

Ex: Let $y = u^{10}$ and $u = x^3 + 7x + 1$, Find $\frac{dy}{dx}$ in two ways.

$$1. \quad \frac{dy}{du} = 10 u^9, \quad \frac{du}{dx} = 3x^2 + 7 \rightarrow \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} = (10 u^9) \cdot (3x^2 + 7)$$

$$\frac{dy}{dx} = [10 (x^3 + 7x + 1)^9] \cdot (3x^2 + 7)$$

$$2. \quad y = (x^3 + 7x + 1)^{10} \rightarrow \frac{dy}{dx} = [10 (x^3 + 7x + 1)^9] \cdot (3x^2 + 7)$$

Ex: Let $y = t^2$ and $x = 5t + 2$ Find $\frac{dy}{dx}$

$$\frac{dy}{dt} = 2t, \quad \frac{dx}{dt} = 5 \rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t}{5} = \frac{2}{5} \left(\frac{x-2}{5} \right)$$

Implicit Differentiation:

Ex: Find $\frac{dy}{dx}$ if $5y^2 + \sin y = x^2$

$$10y \frac{dy}{dx} + \cos y \frac{dy}{dx} = 2x$$

$$(10y + \cos y) \frac{dy}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{2x}{(10y + \cos y)}$$

Ex: find $\frac{dw}{d\lambda}$ if $a^2 w^2 + b^2 \lambda^2 = 1$ where a, b are constants.

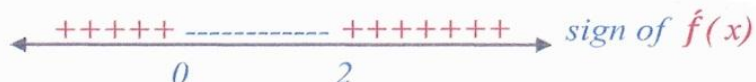
$$2a^2 w \frac{dw}{d\lambda} + 2b^2 \lambda = 0 \rightarrow \frac{dw}{d\lambda} = \frac{-b^2 \lambda}{a^2 w}$$

1.2 Application of Differentiation

Theorem: 1. if $f'(x) > 0$ on (a, b) then f is increase on (a, b) .
2. if $f'(x) < 0$ on (a, b) then f is decrease on (a, b) .

Ex: On which intervals is the function $f(x) = x^3 - 3x^2 + 1$ increase, decrease.

$$\begin{aligned} f'(x) &= 3x^2 - 6x \\ &= 3x(x - 2) = 0 \\ &\quad \downarrow \quad \downarrow \\ &\quad x=0 \quad x=2 \end{aligned}$$



So $f(x)$ is increase on $(-\infty, 0) \cup (2, \infty)$
and decrease on $(0, 2)$

Def.: a function $f(x)$ is concave up on (a, b) if $f''(x) > 0$
and concave down on (a, b) if $f''(x) < 0$

Ex: find the intervals where the fun. $f(x) = x^3 - 3x^2 + 1$ concave up, concave down

$$\begin{aligned} f'(x) &= 3x^2 - 6x \\ f''(x) &= 6x - 6 = 6(x - 1) = 0 \\ &\quad \downarrow \\ &\quad x=1 \end{aligned}$$



f concave up on $(1, \infty)$ and concave down on $(-\infty, 1)$

Def.: Let f be a continuous function on $[a, b]$ and f changes direction of concavity at x_0 then $(x_0, f(x_0))$ called an **inflection point** of f .
or $(x_0, f(x_0))$ is an **inflection point** of f if $f''(x) = 0$

Ex: Find the location of all inflection points of $f(x) = x^4 - 8x^2 + 16$

$$\begin{aligned} f'(x) &= 4x^3 - 16x \\ f''(x) &= 12x^2 - 16 = 4(3x^2 - 4) = 0 \\ &\quad \downarrow \\ &\quad x = \pm \frac{2}{\sqrt{3}} \end{aligned}$$

$(\frac{2}{\sqrt{3}}, 7.1)$ and $(-\frac{2}{\sqrt{3}}, 7.1)$ are inflection points

Ex: Find the intervals in which the function $f(x) = x^{1/3}(x + 4)$

Is increasing, decreasing, concave up, concave down, and inflection point.

$$f(x) = x^{1/3}(x + 4)$$

$$\hat{f}(x) = x^{1/3}(1) + (x + 4)\left(\frac{1}{3}\right)x^{-2/3}$$

$$\begin{aligned}\hat{f}(x) &= x^{1/3} + \frac{(x+4)}{3x^{2/3}} \\ &= \frac{4x+4}{3x^{2/3}} = \frac{4(x+1)}{3x^{2/3}}\end{aligned}$$



f increase on $(-1, 0)$, $(0, \infty)$

f decrease on $(-\infty, -1)$

$$\hat{f}(x) = \frac{4}{3}(x + 1)x^{-2/3}$$

$$\begin{aligned}f''(x) &= \frac{4}{3}\left[(x + 1)\left(-\frac{2}{3}x^{-5/3}\right) + x^{-2/3}\right] \\ &= \frac{4(x-2)}{9x^{5/3}} = 0 \quad \rightarrow \quad x = 2\end{aligned}$$



f concave up on $(-\infty, 0)$, $(2, \infty)$, concave down on $(0, 2)$

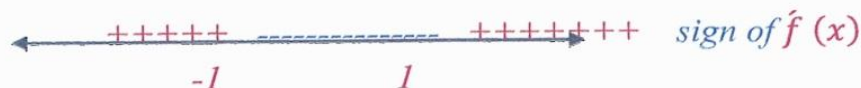
$(2, 7.6)$ is the inflection point

1.3 Sketching Graphs of polynomials and Rational functions

Ex: Sketch the graph of the curve $f(x) = x^3 - 3x + 2$

$$\hat{f}(x) = 3x^2 - 3 = 3(x - 1)(x + 1) = 0$$

$$\begin{array}{cc} \swarrow & \searrow \\ x=1 & x=-1 \end{array}$$



f increase on $(-\infty, -1)$, $(1, \infty)$

f decrease on $(-1, 1)$

$$f(-1) = 4 \quad \rightarrow \quad (-1, 4) \text{ is relative maximum}$$

$$f(1) = 0 \quad \rightarrow \quad (1, 0) \text{ is relative minimum}$$