{n_d} = \frac{h_{tu} - h_{td}}{n_d}$

$$h_{ti} = h_{tu} - n_{di} \times \Delta h = h_{tu} - n_{di} \times \frac{H}{n_d} = h_{tu} - n_{di} \left(\frac{h_{tu} - h_{td}}{n_d} \right)$$

$$\Delta q = K \cdot i \cdot A = K \cdot \frac{\Delta h}{\Delta x} \cdot \Delta y \times 1$$

$But\ since\ \ \Delta x = \Delta y\ for\ squere\ mesh,\ then$

$$\Delta q = K \cdot \Delta h\ =\ K \cdot \frac{H}{n_d}$$

Note that $total\ head = h_t = h_u + h_e$ or $h_u = h_t - h_e$

Where h_u is the water pressure and h_e is the elevation head (Z)

$$q = \Delta q \cdot n_f = K \cdot \frac{H}{n_d} \cdot n_f = K \cdot H \cdot \frac{n_f}{n_d}$$

$$i_{exist} = \frac{\Delta h}{l}\ ,\ i_{cr} = \frac{\gamma_{sub}}{\gamma_w} = \frac{\gamma_{sat} - \gamma_w}{\gamma_w} = \frac{G_s - 1}{1 + e}$$

$$i_{exist}\ \ must\ be\ < \ i_{cr}$$

$$F_s = \frac{i_{cr}}{i_{exist}}$$

Seepage force $j = i\,\gamma_w = seepage\ force\ per\ unit\ volume$

Effective stress at point is $\bar{\sigma} = \sigma - u = \sigma - (h_u \times \gamma_w)$

Where

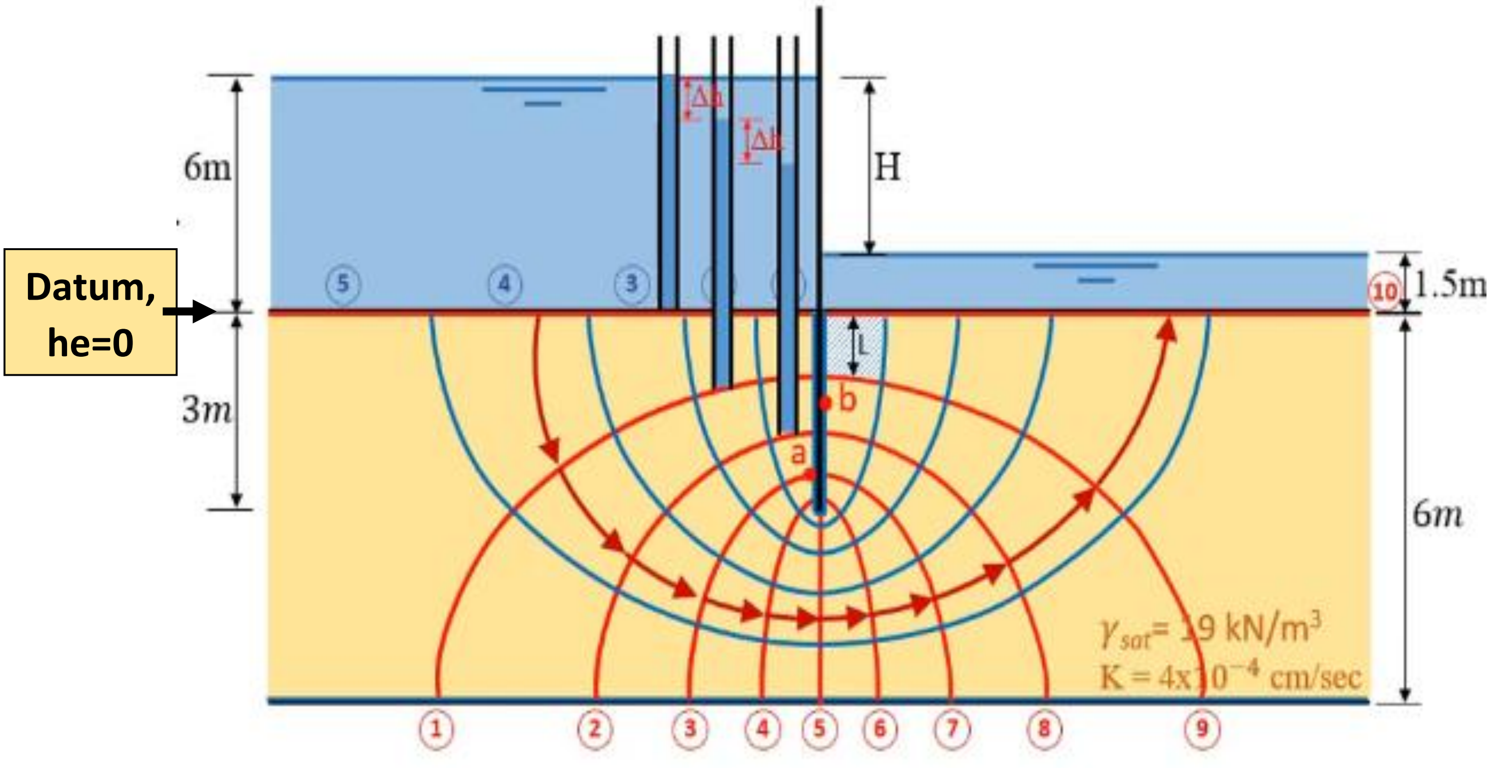
σ = **total stress** = $\gamma_{sat} \times h_s + \gamma_w \times h_w$

u = pore water pressure = ($h_u \times \gamma_w$)

Example 1

For the following flow net :

1) ***Estimate the qountity of seepage flow.***
2) ***Calculate the total flow head and water pressure at points a & b.***
3) ***Check of boiling.***



Solusion: ***Assume the datum at the bottom floor***

$h_{tu} = h_u + h_e = 6 + 0 = 6\ m$
$h_{td} = h_u + h_e = 1.5 + 0 = 1.5\ m$
$H = h_{tu} - h_{td} = 6 - 1.5 = 4.5\ m$

$$\Delta h = \frac{H}{n_d} = \frac{h_{tu} - h_{td}}{n_d} = \frac{4.5}{10} = 0.45\ m$$

$1 -$ *For Total seepage*

$n_f = 5 \quad , \quad n_d = 10, \quad H = 4.5\ m$

$$q = K \cdot H \cdot \frac{n_f}{n_d} = (4 \times 10^{-4}) \frac{60 \times 60 \times 24}{100} \times 4.5 \times \frac{5}{10} = 0.7776\ \frac{m^3}{m\,.day}$$

$2 - $ *For water pressure at point a*

$$\Delta h = \frac{H}{n_d} = \frac{h_{tu} - h_{td}}{n_d} = \frac{4.5}{10} = 0.45\ m$$

$$h_{ti} = h_{tu} - n_{di} \times \Delta h \quad or \quad h_{at} = 6 - 3 \times 0.45 = 4.65\ m$$

$$h_{au} = h_{at} - h_{ae}$$

$$h_{au} = 4.65 - (-3) = 7.65\ m$$

$$P_w = \gamma_w \times h_{au}$$

$$P_w = 9.81 \times 7.65 = 75.0465\ Kpa$$

For water pressure at point b

$$h_{ti} = h_{tu} - n_{di} \times \Delta h \quad or \quad h_{bt} = 6 - 8.5 \times 0.45 = 2.175\ m$$

$$h_{bu} = h_{bt} - h_{be}$$

$$h_{bu} = 2.175 - (-1.5) = 3.675\ m$$

$$P_w = \gamma_w \times h_{bu}$$

$$P_w = 9.81 \times 3.675 = 36.05175\ Kpa$$

$3 - $ *For Check of boiling*

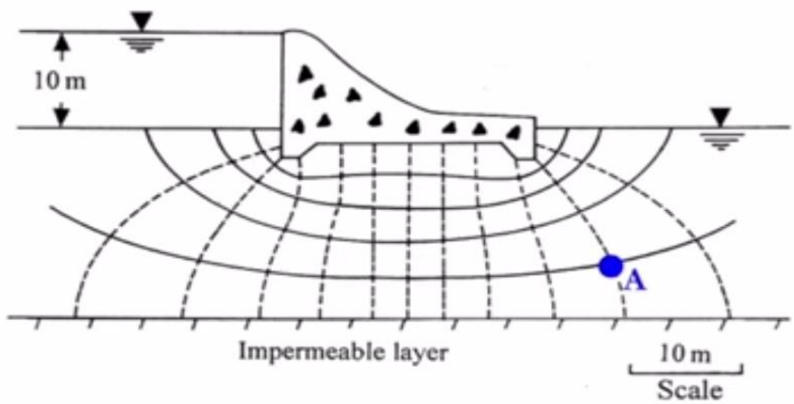
$$i_{exist} = \frac{\Delta h}{l} = \frac{0.45}{0.9} = 0.5\ ,$$

$$i_{cr} = \frac{\gamma_{sub}}{\gamma_w} = \frac{\gamma_{sat} - \gamma_w}{\gamma_w} = \frac{19 - 9.81}{9.81} = 0.9368 > i_{exist}\ (Ok\ Safe)$$

$$F_s = \frac{0.9368}{0.5} = 1.8736 \approx 1.87$$

Example 2

For the given flow net, the hydraulic conductivity of soil k=0.002 cm/sec.
Calculate
(1) The total seepage loss per unit length in $m^3/sec/m$.
(2) The pore pressure at point A



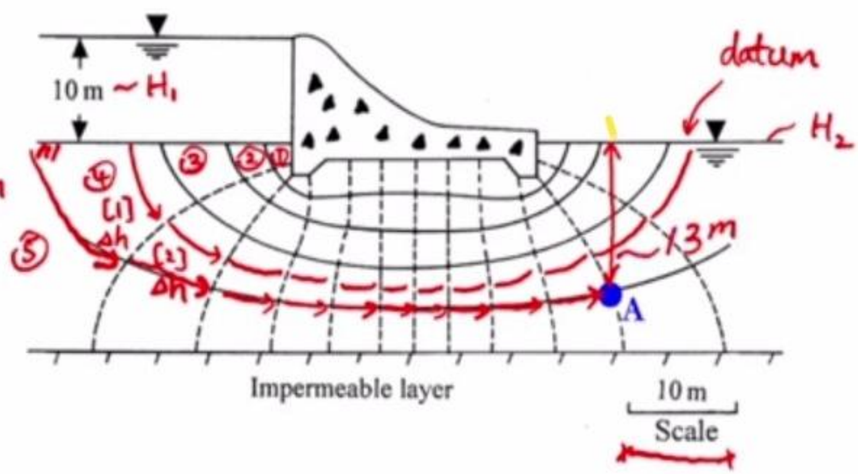
For the given flow net, the hydraulic conductivity of soil k=0.002 cm/sec.
Calculate
(1) The total seepage loss per unit length in $m^3/sec/m$.
(2) The pore pressure at point A

$H = H_1 - H_2 = 10\ m$

$N_f = 5$

$N_d = 12$

✓ $\Delta h = \frac{H}{N_d} = \frac{10}{12} = 0.833\ m$ ✓



$1 - For\ Total\ seepage$

$$n_f = 5 \quad , \quad n_d = 12, \quad H = h_{tu} - h_{td} = 10 - 0 = 10\ m$$

$$q = K \cdot H \cdot \frac{n_f}{n_d} = 0.002 \times \frac{60 \times 60 \times 24}{100} \times 10 \times \frac{5}{12} = 7.2\ \frac{m^3}{m\,.\,day}$$

$2 - For\ water\ pressure\ at\ point\ A$

$$\Delta h = \frac{H}{n_d} = \frac{h_{tu} - h_{td}}{n_d} = \frac{10}{12} = 0.833\ m$$

$$h_{ti} = h_{tu} - n_{di} \times \Delta h \quad or \quad h_{At} = 10 - 10\frac{10}{12} = 1.667\ m$$

$$h_u = h_t - h_e$$

$$h_{Au} = 1.667 - (-13) = 14.667\ m$$

$$P_w = \gamma_w \times h_{Au}$$

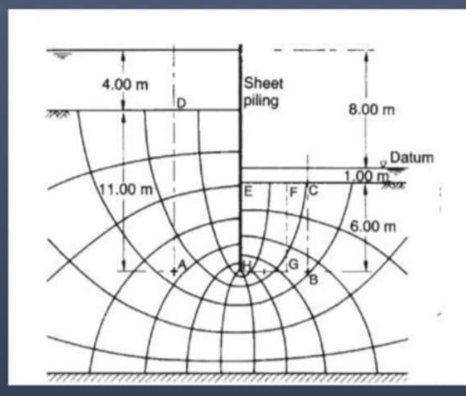
$$P_w = 9.81 \times 14.667 = 143.883\ Kpa$$

Example 3

The flow net for seepage under a sheet pile wall is shown in Figure below, the saturated unit weight of the soil being 20 kN/m³. Calculate

a) The quantity of seepage if the coefficient of permeability of the soil is 2×10^{-5} m/s

b) Determine the values of effective vertical stress at A and B.



MCE

MCE

Solution:

Saturated unit weight of soil = γ = 20 kN/m³

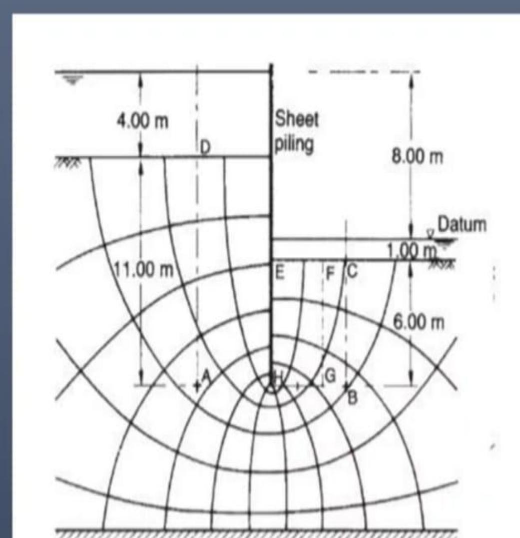
Number of flow channels = $N_f = 5.5$

Number of equipotential drops = $N_d = 12$

Total Head loss = H = 8 m

Head loss per equipotential drop = $h' = H/N_d = 8/12$

Permeability of soil = $k = 2 \times 10^{-5}$ m/s



a) Quantity of seepage = $kH(N_f/N_d)$

$= (2\times10^{-5})(8)(5.5/12)$

$= \mathbf{7.33 \times 10^{-5}\ m^3/s/m}$

MCE

MCE

b) Total head at A = Total head upstream – head loss upto A

$H_a = H - h_{la} = 8 - 3.9(8/12)$

$= 5.4$ m

Elevation head of A = $h_{ea} = -7$ m

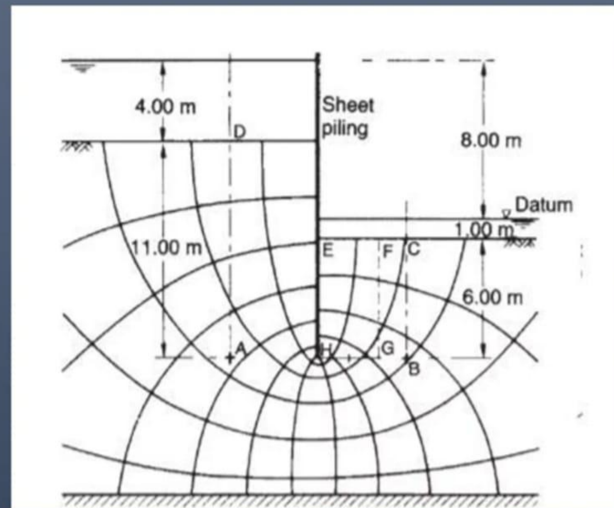
Pressure head at A = $h_{pa} = H_a - h_{ea}$

$= 5.4 - (-7) = 12.4$ m

Pore water pressure at A = $u = h_{pa}\gamma_w$

$= 12.4 \times 9.81$

$= 121.64$ kPa



MCE

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Total head at B = Total head upstream – Head loss upto B

$H_b = H - h_{lb} = 8 - 9.6(8/12)$

$= 1.6$ m

Elevation head of B = $h_{eb} = -7$ m

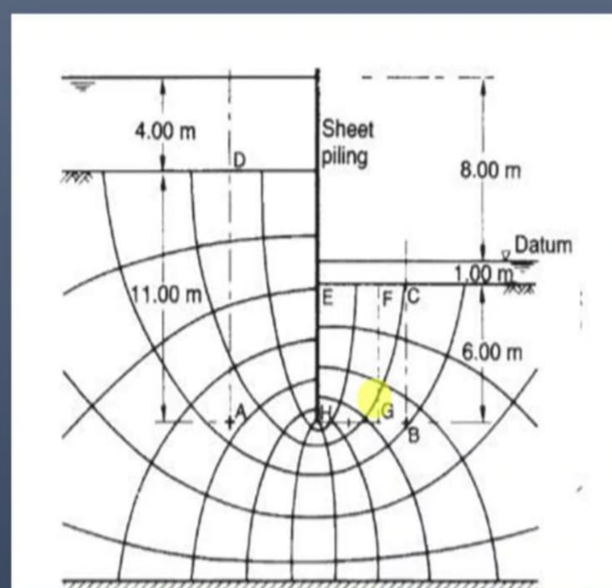
Pressure head at B = $h_{pb} = H_b - h_{eb}$

$= 1.6 - (-7) = 8.6$ m

Pore water pressure at B = $u = h_{pb}\gamma_w$

$= 8.6 \times 9.81$

$= 84.37$ kPa



MCE

MCE

Total stress at point B is given as

$\sigma = \gamma_{sat} x6 + \gamma_w x1$

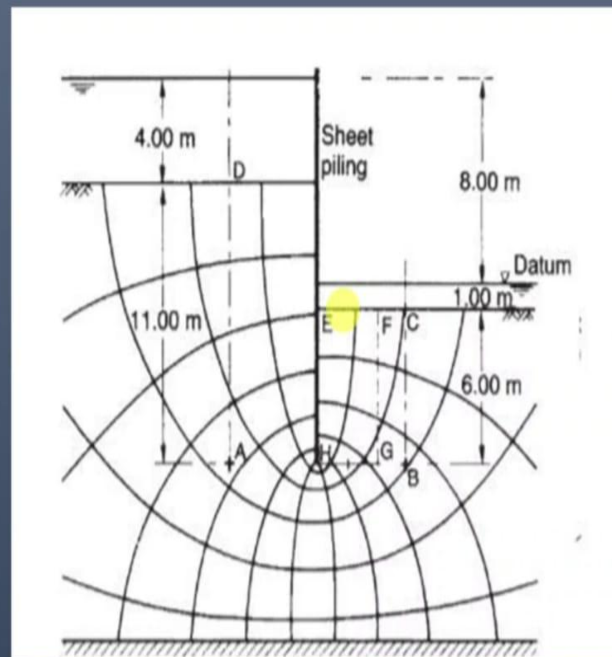
$= 20x6 + 9.81x1$

$= 129.81$ kPa

Effective stress at point B is given as

$\sigma' = \sigma - u = 129.81 - 84.37$

= 45.44 kPa



2- Numerical Solution

The following equation is considered as Laplace equation of two dimensions steady state condition un isotropic case

$$K_x \frac{\partial^2 h^2}{\partial x^2} + K_y \frac{\partial^2 h^2}{\partial y^2} = 0$$

Assume homogenous and Isotropic; $K_x = K_y$

$$\frac{\partial^2 h^2}{\partial x^2} + \frac{\partial^2 h^2}{\partial y^2} = 0$$

Which is Laplace equation of two dimensions steady state condition.

Partial Difference operation in Cartesian coordinates

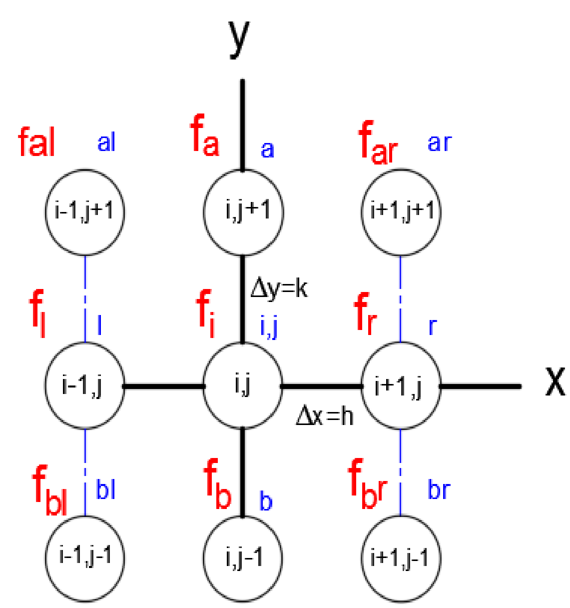
$$let \frac{\partial}{\partial x} = Dx \ : \frac{\partial}{\partial y} = Dy : \frac{\partial}{\partial y \partial x} = DyDx = Dyx$$

$$\frac{\partial^2}{\partial x^2} = D^2x \ : \frac{\partial^2}{\partial y^2} = D^2y \ : \frac{\partial}{\partial x \partial y} = DxDy = Dxy$$

$$\frac{\partial f}{\partial x_i} = D_x f_i = \frac{f_{i+1,j} - f_{i-1,j}}{2\Delta x = 2h} = \frac{f_r - f_l}{2\Delta x = 2h}$$

$$\frac{\partial f}{\partial y_i} = D_y f_i = \frac{f_{i,j+1} - f_{i,j-1}}{2\Delta y = 2k} = \frac{f_a - f_b}{2\Delta y = 2k}$$

$$let \ \Delta x = \Delta y \ \ or \ \ h = k$$



$\therefore$ *By the same way*

$$\frac{\partial^2 f}{\partial x^2} = D_x \left|\frac{f_{i+1,j} - f_{i\ ,j}}{h}\right| = \frac{1}{h}\left[D_x f_{i+1\ ,j} - D_x f_{i,j}\right]$$

$$= \frac{1}{h}\left[\frac{f_{i+1\ ,j} - f_{i\ ,j}}{h} - \frac{f_{i\ ,j} - f_{i-1\ ,j}}{h}\right] = \frac{1}{h}\left[\frac{f_{i+1\ ,j} - 2f_{i\ ,j} + f_{i-1\ ,j}}{h}\right]$$

$$\rightarrow \frac{\partial^2 f}{\partial x^2} = D_x^2 f = \frac{1}{h^2}\left[f_{i+1\ ,j} - 2f_{i\ ,j} + f_{i-1\ ,j}\right] = \frac{1}{h^2}\left[f_r - 2f_i + f_l\right]$$

$Also, we \ \ can \ see \ \ that$

$$\frac{\partial^2 f}{\partial y^2} = \frac{1}{h^2}\left[f_{i\ ,j+1} - 2f_{i\ ,j} + f_{i,j-1}\right] = D^2 y f$$

$$for \frac{\partial^2 f}{\partial y \partial x} = D_{xy} f \ = D_y \left[\frac{f_{i+1\ ,j} - f_{i-1\ ,j}}{2h}\right]$$

$$\rightarrow \frac{1}{2h}\left[\frac{f_{i+1,j+1} - f_{i+1,j-1}}{2h} - \frac{f_{i-1,j+1} - f_{i-1,j-1}}{2h}\right]$$

$$\frac{1}{4h^2}\left[f_{i+1,j+1} - f_{i+1,j-1} - f_{i-1,j+1} + f_{i-1,j-1}\right]$$

$$= \frac{1}{4h^2}\left[f_{ar} - f_{br} - f_{al} + f_{bl}\right]$$

Abstract

$$\frac{\partial f}{\partial x_i} = \frac{f_{i+1,j} - f_{i-1,j}}{2h} = \frac{f_r - f_l}{2h}$$

$$\frac{\partial f}{\partial y_i} = \frac{f_{i,j+1} - f_{i,j-1}}{2k} = \frac{f_a - f_b}{2k}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{h^2}\left[f_{i+1,j} - 2f_{i,j} + f_{i-1,j}\right] = \frac{1}{h^2}\left[f_r - 2f_i + f_l\right]$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{1}{k^2}\left[f_{i,j+1} - 2f_{i,j} + f_{i,j-1}\right] = \frac{1}{k^2}\left[f_a - 2f_i + f_b\right]$$

$$\nabla^2 f \ = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = D_x^2 f + D_y^2 f \ = 0$$

$$or \ \ in \ general \ \ \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = D_x^2 + D^2 y = 0$$

$$\rightarrow \frac{f_r - 2f_i + f_l}{h^2} + \frac{f_a - 2f_i + f_b}{k^2} = 0$$

$let \ \ \ h = k \rightarrow$

$$f_a + f_b + f_r + f_l - 4f_i = 0$$

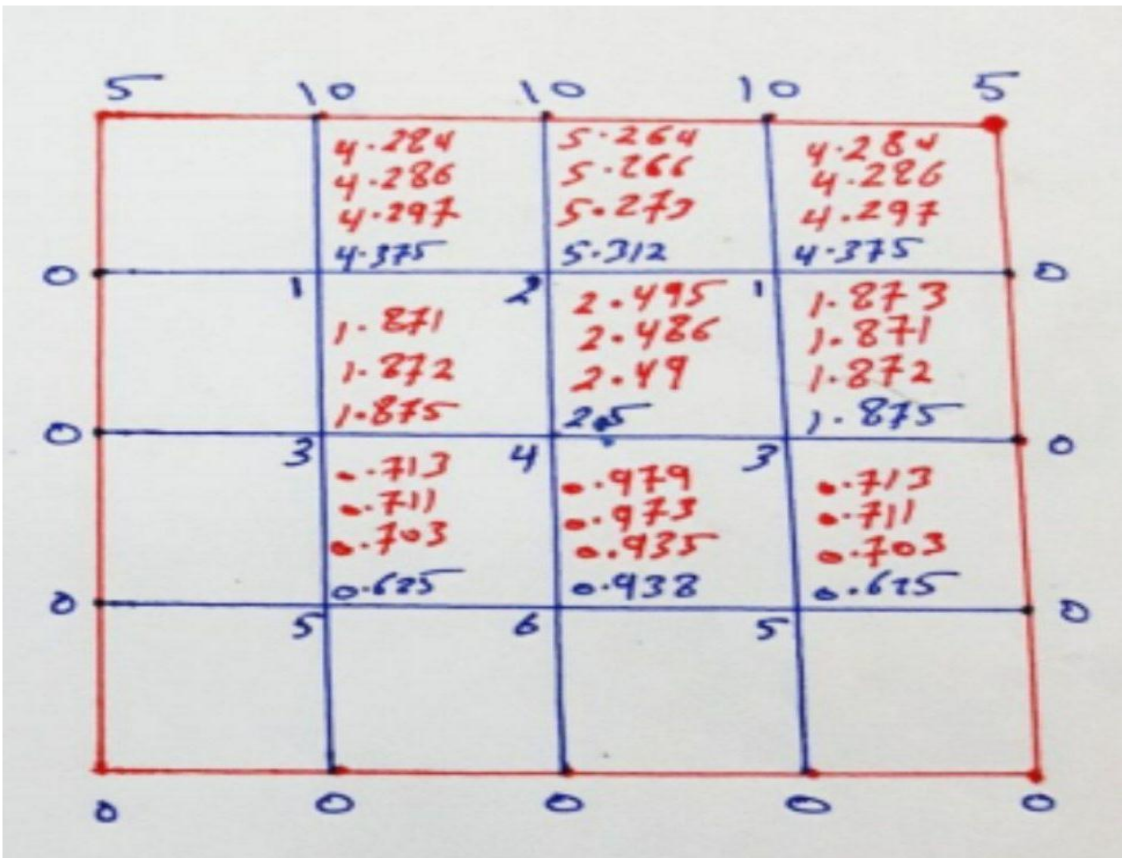
$$or \ \ f_i = \frac{f_a + f_b + f_r + f_l}{4}$$

Not: there are many way to solve the above equation but we will discuss the iteration method only

- Iteration method

In this method, the procedure depend on iterative the equation

Example:



special points

1- **Point i** $\rightarrow f_i = \frac{f_a+f_b+f_r+f_l}{4}$

2- **Point A** $\rightarrow f_A = \frac{f_r+f_l+f_a+f_b}{4} = \frac{2f_r+f_a+f_b}{4}$

3- **Point G** $\rightarrow f_A = \frac{f_r+f_l+f_a+f_b}{4} = \frac{2f_l+f_a+f_b}{4}$

4- **Point B** $\rightarrow f_B = \frac{f_r+f_l+f_a+f_b}{4} = \frac{2f_r+2f_a}{4}$

5- **Point J** $\rightarrow f_J = \frac{f_r+f_l+f_a+f_b}{4} = \frac{2f_l+2f_a}{4}$

6- **Point E** $\rightarrow f_E = \frac{f_r+f_l+f_a+f_b}{4} = \frac{f_r+f_l+2f_a}{4}$

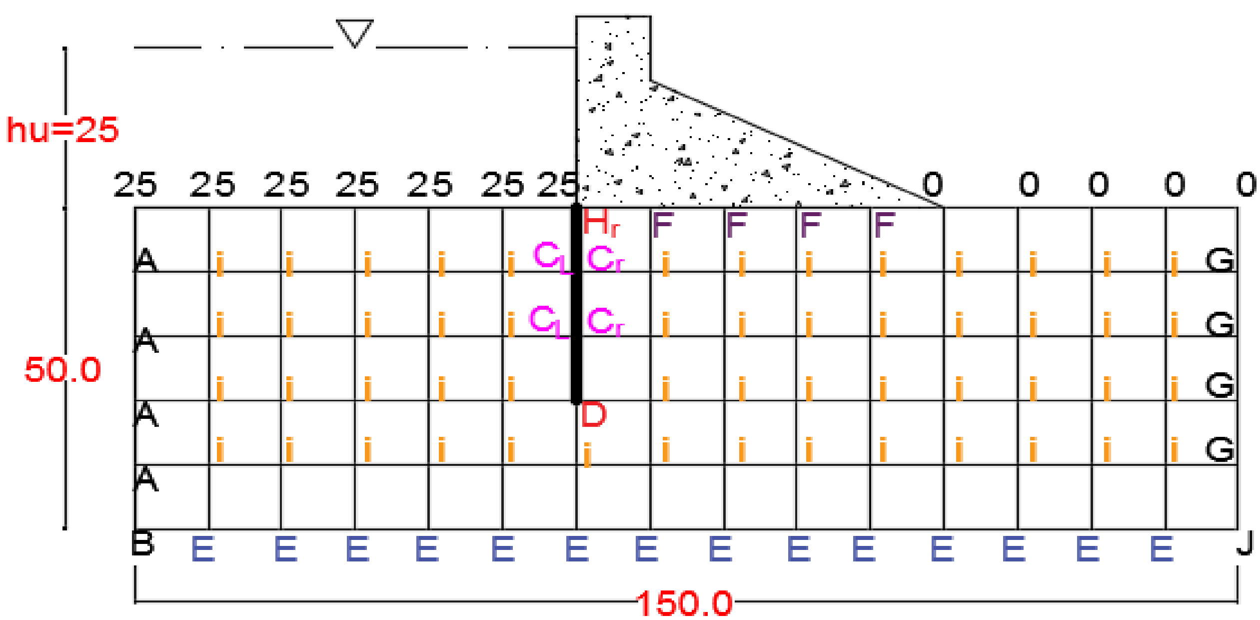
7- **Point F** $\rightarrow f_F = \frac{f_r+f_l+f_a+f_b}{4} = \frac{f_r+f_l+2f_b}{4}$

8- **Point H_r** $\rightarrow f_{H_r} = \frac{f_r+f_l+f_a+f_b}{4} = \frac{2f_r+2f_b}{4}$

9- **Point C_r** $\rightarrow f_{C_r} = \frac{f_r+f_l+f_a+f_b}{4} = \frac{2f_r+f_a+f_b}{4}$

10- **Point C_L** $\rightarrow f_{C_L} = \frac{f_r+f_l+f_a+f_b}{4} = \frac{2f_l+f_a+f_b}{4}$

11- **Point D** $\rightarrow f_D = \frac{f_r+f_l+f_a+f_b}{4} = \frac{f_r+f_l+(0.5f_{C_r}+0.5f_{C_L})+f_b}{4}$



Example: For the dam above, find the distribution of total head under the dam by numerical solution using iteration method:

Solution:

	0	10	20	30	40	50	60	60	70	80	90	100	110	120	130	140	150
	hu	hu	hu	hu	hu	hu	L	R	F	F	F	F	hd	hd	hd	hd	hd
0	25.00	25.00	25.00	25.00	25.00	25.00	25.00	9.56	9.10	7.95	6.24	3.87	0.00	0.00	0.00	0.00	0.00
10	24.03	23.99	23.85	23.61	23.29	22.91	22.67	10.01	9.45	8.22	6.57	4.62	2.56	1.68	1.26	1.06	1.00
20	23.16	23.07	22.80	22.32	21.62	20.70	19.84	11.57	10.48	8.91	7.21	5.49	3.94	2.92	2.30	1.97	1.87
30	22.46	22.34	21.95	21.26	20.16	18.42	15.30		12.00	9.74	7.85	6.19	4.78	3.74	3.05	2.66	2.53
40	22.01	21.87	21.41	20.60	19.35	17.53	15.07		12.46	10.21	8.28	6.62	5.26	4.22	3.50	3.08	2.95
50	21.86	21.70	21.23	20.38	19.09	17.29	15.00		12.57	10.35	8.42	6.77	5.42	4.38	3.65	3.23	3.09

H.w1 : Resolve the problem with hu = 25 m & hd = 5 m

	0	10	20	30	40	50	60	60	70	80	90	100	110	120	130	140	150
	hu	hu	hu	hu	hu	hu	L	R	F	F	F	F	hd	hd	hd	hd	hd
0	25.00	25.00	25.00	25.00	25.00	25.00	25.00	12.64	12.28	11.36	9.99	8.10	5.00	5.00	5.00	5.00	5.00
10	24.23	24.19	24.08	23.89	23.63	23.33	23.13	13.01	12.56	11.58	10.26	8.70	7.05	6.35	6.01	5.85	5.80
20	23.53	23.46	23.24	22.86	22.29	21.56	20.87	14.26	13.39	12.13	10.77	9.39	8.15	7.33	6.84	6.58	6.49
30	22.97	22.87	22.56	22.01	21.13	19.74	17.24		14.60	12.79	11.28	9.95	8.83	7.99	7.44	7.13	7.03
40	22.61	22.49	22.13	21.48	20.48	19.03	17.06		14.97	13.17	11.62	10.30	9.21	8.38	7.80	7.47	7.36
50	22.49	22.36	21.98	21.30	20.27	18.83	17.00		15.06	13.28	11.73	10.42	9.33	8.50	7.92	7.58	7.47

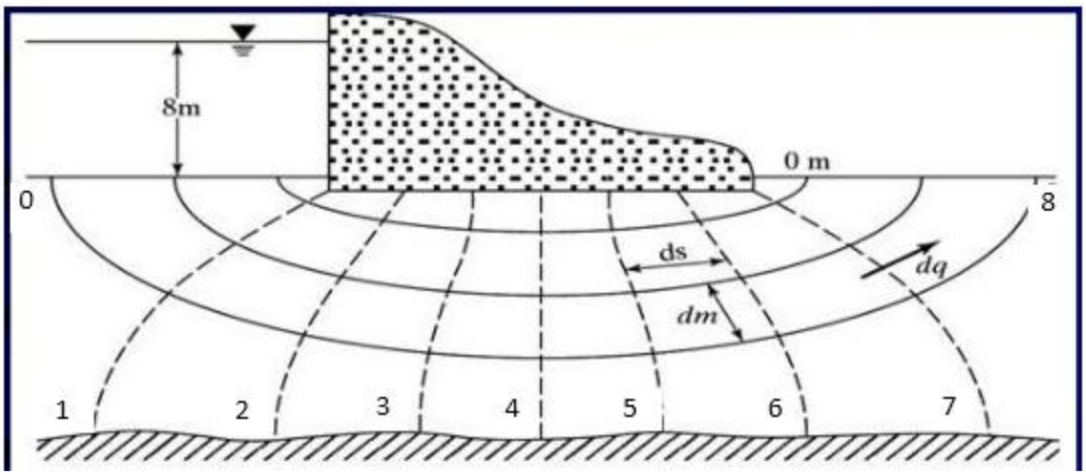
H.w1 : Resolve the problem with hu = 35 m & hd = 2 m

	0	10	20	30	40	50	60	60	70	80	90	100	110	120	130	140	150
	hu	hu	hu	hu	hu	hu	L	R	F	F	F	F	hd	hd	hd	hd	hd
0	35.00	35.00	35.00	35.00	35.00	35.00	35.00	14.61	14.02	12.49	10.24	7.11	2.00	2.00	2.00	2.00	2.00
10	33.73	33.67	33.48	33.17	32.74	32.24	31.92	15.21	14.48	12.85	10.68	8.10	5.38	4.22	3.66	3.39	3.31
20	32.57	32.45	32.09	31.47	30.53	29.32	28.19	17.27	15.84	13.77	11.51	9.25	7.20	5.85	5.04	4.60	4.47
30	31.65	31.49	30.98	30.06	28.61	26.32	22.20		17.84	14.86	12.37	10.17	8.31	6.94	6.03	5.51	5.34
40	31.06	30.86	30.26	29.19	27.54	25.14	21.90		18.45	15.47	12.92	10.74	8.94	7.57	6.62	6.07	5.89
50	30.85	30.65	30.02	28.90	27.20	24.82	21.80		18.59	15.66	13.11	10.94	9.15	7.78	6.82	6.26	6.07

H.W:

For the dams below, find the distribution of total head under the dam by numerical solution using iteration method and compare the results with that of the flow net method:

1-



2-