

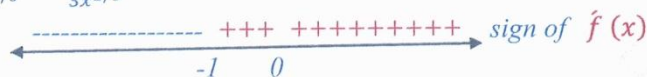
Ex: Find the intervals in which the function $f(x) = x^{1/3}(x+4)$

Is increasing, decreasing, concave up, concave down, and inflection point.

$$f(x) = x^{1/3}(x+4)$$

$$\hat{f}(x) = x^{1/3}(1) + (x+4)\left(\frac{1}{3}\right)x^{-2/3}$$

$$\begin{aligned}\hat{f}(x) &= x^{1/3} + \frac{(x+4)}{3x^{2/3}} \\ &= \frac{4x+4}{3x^{2/3}} = \frac{4(x+1)}{3x^{2/3}}\end{aligned}$$



f increase on $(-1, 0)$, $(0, \infty)$

f decrease on $(-\infty, -1)$

$$\hat{f}(x) = \frac{4}{3}(x+1)x^{-2/3}$$

$$\begin{aligned}f''(x) &= \frac{4}{3}\left[(x+1)\left(\frac{-2}{3}x^{-5/3}\right) + x^{-2/3}\right] \\ &= \frac{4(x-2)}{9x^{5/3}} = 0 \quad \rightarrow \quad x = 2\end{aligned}$$



f concave up on $(-\infty, 0)$, $(2, \infty)$, concave down on $(0, 2)$

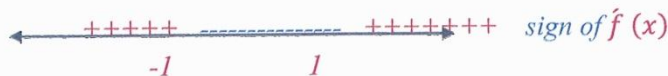
$(2, 7.6)$ is the inflection point

1.3 Sketching Graphs of polynomials and Rational functions

Ex: Sketch the graph of the curve $f(x) = x^3 - 3x + 2$

$$\hat{f}(x) = 3x^2 - 3 = 3(x-1)(x+1) = 0$$

$$\begin{array}{cc} \swarrow & \searrow \\ x=1 & x=-1 \end{array}$$



f increase on $(-\infty, -1)$, $(1, \infty)$

f decrease on $(-1, 1)$

$f(-1) = 4 \quad \rightarrow \quad (-1, 4)$ is relative maximum

$f(1) = 0 \quad \rightarrow \quad (1, 0)$ is relative minimum

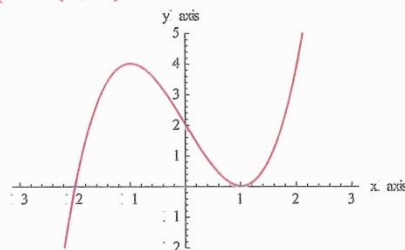
Mathematics I 1st classes

$$f''(x) = 6x = 0 \rightarrow x = 0 \rightarrow (0, 2) \text{ is inflection point}$$

$$\begin{array}{c} \text{-----} \quad \text{++++++} \\ \leftarrow \hspace{1.5cm} \hspace{1.5cm} \rightarrow \end{array} \quad \text{sign of } f''(x)$$

0

f is concave up on $(0, \infty)$ and concave down on $(-\infty, 0)$



Ex: sketch the graph of $f(x) = \frac{x^2}{x^2 - 1}$

$$x^2 - 1 = 0$$

$$(x-1)(x+1) = 0 \rightarrow x = 1 \text{ and } x = -1 \text{ are vertical asymptote.}$$

$$\lim_{x \rightarrow \infty} \frac{x^2}{x^2 - 1} = \lim_{x \rightarrow \infty} \frac{1}{1 - \frac{1}{x^2}} = 1 \quad y = 1 \text{ is Horizontal asymptote}$$

$$f(x) = \frac{x^2}{x^2 - 1}$$

$$f'(x) = \frac{(x^2 - 1) \cdot (2x) - (x^2) \cdot (2x)}{(x^2 - 1)^2} = \frac{-2x}{(x^2 - 1)^2} = 0 \rightarrow x = 0$$

$$\begin{array}{c} \text{++++} \quad \text{++++} \quad \text{-----} \quad \text{-----} \\ \leftarrow \hspace{1.5cm} \hspace{1.5cm} \hspace{1.5cm} \hspace{1.5cm} \rightarrow \end{array} \quad \text{sign of } f'(x)$$

-1 0 1

f increase on $(-\infty, -1)$, $(-1, 0)$

f decrease on $(0, 1)$, $(1, \infty)$

$$f'(x) = \frac{-2x}{(x^2 - 1)^2}$$

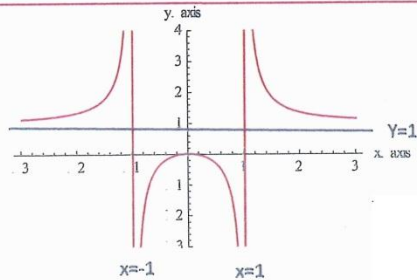
$$f''(x) = \frac{(x^2 - 1)^2(-2) - (-2x)(2((x^2 - 1)(2x)))}{(x^2 - 1)^4} = \frac{6(x^2 + 1)}{(x^2 - 1)^3} \neq 0$$

$$\begin{array}{c} \text{+++++} \quad \text{-----} \quad \text{+++++} \\ \leftarrow \hspace{1.5cm} \hspace{1.5cm} \hspace{1.5cm} \rightarrow \end{array} \quad \text{sign of } f''(x)$$

-1 1

f is concave up on $(-\infty, -1)$, $(1, \infty)$ and concave down on $(-1, 1)$

Mathematics I 1st. classes



Ex: Sketch the graph of the rational function $f(x) = \frac{4-x^3}{x^2}$

$x^2 = 0 \rightarrow x = 0$ is the **vertical asymptote**

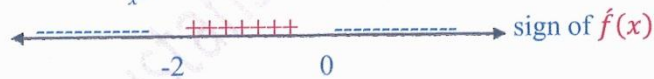
There is **no Horizontal asymptote**

$$f(x) = \frac{4-x^3}{x^2} = -x + \frac{4}{x^2}$$

$y = -x$ is the **oblique asymptote**

$$\begin{array}{r} x^2 \overline{) -x^3 - 4} \\ \underline{-x^3} \\ +4 \end{array}$$

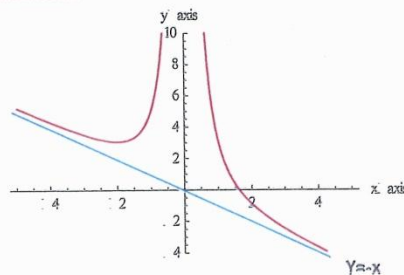
$$\begin{aligned} f'(x) &= \frac{(x^2)(-3x^2) - (4-x^3)(2x)}{x^4} \\ &= \frac{-3x^4 - 8x + 2x^4}{x^4} = \frac{-x^4 - 8x}{x^4} = \frac{-(x^3 + 8)}{x^3} \\ &= \frac{-(x+2)(x^2-2x+4)}{x^3} = 0 \rightarrow x = -2 \end{aligned}$$



f decrease on $(-\infty, -2), (0, \infty)$

f increase on $(-2, 0)$

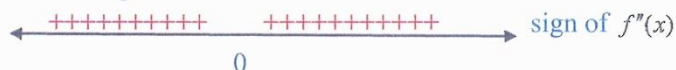
$(-2, 3)$ is relative minimum



$$f'(x) = \frac{-(x^3+8)}{x^3}$$

$$f''(x) = \frac{(x^3)(-3x^2) + (x^3+8)(3x^2)}{x^6} = \frac{24}{x^4} \neq 0$$

There is no inflection point



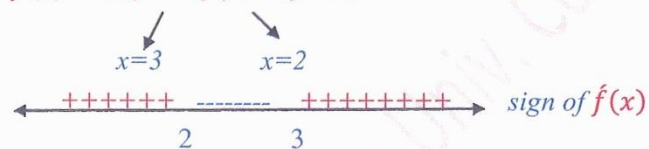
f is concave up on $(-\infty, 0), (0, \infty)$

Ex : Find the absolute max. and absolute min. values of
 $f(x) = 2x^3 - 15x^2 + 36x$ on the interval $[1, 5]$

$$f(x) = 2x^3 - 15x^2 + 36x$$

$$f'(x) = 6x^2 - 30x + 36$$

$$f'(x) = 6(x-3)(x-2) = 0$$



$$f(1)=23$$

$$f(2)=28$$

$$f(3)=27$$

$$f(5)=55$$

$(2, 28)$ is relative max. but $(5, 55)$ is absolute max.

$(3, 27)$ is relative min. but $(1, 23)$ is absolute min.

1.4 Newton method:

used to find the roots of the function $f(x)$
 Numerically.

$$x_{n+1} = x_n - \frac{f(x)}{f'(x)}$$

