

Composition of functions

If $f: A \rightarrow B$ and $g: B \rightarrow C$ then $g \circ f(x) = g(f(x)) : A \rightarrow C$

Ex: let $f(x) = \sqrt{x}$ and $g(x) = 3x^2 + 5$

Find $f \circ g$, $g \circ f$, $f \circ f$, $g \circ g$

$$f \circ g(x) = f(g(x)) = f(3x^2 + 5) = \sqrt{3x^2 + 5}$$

$$g \circ f(x) = g(f(x)) = g(\sqrt{x}) = 3(\sqrt{x})^2 + 5 = 3x + 5$$

$$f \circ f(x) = f(f(x)) = f(\sqrt{x}) = \sqrt{\sqrt{x}} = \sqrt[4]{x}$$

$$g \circ g(x) = g(g(x)) = g(3x^2 + 5) = 3(3x^2 + 5)^2 + 5$$

Ex: find $g(x)$ if $f(x) = \sqrt{x+5}$ and $f \circ g(x) = 3|x|$

$$f \circ g(x) = f(g(x))$$

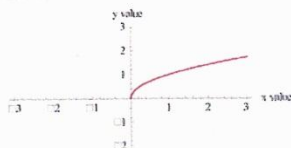
$$3|x| = \sqrt{g(x) + 5}$$

$$9|x^2| = g(x) + 5$$

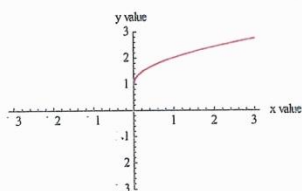
$$\therefore g(x) = 9|x^2| - 5$$

0.3 Shifting graphs

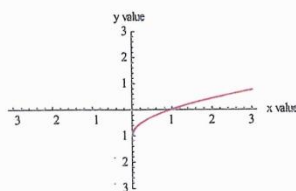
$$f(x) = \sqrt{x}$$



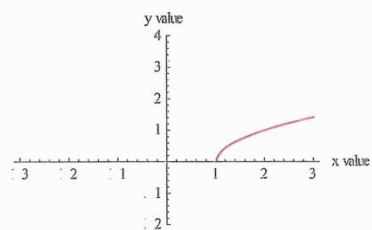
$$1) f(x) = \sqrt{x} + 1$$



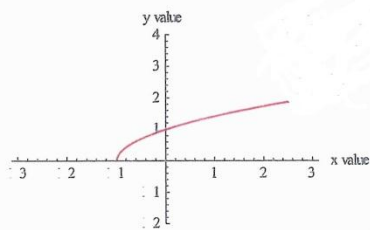
$$2) f(x) = \sqrt{x} - 1$$



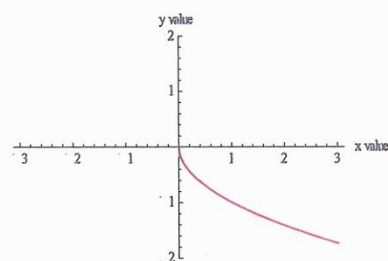
3) $f(x) = \sqrt{x-1}$



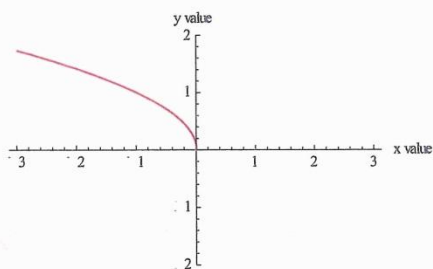
4) $f(x) = \sqrt{x+1}$



5) $f(x) = -\sqrt{x}$



6) $f(x) = \sqrt{-x}$



Ex: Sketch the graph of the function.

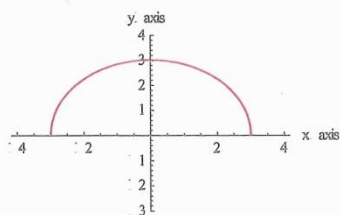
$f(x) = \sqrt{9-x^2}$, find domain and range

$$y = \sqrt{9-x^2}$$

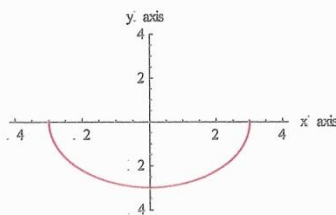
$$y^2 = 9-x^2$$

$$x^2 + y^2 = 9 \quad \text{is a circle of center (0,0) and radius } r = 3$$

$$y = \pm\sqrt{9-x^2}$$



$D_f = [-3, 3]$, $R_f = [0, 3]$

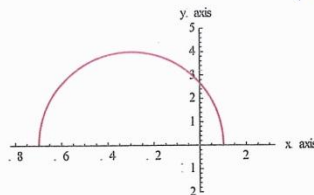


Mathematics I 1st classes

Ex: sketch the graph of $y = \sqrt{7 - 6x - x^2}$

$$\begin{aligned} 7 - 6x - x^2 &= -[x^2 + 6x] + 7 \\ &= -[x^2 + 6x + 9 - 9] + 7 \\ &= -[(x+3)^2 - 9] + 7 \\ &= -(x+3)^2 + 9 + 7 \\ &= 16 - (x+3)^2 \end{aligned}$$

$$\therefore y = \sqrt{16 - (x+3)^2} \quad \text{or} \quad \begin{aligned} y^2 &= 16 - (x+3)^2 \\ (x+3)^2 + y^2 &= 16 \end{aligned}$$

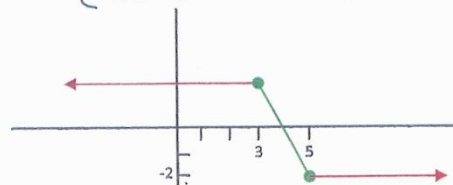
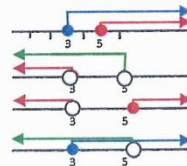


Ex: express the function in piecewise from without using absolute values.

Sketch the graph of

$$f(x) = |x - 5| - |x - 3|$$

$$\begin{aligned} &= \begin{cases} (x-5) - (x-3) & x-5 \geq 0, x-3 \geq 0 \\ -(x-5) + (x-3) & x-5 < 0, x-3 < 0 \\ (x-5) + (x-3) & x-5 \geq 0, x-3 < 0 \\ -(x-5) - (x-3) & x-5 < 0, x-3 \geq 0 \end{cases} \\ &= \begin{cases} -2 & x \geq 5, x \geq 3 \\ 2 & x < 5, x < 3 \\ 2x - 8 & x \geq 5, x < 3 \text{ neglect because no sol.} \\ -2x + 8 & x < 5, x \geq 3 \end{cases} \\ &= \begin{cases} -2 & x \geq 5 \\ 2 & x < 3 \\ -2x + 8 & 3 \leq x < 5 \end{cases} \end{aligned}$$



Hw.:

1) use the graph of $y = |x|$ to sketch the graph the following

a. $y = |x - 4|$

b. $y = |x| + 4$

2) use the graph of $y = \sqrt{x}$ to sketch the graph the following

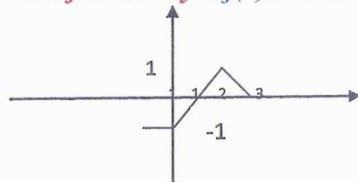
a. $y = \sqrt{x - 3}$

b. $y = \sqrt{x} + 3$

c. $y = \sqrt{x - 3} + 3$

d. $y = \sqrt{x + 1} - 2$

ex: a function $y = f(x)$ has the graph

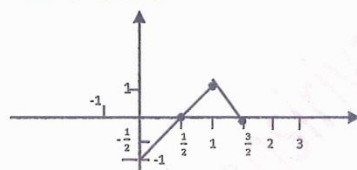


$$D_f = [-1, 3]$$

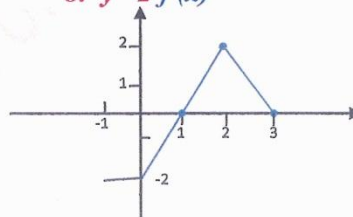
$$R_f = [-1, 1]$$

Use this graph to obtain the graphs of:

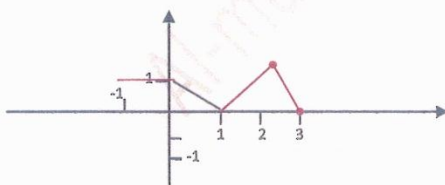
a. $y = f(2x)$



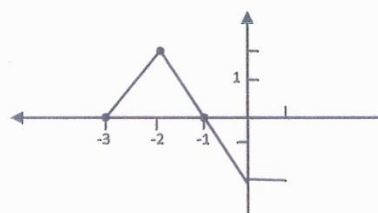
b. $y = 2f(x)$



c. $y = |f(x)|$



d. $y = 2f(-x)$



Def: a function $f(x)$ is said to be even function If $f(-x) = f(x)$ and it is symmetric about y - axis

Mathematics I 1st. classes

Ex: the function $f(x) = x^4 + x^2 - 3$ is even function.

$$\begin{aligned} f(-x) &= (-x)^4 + (-x)^2 - 3 \\ &= x^4 + x^2 - 3 \\ &= f(x) \end{aligned}$$

Since $f(-x) = f(x) \quad \therefore f(x)$ is even fun.

Def: a function $f(x)$ is said to be **odd function** If $f(-x) = -f(x)$ and it is symmetric about the origin

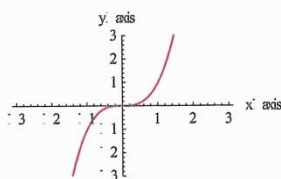
Ex: The function $f(x) = \sin x - 3x^3 + x$ is odd function.

$$\begin{aligned} f(-x) &= \sin(-x) - 3(-x)^3 + (-x) \\ &= -\sin x + 3x^3 - x = -[\sin x - 3x^3 + x] = -f(x) \end{aligned}$$

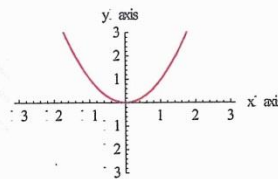
Since $f(-x) = -f(x) \quad \therefore f$ is odd fun.

Ex: $f(x) = x^3$ is odd fun.

Ex: $f(x) = x^2$ is even fun.



Symm. About the origin



Symm. About y - axis

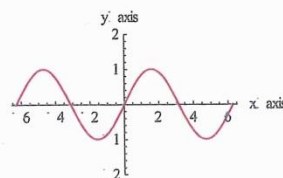
0.4 Trigonometric Functions

1) $y = f(x) = \sin x$

$$D_f = \mathbb{R}$$

$$R_f = [-1, 1]$$

$$\therefore \sin(-x) = -\sin x$$



$\therefore \sin x$ is odd fun. and sym. about the origin

Def: a function $f(x)$ is said to be **periodic** with period p if p is the smallest number such that $f(x + p) = f(x)$.