



Inverters

Lecture 13

Electrical Engineering Department
Power Electronics and Special Machine

By

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Important Notice

Note to Students:

These lecture materials are provided as supplementary support to help clarify and reinforce the concepts discussed in the main course lectures. They are not a substitute for the official course materials or the primary references assigned. Students are encouraged to refer to the original sources and attend the main lectures to ensure full understanding of the subject.

Inverters (DC-AC Converters)

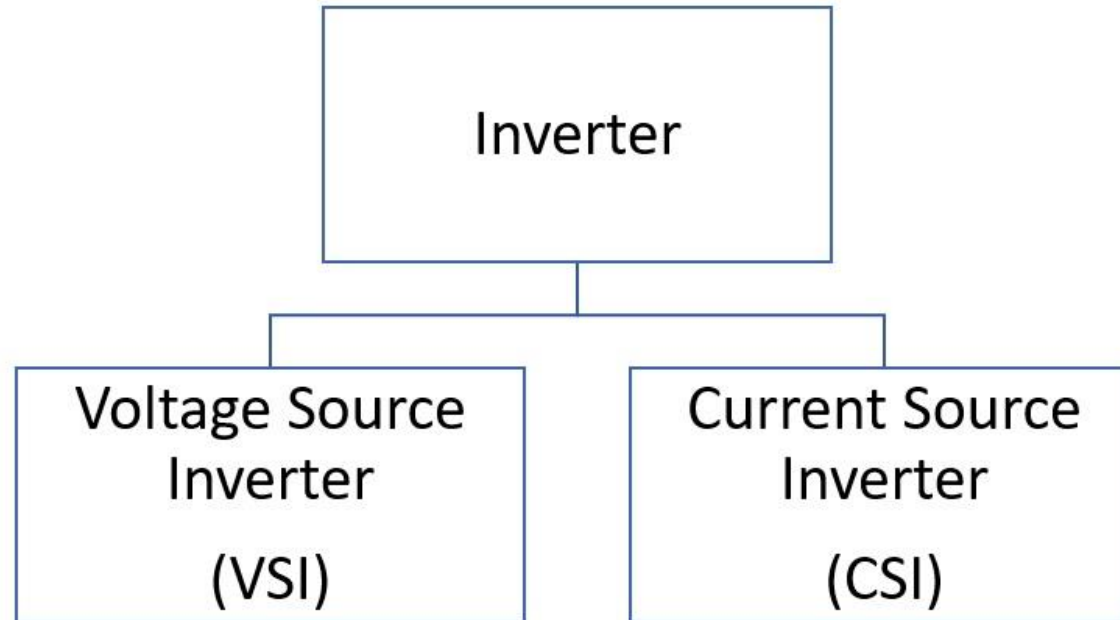
Inverter circuits convert the DC power into AC power at a desired output voltage or current and frequency.

Conversion is achieved using:

- Thyristors
- BJT
- Power MOSFET
- IGBT

Inverters (DC-AC Converters)

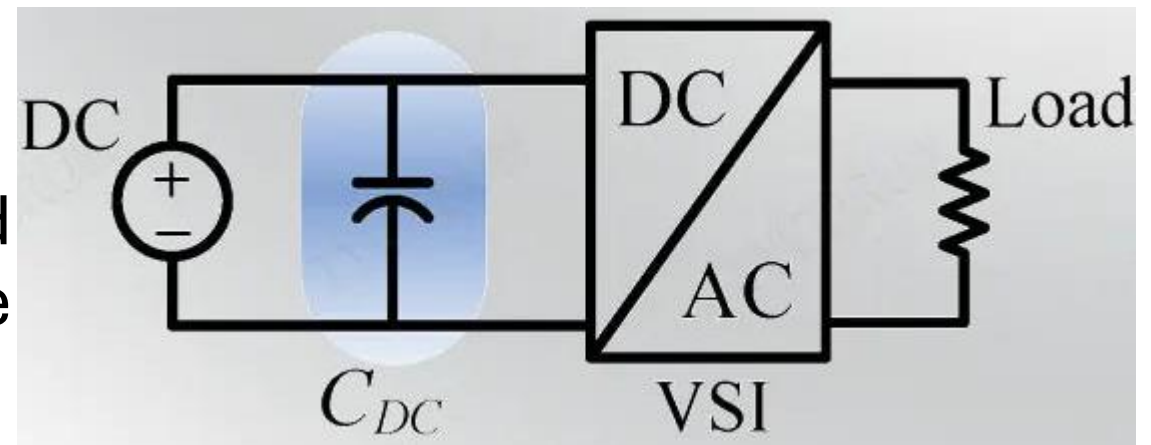
There are two types of inverters



Inverters (DC-AC Converters)

1. Voltage-source inverter (VSI)

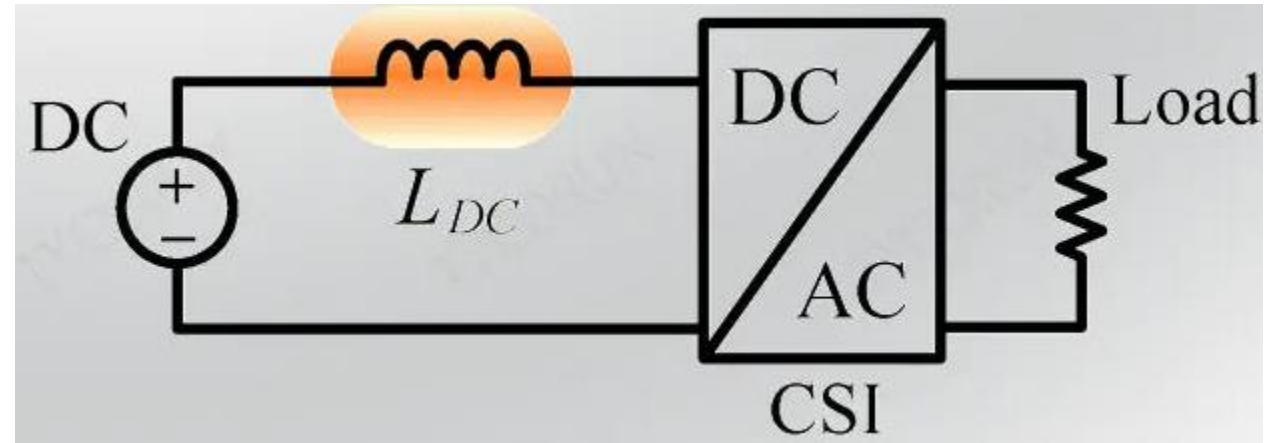
- The DC supply voltage is fixed.
- The supply has low impedance.
- The inverter specifies the load voltage while the current shape is dictated by the load.



Inverters (DC-AC Converters)

1. Current-source inverter (CSI)

- The DC supply voltage is varied to control the magnitude of load current.
- Large series impedance is connected in series with the supply to prevent sudden changes in current.



Inverters (DC-AC Converters)

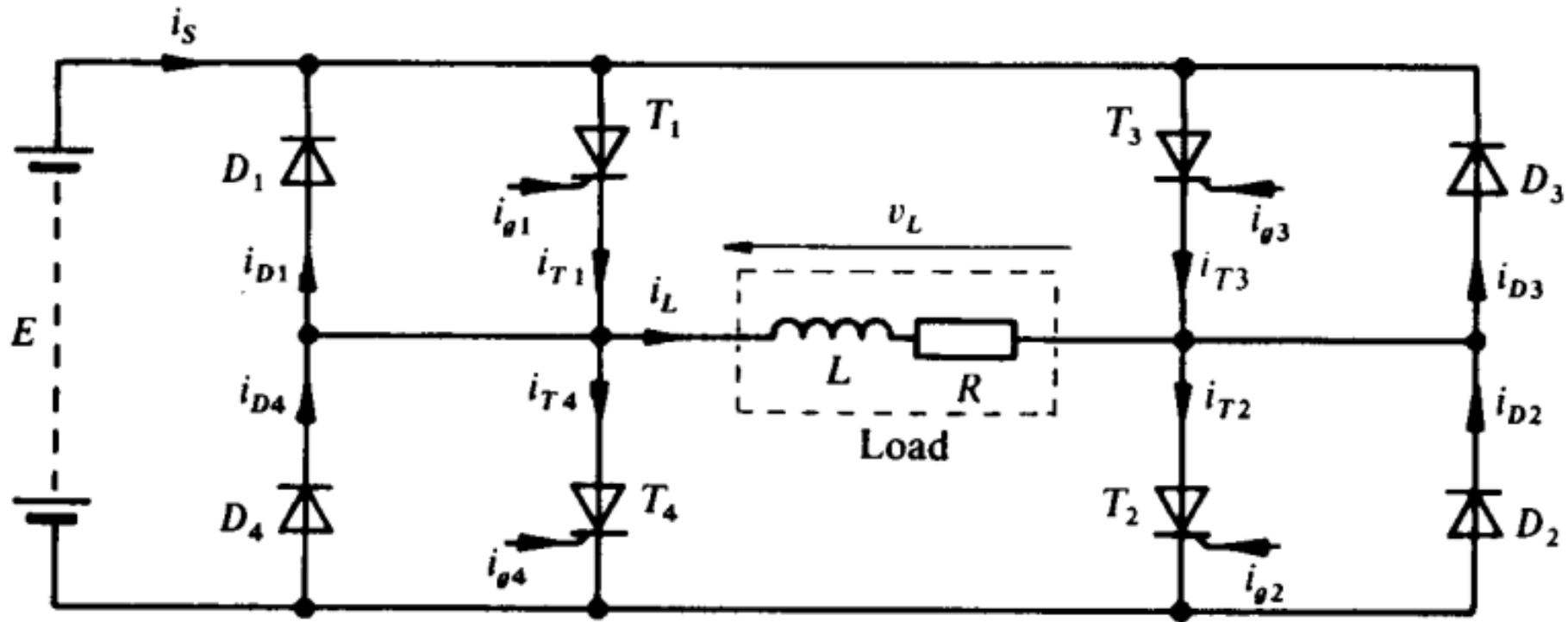
Inverter circuits applications

- Uninterruptable power supplies (UPS).
- Induction heating.
- Variable-speed AC motor drives



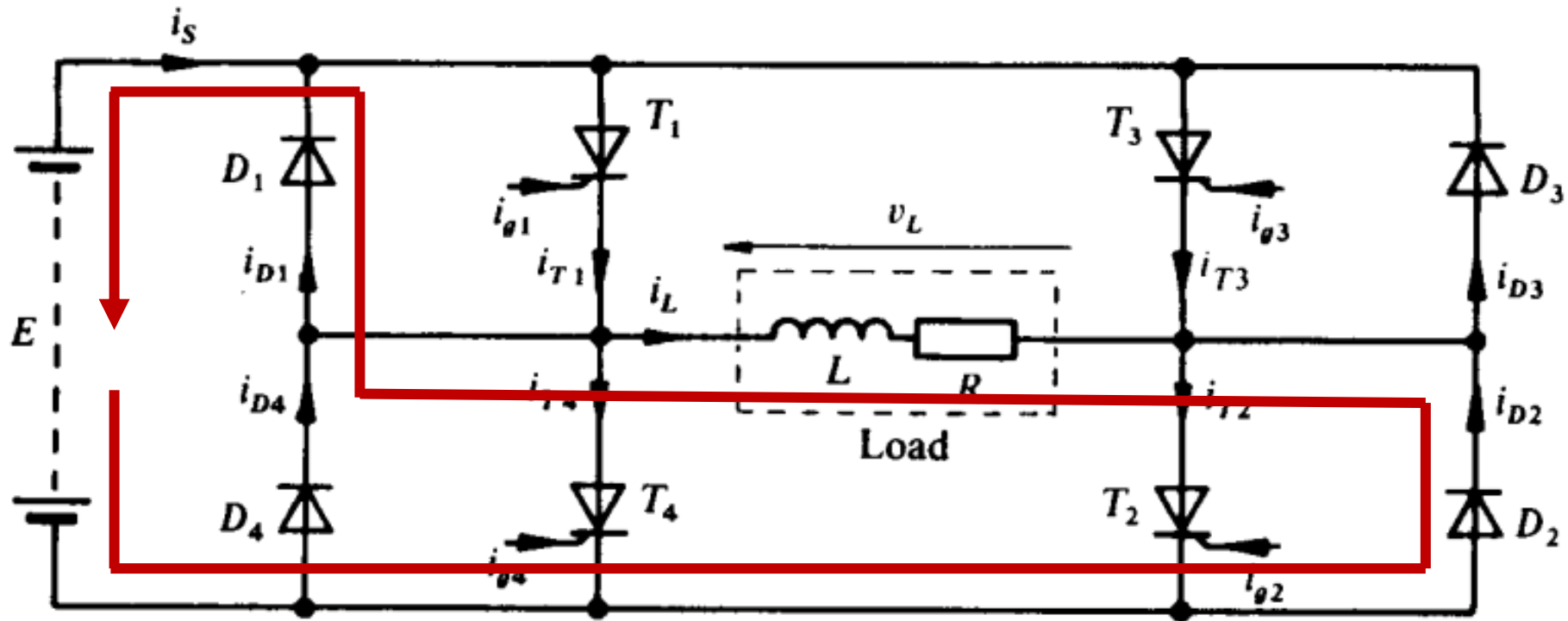
Single-phase inverter

Single phase voltage-source inverter



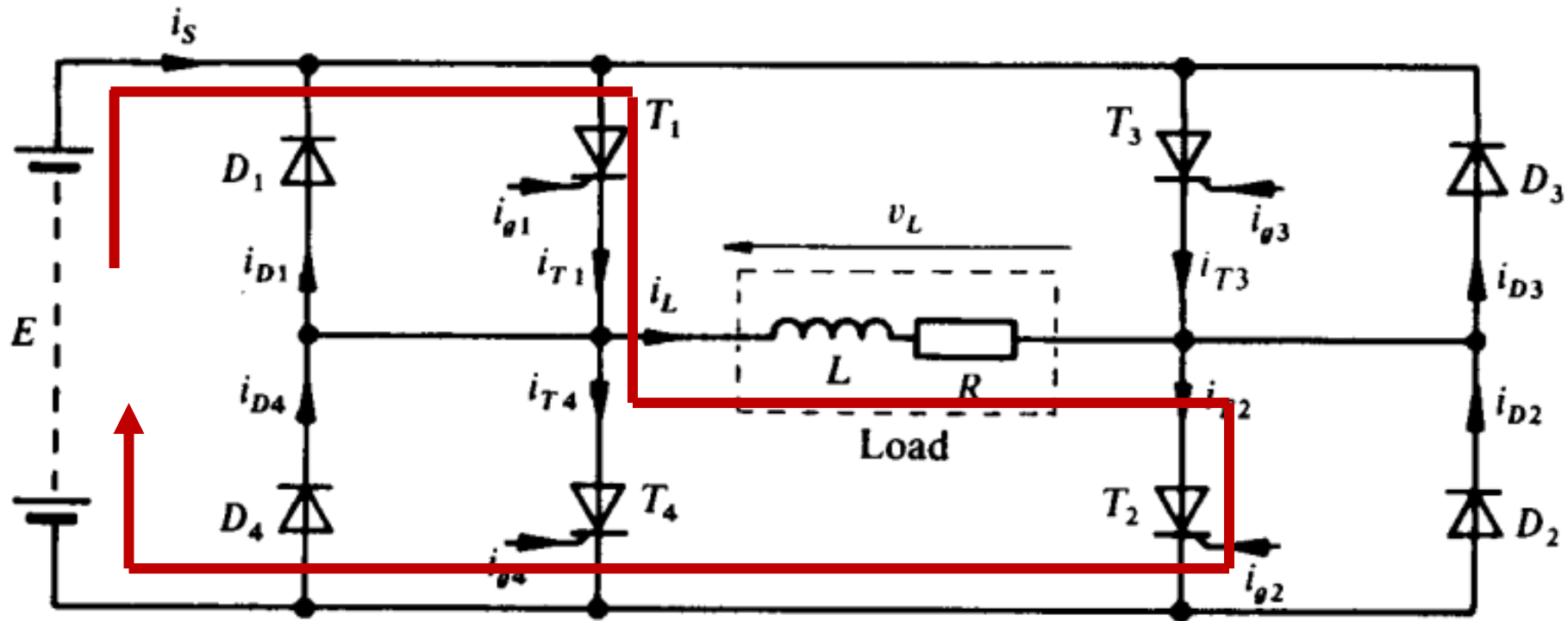
Single-phase inverter

Single phase voltage-source inverter (Case 1)



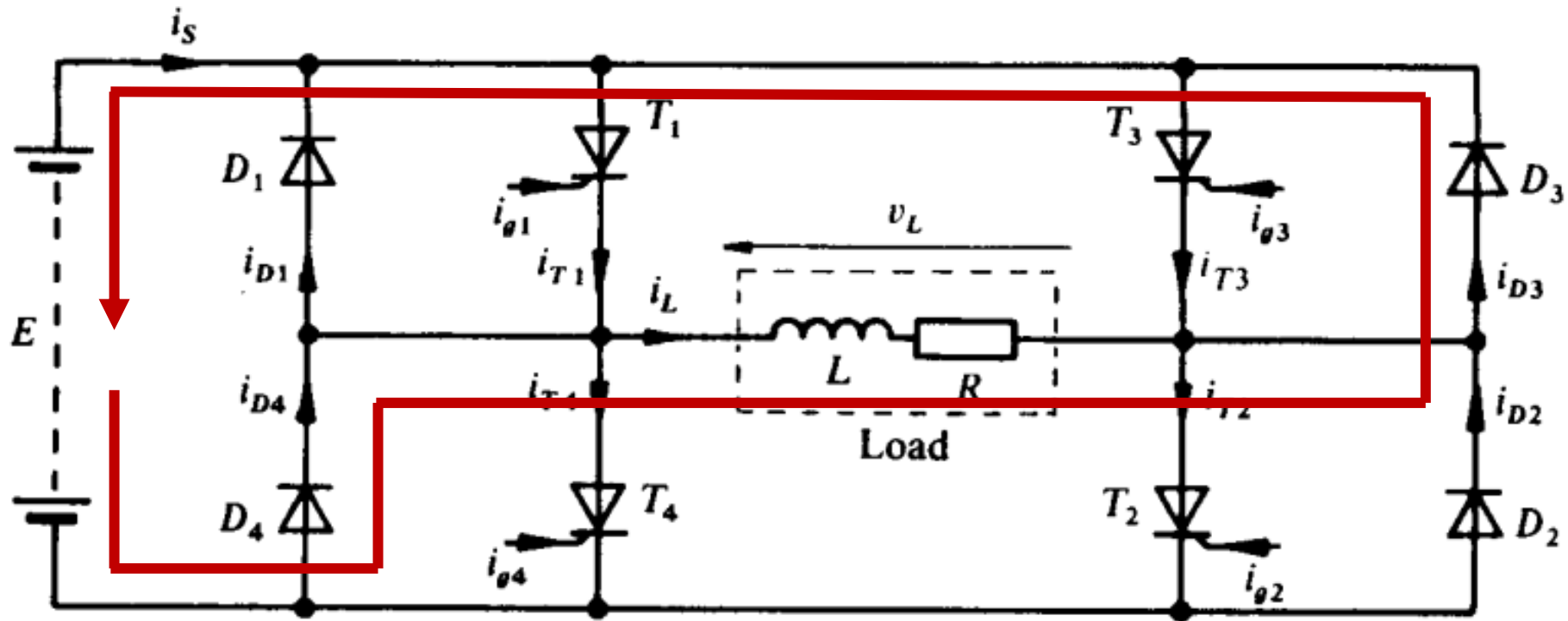
Single-phase inverter

Single phase voltage-source inverter (Case 2)



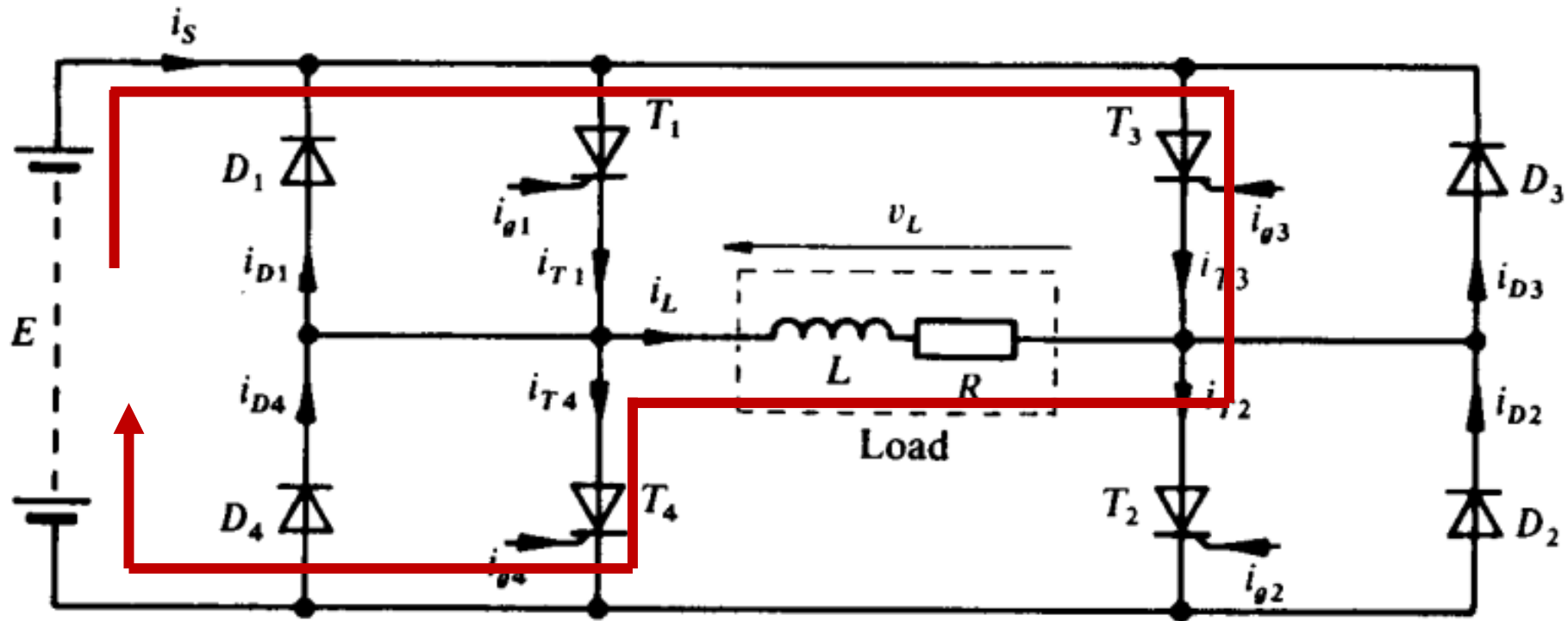
Single-phase inverter

Single phase voltage-source inverter (Case 3)



Single-phase inverter

Single phase voltage-source inverter (Case 4)



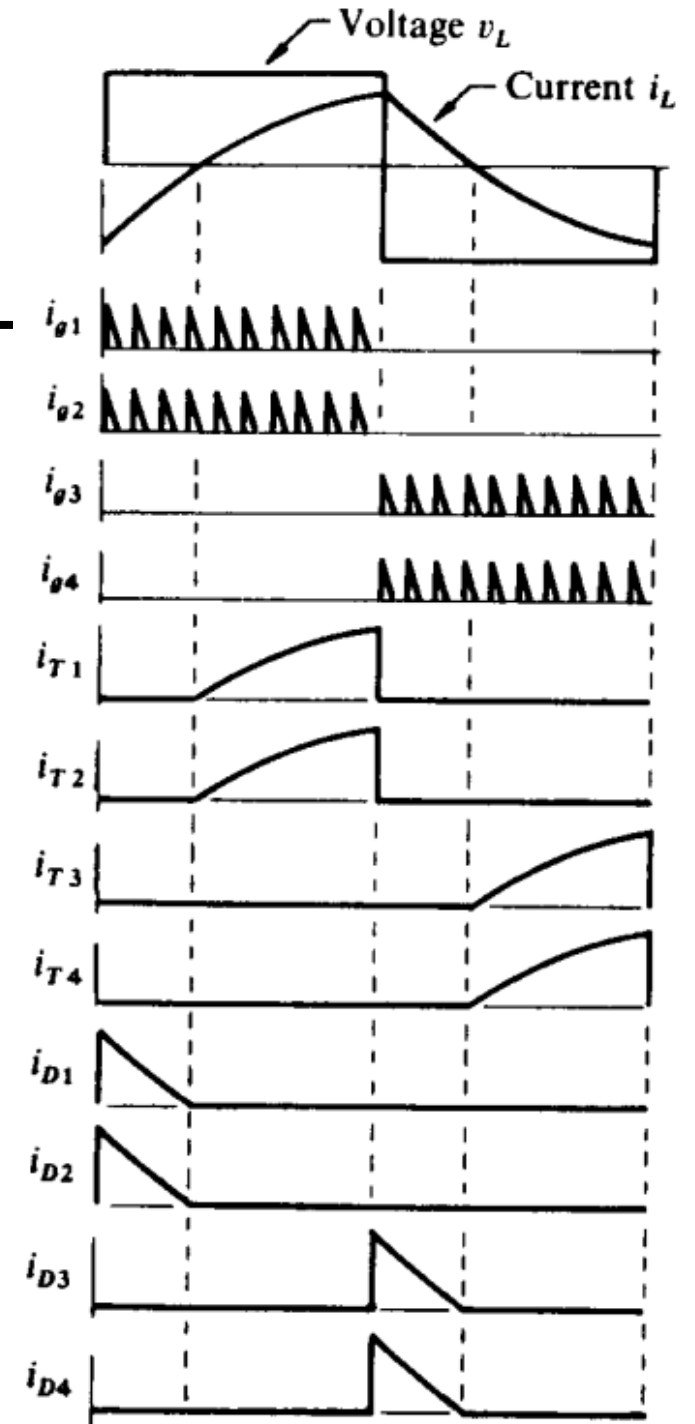
Single-phase inverter

Single phase voltage-source inverter

- Square-wave output signal.

Case	Conducting devices	V_L	I_L
1	$D_1 D_2$	$+V_S$	-
2	$T_1 T_2$	$+V_S$	+
3	$D_3 D_4$	$-V_S$	+
4	$T_3 T_4$	$-V_S$	-

$$V_S = L \frac{d i_L}{dt} + i_L R, \text{ when } T_1 \text{ and } T_2 \text{ conduct}$$



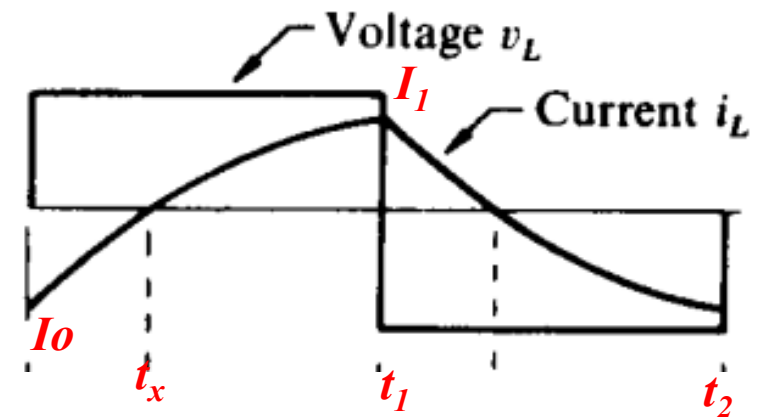
Single-phase inverter

$$-V_s = L \frac{d i_L}{dt} + i_L R, \text{ when } T_3 \text{ and } T_4 \text{ conduct}$$

Using Laplace transform

$$i_L = \frac{V_s}{R} (1 - e^{-tR/L}) \text{ with no load initial current}$$

$$i_L = \frac{V_s}{R} - \left(\frac{V_s}{R} - I_o \right) e^{-\frac{tR}{L}} \quad 0 \leq t \leq t_1 \text{ (steady - state)}$$



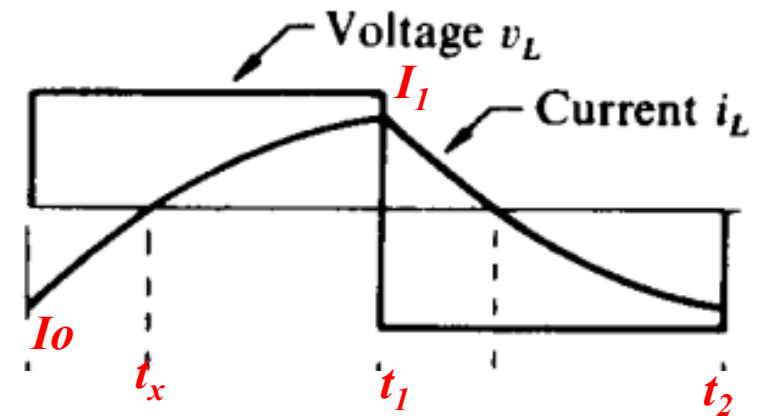
Single-phase inverter

$$i_L = -\frac{V_s}{R} + \left(\frac{V_s}{R} - I_1\right) e^{-\frac{tR}{L}} \quad 0 \leq t \leq t_2 - t_1$$

$$I_1 = \frac{V_s}{R} \frac{(1 - e^{-\frac{R}{L}t_1})}{(1 + e^{-\frac{R}{L}t_1})} = -I_o$$

$$t_x = \frac{L}{R} \ln \left(1 - \frac{I_o R}{V_s}\right)$$

$$P_L = \frac{1}{t_1} \int_0^{t_1} V_s i_L(t) dt$$



Single-phase inverter

Example: A single-phase bridge inverter supplies a 10 ohm resistor with inductance 50 mH from a 340 V DC source. If the bridge is operating at 50 Hz, determine the load RMS voltage, the steady-state mean power delivered to the load, and steady-state current waveforms with a square-waveform output.

Solution:

The time constant of the load,

$$\tau = \frac{L}{R} = \frac{0.05}{10} = 5 \text{ ms}$$

The output voltage RMS value is 340 V

Single-phase inverter

Solution:

$$\text{For } f=50 \text{ Hz, } T = \frac{1}{f} = \frac{1}{50} = 20 \text{ ms, } t_1 = \frac{T}{2} = 10 \text{ ms}$$

$$I_1 = \frac{V_s}{R} \frac{(1 - e^{-\frac{R}{L}t_1})}{(1 + e^{-\frac{R}{L}t_1})} = \frac{340}{10} \times \frac{(1 - e^{-2})}{(1 + e^{-2})} = 25.9 \text{ A} = -I_o$$

$$i_L = \frac{V_s}{R} - \left(\frac{V_s}{R} - I_o \right) e^{-\frac{tR}{L}} \quad 0 \leq t \leq t_1 (\text{steady - state})$$
$$= 34 - (34 + 25.9)e^{-200t}$$

Single-phase inverter

Solution:

The zero current cross-over time (t_x) is

$$t_x = \frac{L}{R} \ln \left(1 - \frac{I_o R}{V_S} \right) = 5 \times 10^{-3} \ln \left(1 - \frac{25.9 \times 10}{340} \right) = 2.83 \text{ ms}$$

For $V_L = -340 \text{ V}$

$$i_L = -\frac{V_S}{R} + \left(\frac{V_S}{R} - I_o \right) e^{-\frac{tR}{L}} \quad 0 \leq t \leq t_2 - t_1$$
$$= -34 + (34 + 25.9)e^{-200t}$$

Single-phase inverter

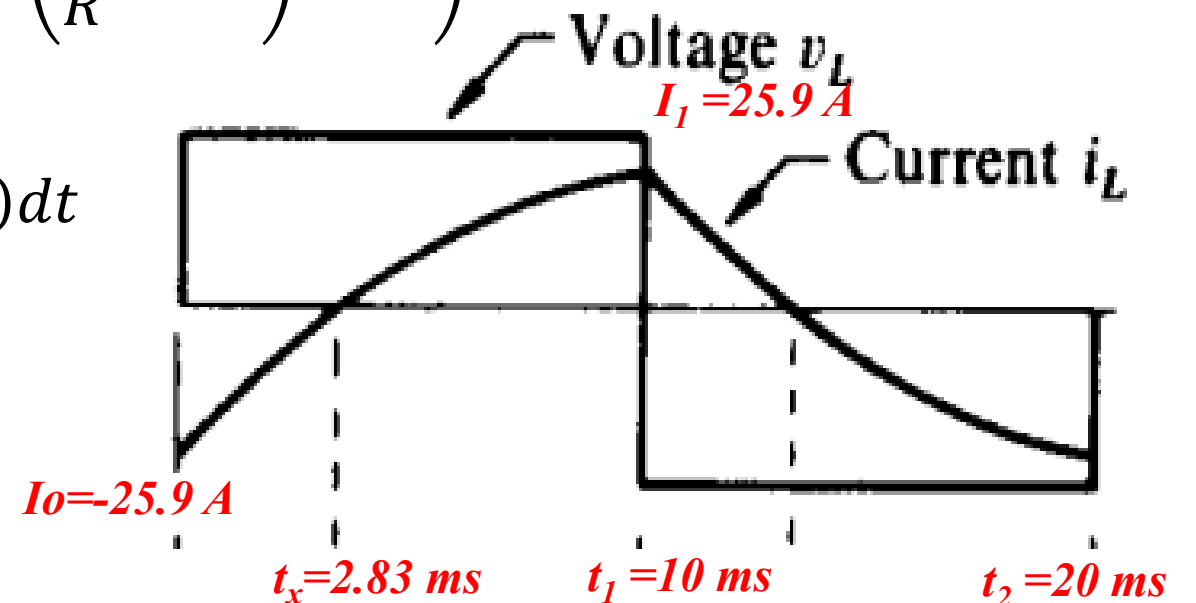
Solution:

The steady-state mean power is given by:

$$P_L = \frac{1}{t_1} \int_0^{t_1} V_S i_L(t) dt = \frac{1}{t_1} \int_0^{t_1} V_S \left(\frac{V_S}{R} - \left(\frac{V_S}{R} - I_o \right) e^{-\frac{tR}{L}} \right) dt$$

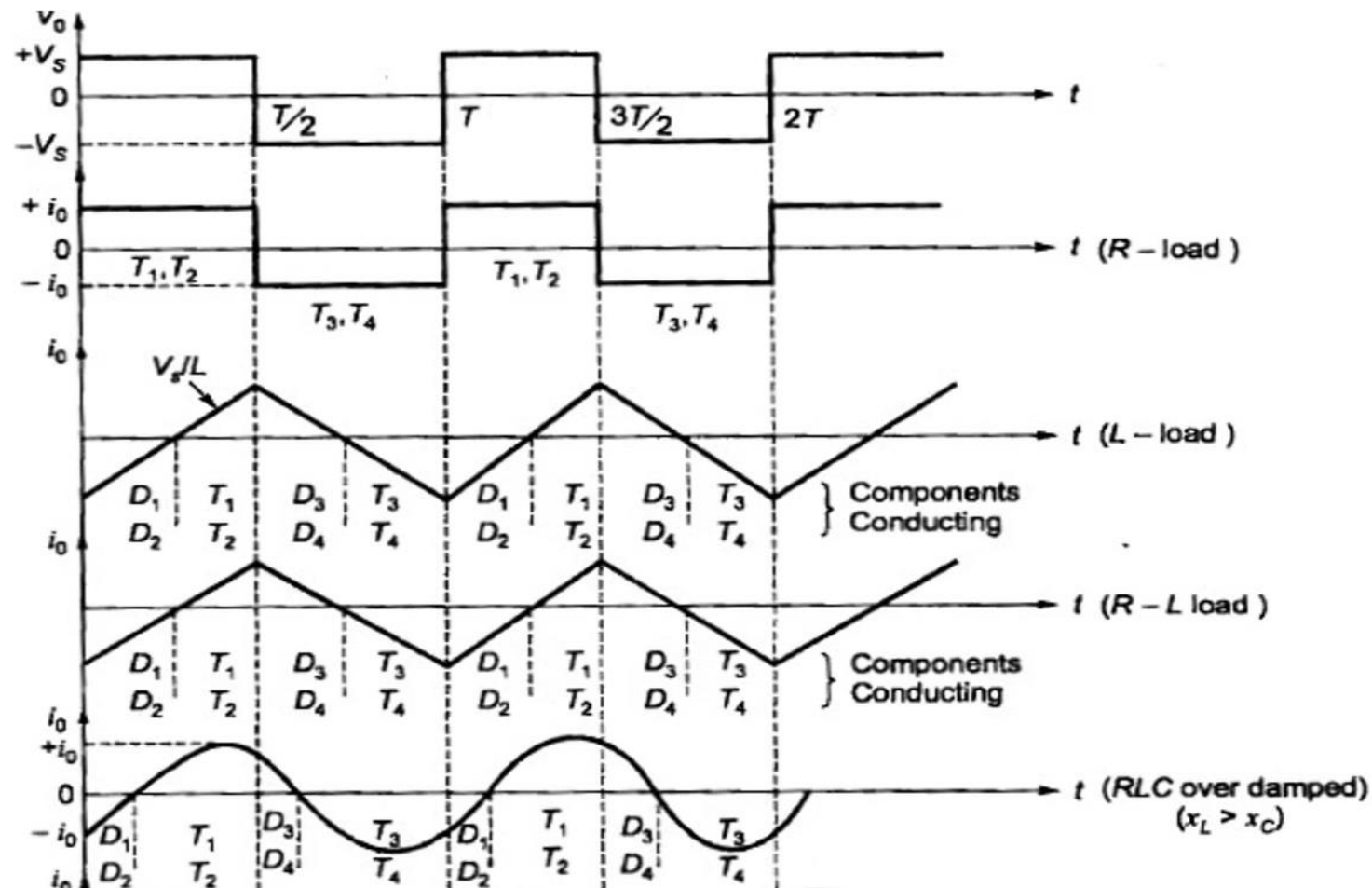
$$P_L = \frac{1}{0.01} \int_0^{0.01} 340 \times (34 - 59.9 e^{-200t}) dt$$

$$P_L = 2755 \text{ W}$$



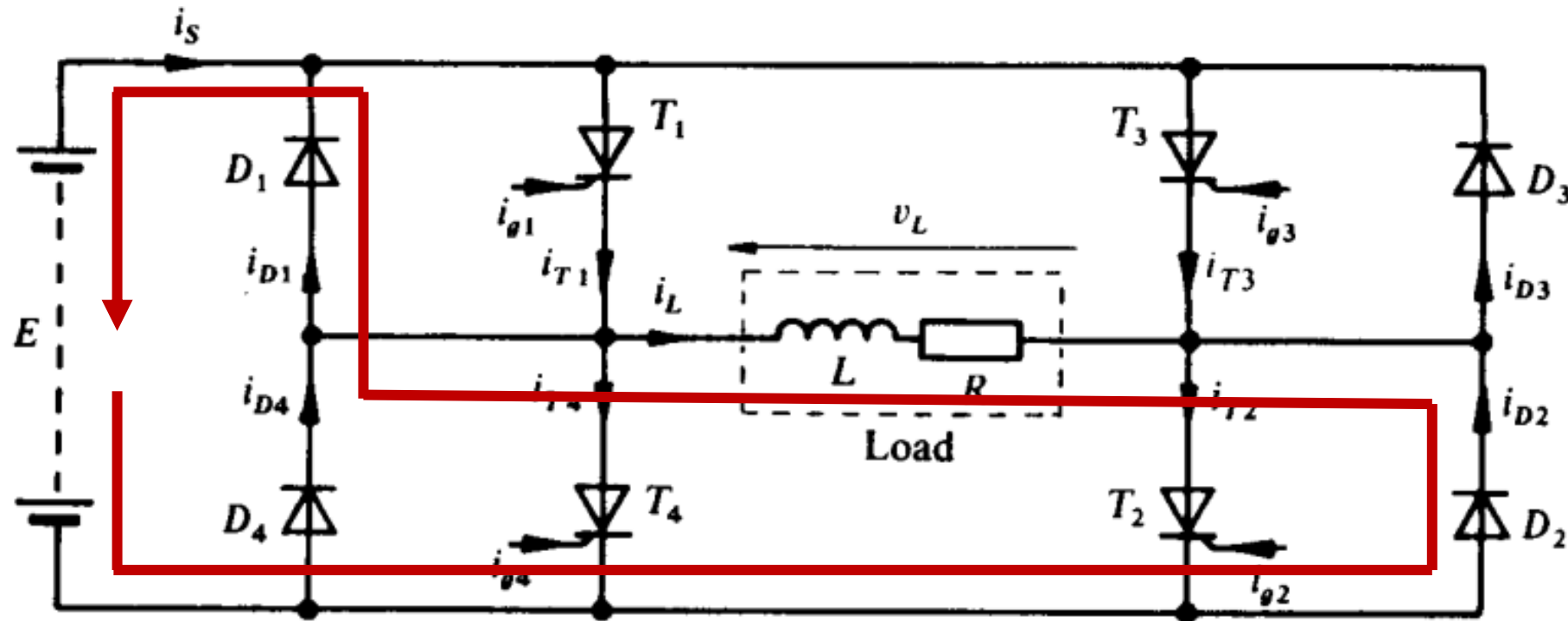
Single-phase inverter

Single phase voltage-source inverter for different load types



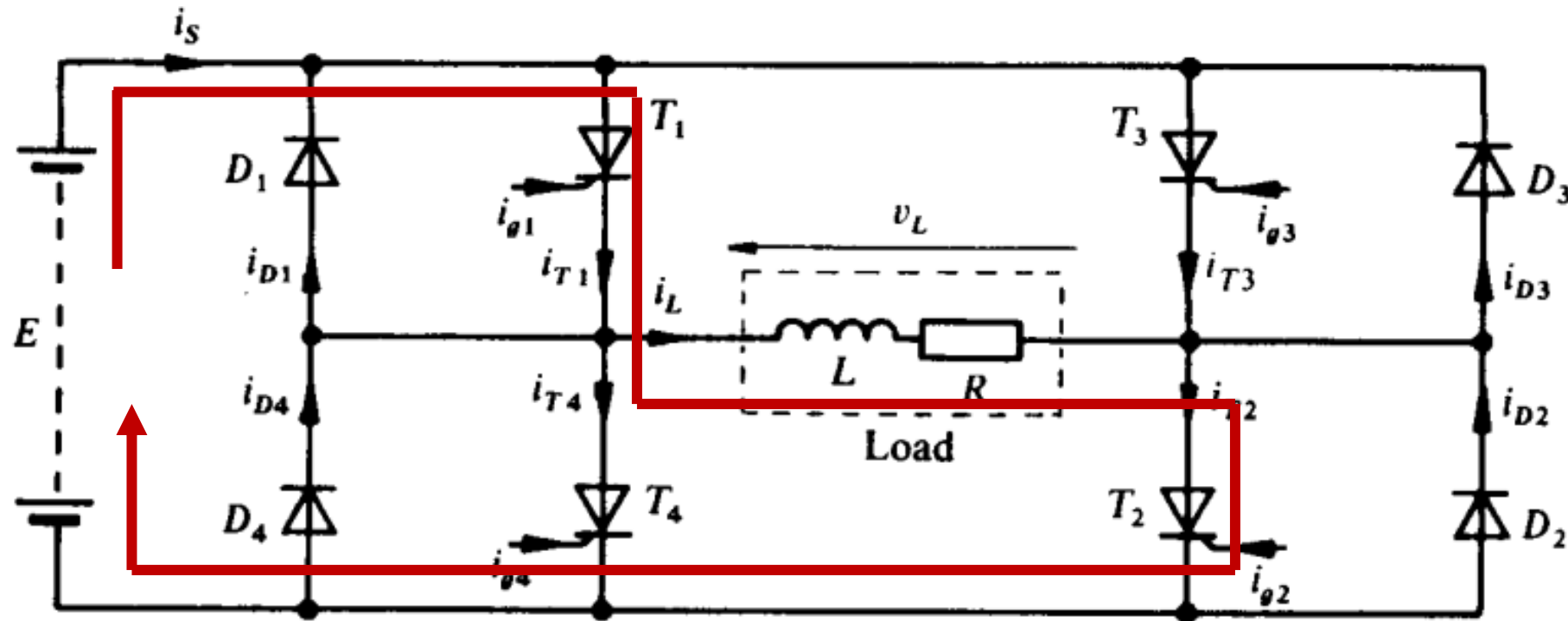
Single-phase inverter

Single phase voltage-source inverter (Quasi-square wave output signal) (Case 1)



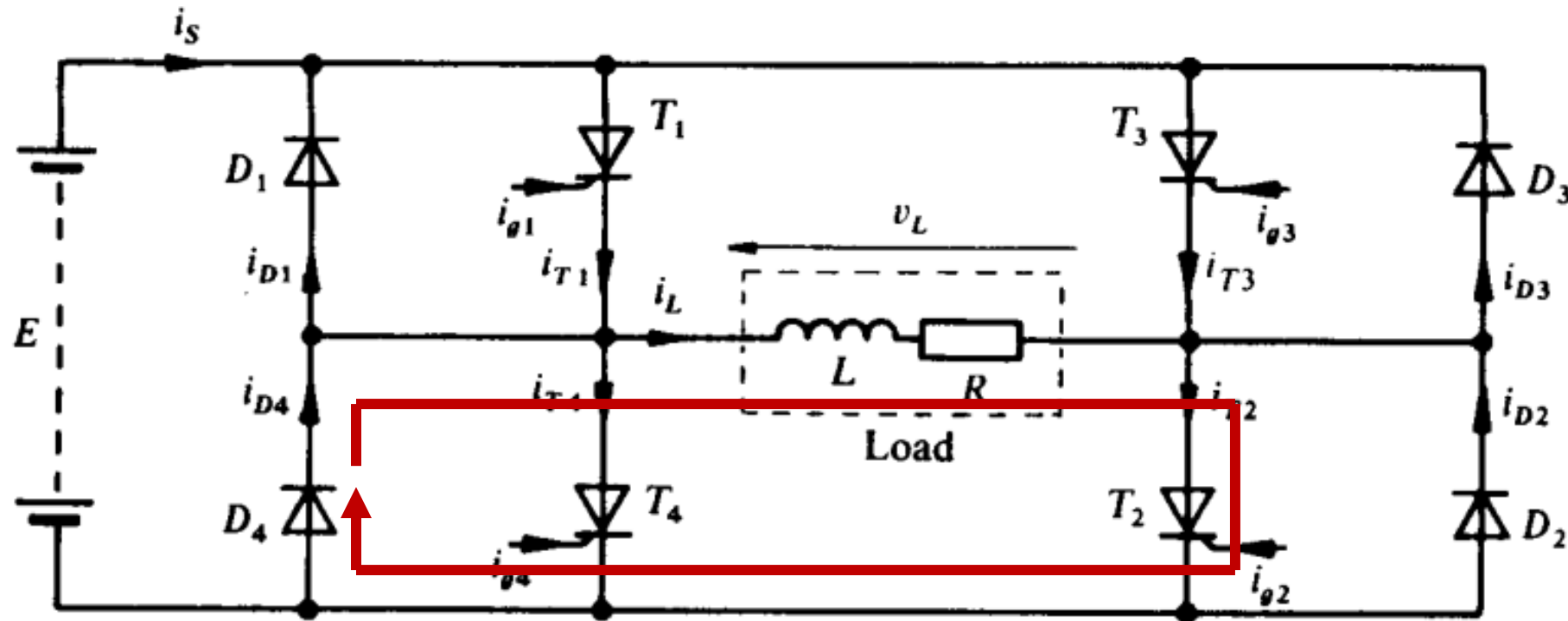
Single-phase inverter

Single phase voltage-source inverter (Quasi-square wave output signal) (Case 2)



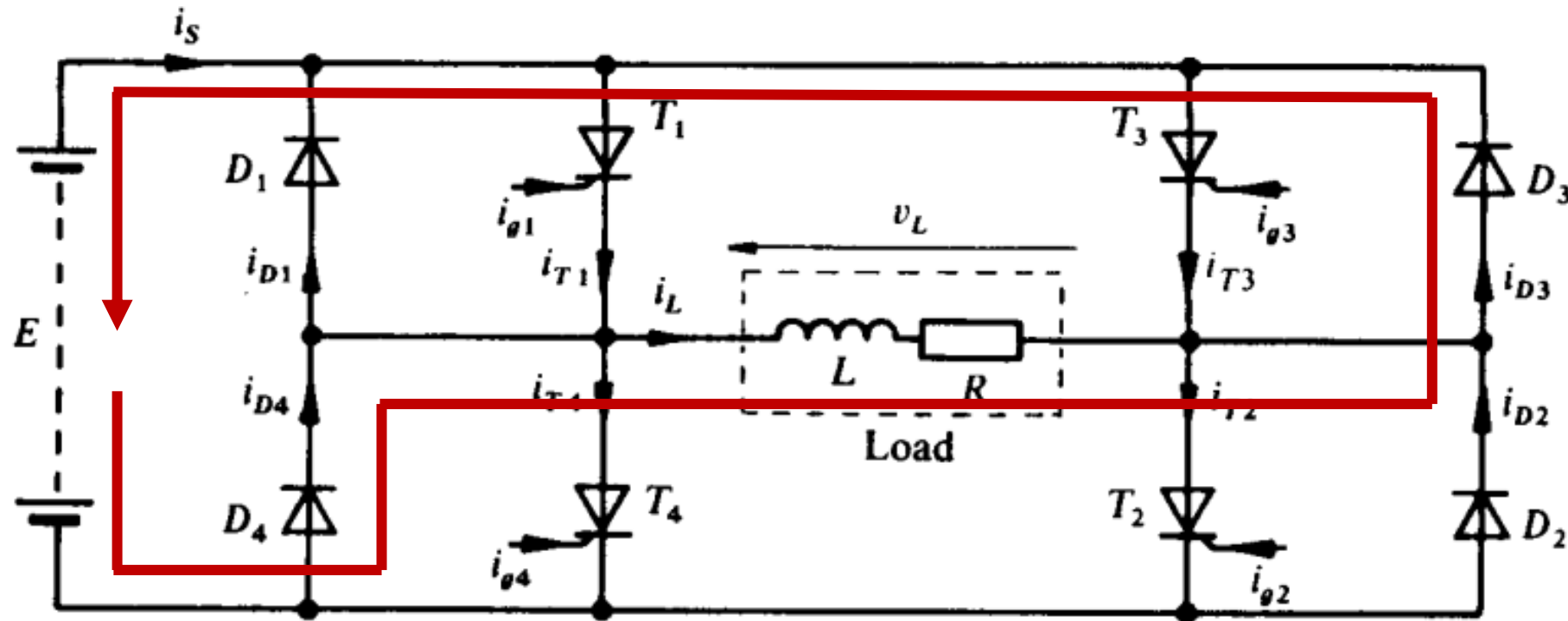
Single-phase inverter

Single phase voltage-source inverter (Quasi-square wave output signal) (Case 3)



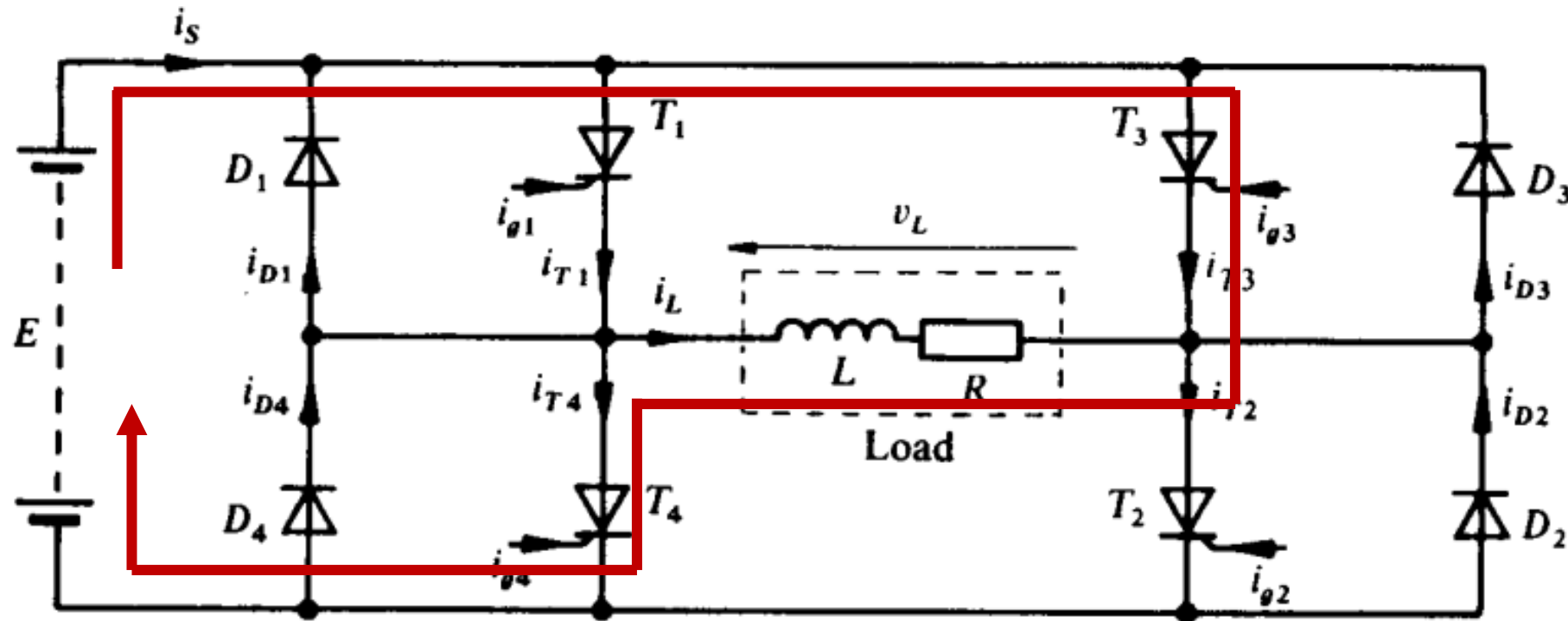
Single-phase inverter

Single phase voltage-source inverter (Quasi-square wave output signal) (Case 4)



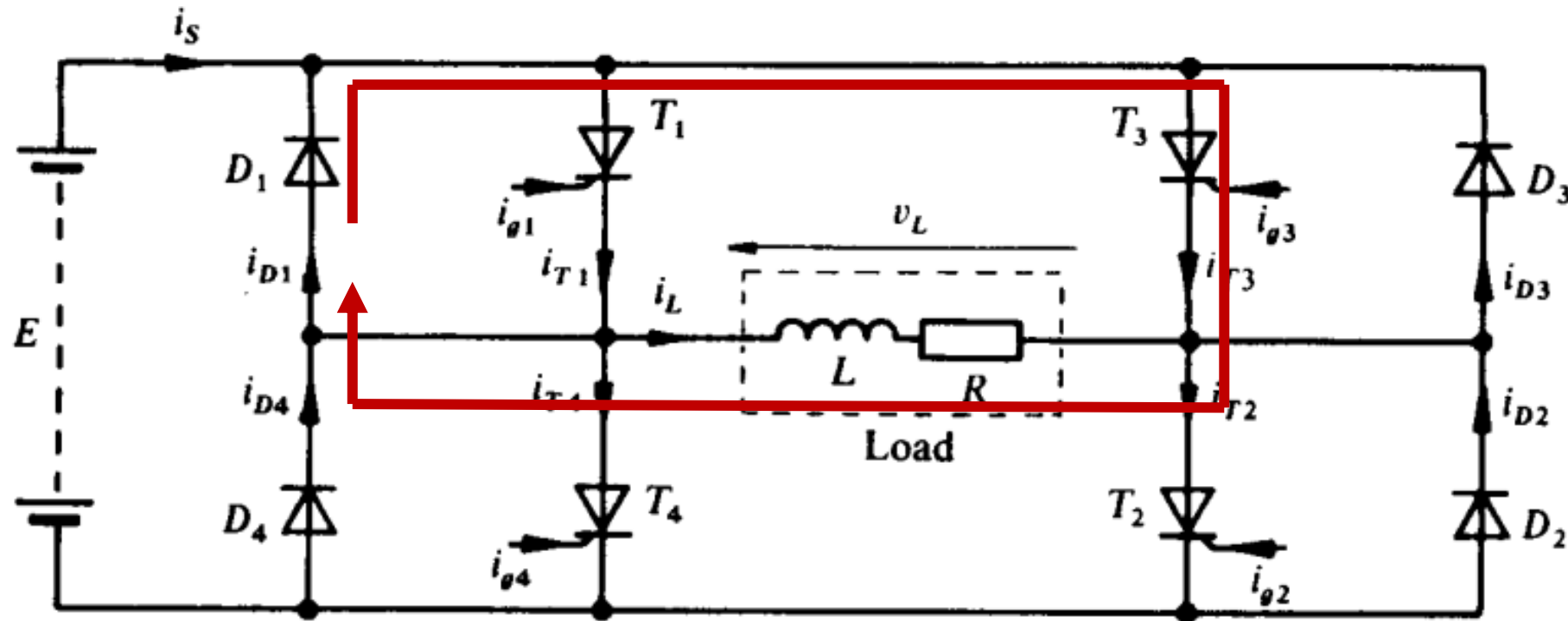
Single-phase inverter

Single phase voltage-source inverter (Quasi-square wave output signal) (Case 5)



Single-phase inverter

Single phase voltage-source inverter (Quasi-square wave output signal) (Case 6)

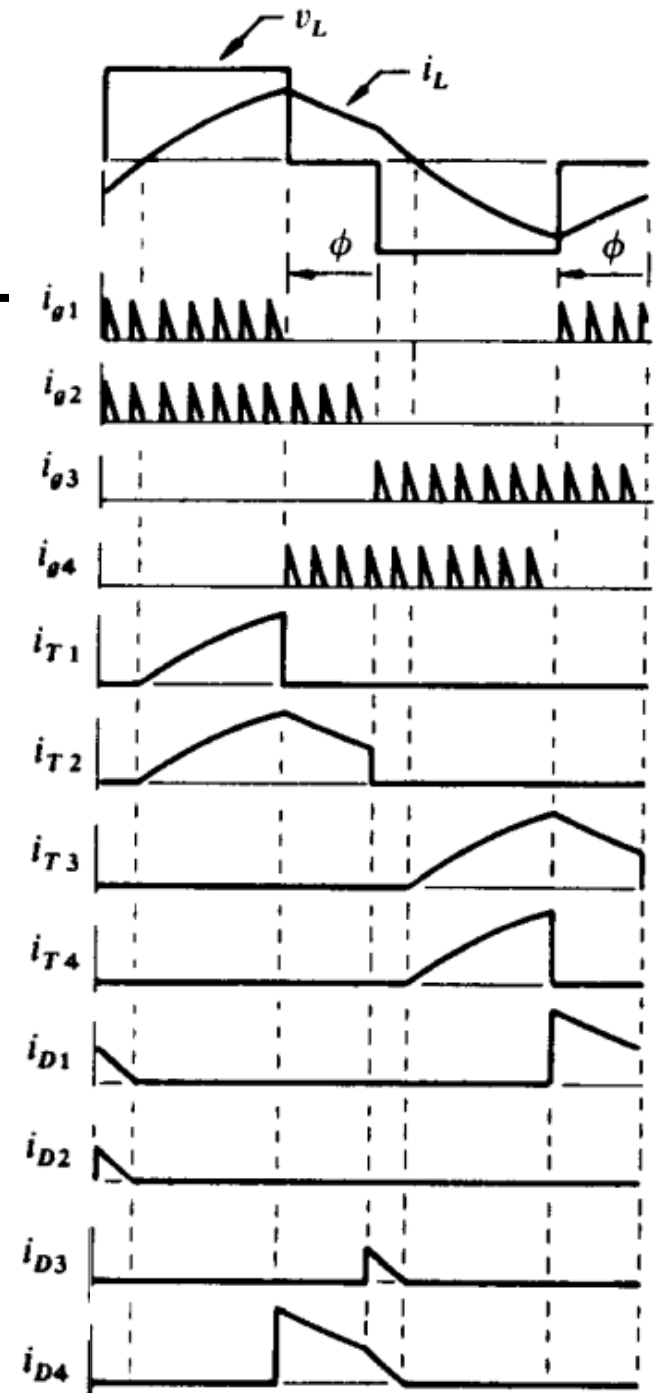


Single-phase inverter

Single phase voltage-source inverter

- Quasi-square wave output signal.

Case	Conducting devices	V_L	I_L
1	$D_1 D_2$	$+V_S$	-
2	$T_1 T_2$	$+V_S$	+
3	$T_2 D_4$	0	+
4	$D_3 D_4$	$-V_S$	+
5	$T_3 T_4$	$-V_S$	-
6	$T_3 D_1$	0	-



Single-phase inverter

For $V_L > 0$ and $I_o \leq 0$

$$i_L = \frac{V_s}{R} - \left(\frac{V_s}{R} - I_o \right) e^{-\frac{tR}{L}} \quad 0 \leq t \leq t_o \text{ (steady - state)}$$

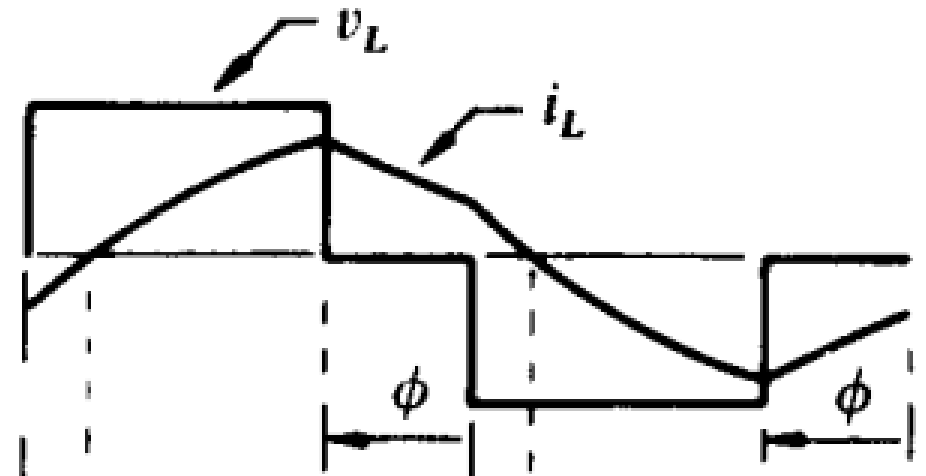
For $V_L = 0$ and $I_2 \geq 0$

$$i_L = I_1 e^{-\frac{tR}{L}} \quad 0 \leq t \leq t_1 - t_o$$

For $V_L < 0$ and $I_2 \geq 0$

$$i_L = -\frac{V_s}{R} + \left(\frac{V_s}{R} + I_2 \right) e^{-\frac{tR}{L}} \quad 0 \leq t \leq t_o$$

$$P_L = \frac{1}{t_1} \int_0^{t_o} V_s i_L(t) dt$$



Single-phase inverter

Example: A single-phase bridge inverter supplies a 10 ohm resistor with inductance 50 mH from a 340 V DC source. If the bridge is operating at 50 Hz, determine the load RMS voltage, the steady-state mean power delivered to the load, and steady-state current waveforms with a quasi-square-wave output with a 50 percent on-time.

Solution:

The quasi-square output voltage has (50 % on-time) a 5 ms on-time, and 5 ms off-time (zero volt), thus $\alpha = \frac{\pi}{2}$.

$$\text{The RMS output voltage} = V_S \sqrt{\left(1 - \frac{\alpha}{\pi}\right)} = 340 \times \sqrt{\left(1 - \frac{\pi/2}{\pi}\right)} = 240.4 \text{ V}$$

Single-phase inverter

Solution:

For first 5 ms on-period when $V_L=340$ V and initially $i_L=0$ A

$$i_L = \frac{V_s}{R} - \left(\frac{V_s}{R} - I_o \right) e^{-\frac{tR}{L}} = \frac{340}{10} - \left(\frac{340}{10} - 0 \right) e^{-\frac{t10}{0.05}} = 34 - 34e^{-200t}$$

And at $t=5$ ms, $i_L=21.5$ A = I_1

For first 5 ms zero-period when $V_L=0$ V

$$i_L = I_1 e^{-\frac{tR}{L}} = 21.5e^{-200t}$$

And at $t=5$ ms, $i_L=7.9$ A

Single-phase inverter

Solution:

Second 5 ms on-period when $V_L = -340$ V

$$i_L = -\frac{V_s}{R} + \left(\frac{V_s}{R} + I_o \right) e^{-\frac{tR}{L}} = -\frac{340}{10} + \left(\frac{340}{10} + 0 \right) e^{-\frac{t10}{0.05}} = -34 + (34 + 7.9)e^{-200t}$$

And at $t=1$ ms, $i_L = -18.6$ A = I_1

For second 5 ms zero-period when $V_L = 0$ V

$$i_L = I_1 e^{-\frac{tR}{L}} = -18.6e^{-200t}$$

And at $t=5$ ms, $i_L = -6.8$ A

Single-phase inverter

Solution:

Second 5 ms on-period when $V_L=340$ V

$$i_L = \frac{V_s}{R} - \left(\frac{V_s}{R} - I_o \right) e^{-\frac{tR}{L}} = \frac{340}{10} - \left(\frac{340}{10} - (-6.8) \right) e^{-\frac{t10}{0.05}} = 34 - (34 + 6.8)e^{-200t}$$

And at $t=0.9$ ms, $i_L=0$ A ending with $i_L = 19$ A

For third 5 ms zero-period when $V_L=0$ V

$$i_L = I_1 e^{-\frac{tR}{L}} = 19e^{-200t}$$

Ending with $i_L=7$ A

Single-phase inverter

Solution:

Fourth 5 ms on-period when $V_L = -340$ V

$$i_L = -\frac{V_s}{R} + \left(\frac{V_s}{R} + I_o \right) e^{-\frac{tR}{L}} = -\frac{340}{10} + \left(\frac{340}{10} + 7 \right) e^{-\frac{t10}{0.05}} = 34 + (34 + 7)e^{-200t}$$

And at $t = 0.9$ ms, $i_L = 0$ A ending with $i_L = -18.9$ A

For fourth 5 ms zero-period when $V_L = 0$ V

$$i_L = I_1 e^{-\frac{tR}{L}} = -18.9e^{-200t}$$

Ending with $i_L = -7$ A

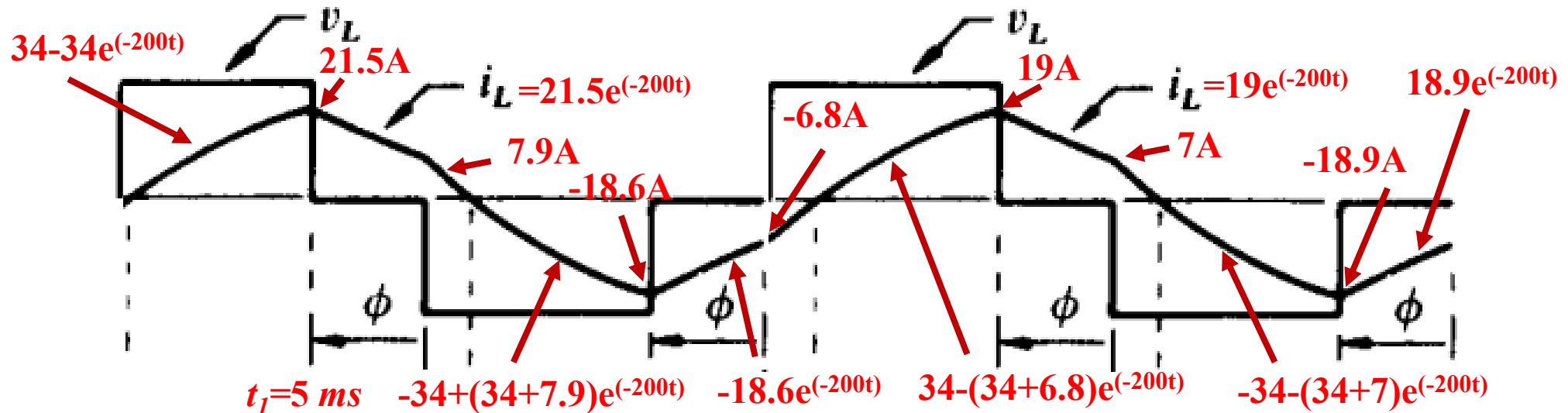
Single-phase inverter

Solution:

The mean load power for the quasi-square wave is given by:

$$P_L = \frac{1}{t_1} \int_0^{t_0} V_S i_L(t) dt = \frac{1}{0.01} \int_0^{0.005} 340 \times [34 - (34 + 7)e^{-200t}] dt$$

$$= 1374 \text{ W}$$



The image shows a long, two-story facade of a building with a series of pointed arches. The upper level features a row of smaller arches, while the lower level has larger, more prominent ones. The walls are covered in intricate, repeating geometric patterns, likely made of brick or stone. The central archway is the largest and is currently dark, suggesting an interior courtyard or a deep shadow. The sky is a clear, pale blue. The text "Thanks for listening!" is centered over the middle of the image in a white, bold, sans-serif font.

Thanks for listening!