

Ex: the function $f(x) = x^4 + x^2 - 3$ is even function.

$$\begin{aligned} f(-x) &= (-x)^4 + (-x)^2 - 3 \\ &= x^4 + x^2 - 3 \\ &= f(x) \end{aligned}$$

Since $f(-x) = f(x) \quad \therefore f(x)$ is even fun.

Def: a function $f(x)$ is said to be **odd function** If $f(-x) = -f(x)$ and it is symmetric about the origin

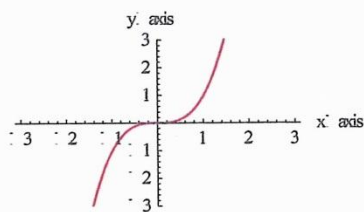
Ex: The function $f(x) = \sin x - 3x^3 + x$ is odd function.

$$\begin{aligned} f(-x) &= \sin(-x) - 3(-x)^3 + (-x) \\ &= -\sin x + 3x^3 - x = -[\sin x - 3x^3 + x] = -f(x) \end{aligned}$$

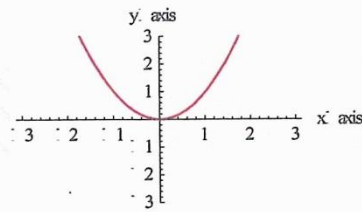
Since $f(-x) = -f(x) \quad \therefore f$ is odd fun.

Ex: $f(x) = x^3$ is odd fun.

Ex: $f(x) = x^2$ is even fun.



Symm. About the origin



Symm. About y - axis

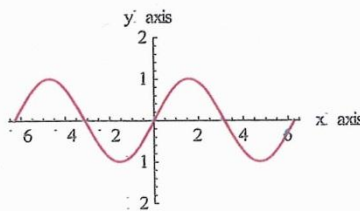
0.4 Trigonometric Functions

1) $y = f(x) = \sin x$

$$D_f = \mathbb{R}$$

$$R_f = [-1, 1]$$

$$\therefore \sin(-x) = -\sin x$$



$\therefore \sin x$ is odd fun. and sym. about the origin

Def: a function $f(x)$ is said to be **periodic** with period p if p is the smallest number such that $f(x + p) = f(x)$.

Ex: $f(x) = \sin x$ is periodic function with period 2π

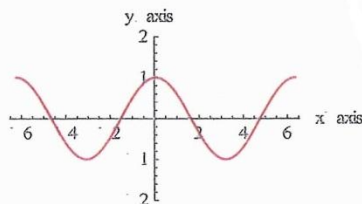
since $\sin(x + 2\pi) = \sin x$

2) $y = f(x) = \cos x$

$D_f = \mathbb{R}$

$R_f = [-1, 1]$

$\therefore \cos(-x) = \cos x$



$\cos x$ is even function and symmetric about the y-axis

Ex: $f(x) = \cos x$ is periodic function with period 2π .

Since $\cos(x + 2\pi) = \cos x$

3) $y = f(x) = \tan x$

$D_f = \mathbb{R} \setminus \{x : x = \frac{n\pi}{2}, n = \pm 1, \pm 3, \pm 5, \dots\}$

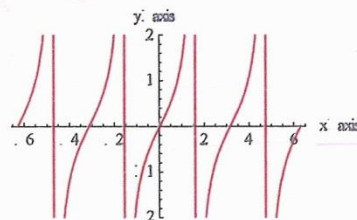
$R_f = \mathbb{R}$

$\therefore \tan(-x) = -\tan x$

$\therefore \tan x$ is odd function and symmetric about the origin

$\therefore \tan(x + \pi) = \tan x$

$\therefore \tan x$ is periodic with period π



4) $y = f(x) = \cot x = \frac{\cos x}{\sin x}$

$D_f = \mathbb{R} \setminus \{x : x = n\pi, n = 0, \pm 1, \pm 2, \dots\}$

$R_f = \mathbb{R}$

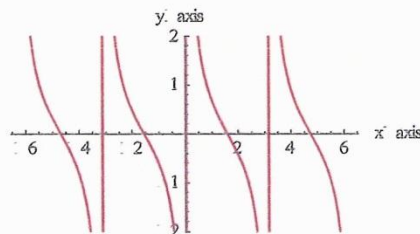
$\therefore \cot(-x) = -\cot x$

$\therefore \cot x$ is **odd** function and

Symmetric about the origin

$\therefore \cot(x + \pi) = \cot x$

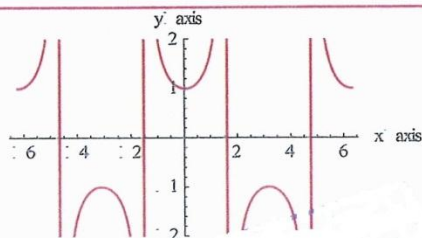
$\therefore \cot x$ is **periodic** with period π .



5) $y = f(x) = \sec x = \frac{1}{\cos x}$

$D_f = \mathbb{R} \setminus \{x : x = \frac{n\pi}{2}, n = \pm 1, \pm 3, \dots\}$

$R_f = \mathbb{R} \setminus (-1, 1)$



$\therefore \sec(-x) = \sec x$

$\therefore \sec x$ is **even** function.

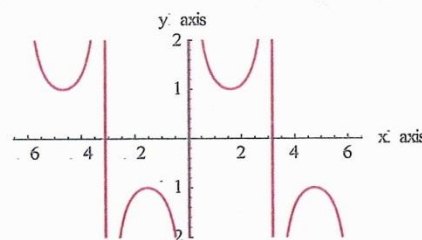
$\therefore \sec(x + 2\pi) = \sec x$

$\therefore \sec x$ is **periodic** function with period 2π

6) $y = f(x) = \csc x = \frac{1}{\sin x}$

$D_f = \mathbb{R} \setminus \{x : x = n\pi, n = 0, \pm 1, \pm 2, \dots\}$

$R_f = \mathbb{R} \setminus (-1, 1)$



$\therefore \csc(-x) = -\csc x$

$\therefore \csc x$ is **odd** function.

$\therefore \csc(x + 2\pi) = \csc x$

$\therefore \csc x$ is **periodic** function with period 2π

Trigonometric identities:

1. $\cos^2 x + \sin^2 x = 1$

2. $\cot^2 x + 1 = \csc^2 x$

3. $1 + \tan^2 x = \sec^2 x$

4. $\sin 2x = 2 \sin x \cos x$

5. $\cos 2x = \cos^2 x - \sin^2 x$

$= 2 \cos^2 x - 1$

$= 1 - 2 \sin^2 x$

6. $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

7. $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

8. $\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

9. $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$$10. \cos^2 x = \frac{1}{2} (1 + \cos 2x)$$

$$11. \sin x \cos y = \frac{1}{2} [\sin (x-y) + \sin (x+y)]$$

$$12. \sin x \sin y = \frac{1}{2} [\cos (x-y) - \cos (x+y)]$$

$$13. \cos x \cos y = \frac{1}{2} [\cos (x-y) + \cos (x+y)]$$

$$14. \cos \left(\frac{\pi}{2} - x \right) = \sin x$$

$$15. \sin \left(\frac{\pi}{2} - x \right) = \cos x$$

$$16. \sin (\pi - x) = \sin x$$

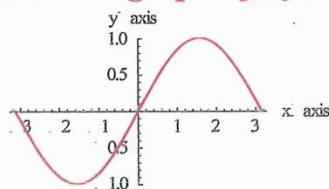
Ex: prove that $\frac{\sin Q + \cos 2Q - 1}{\cos Q - \sin 2Q} = \tan Q$

$$\frac{\sin Q + \cos 2Q - 1}{\cos Q - \sin 2Q} = \frac{\sin Q + (1 - 2 \sin^2 Q) - 1}{\cos Q - 2 \sin Q \cos Q} = \frac{\sin Q (1 - 2 \sin Q)}{\cos Q (1 - 2 \sin Q)} = \frac{\sin Q}{\cos Q} = \tan Q$$

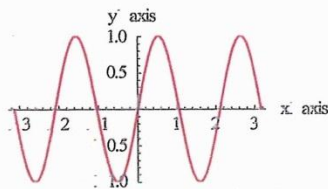
Ex: Prove that $\frac{\sin 2Q}{\sin Q} - \frac{\cos 2Q}{\cos Q} = \sec Q$

$$\begin{aligned} \text{L.H.S.} &= \frac{\cos Q \sin 2Q - \sin Q \cos 2Q}{\sin Q \cos Q} = \frac{\cos Q (2 \sin Q \cos Q) - \sin Q (2 \cos^2 Q - 1)}{\sin Q \cos Q} \\ &= \frac{2 \sin Q \cos^2 Q - 2 \sin Q \cos^2 Q + \sin Q}{\sin Q \cos Q} \\ &= \frac{\sin Q}{\sin Q \cos Q} = \frac{1}{\cos Q} = \sec Q = \text{R.H.S.} \end{aligned}$$

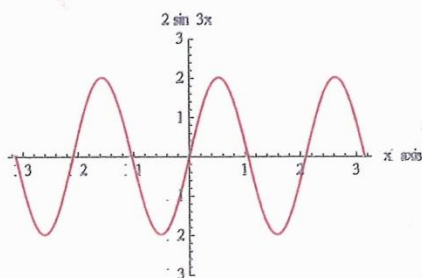
Ex: Sketch the graph of $y = 2 \sin (3x) + 1$



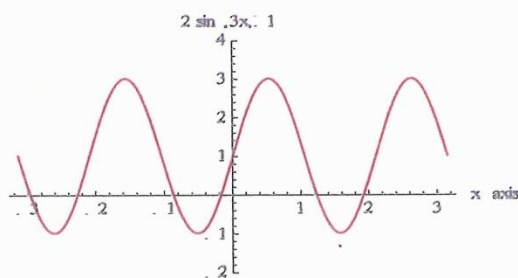
$$y = \sin(x)$$



$$y = \sin(3x)$$



$$y = 2 \sin(3x)$$



$$y = 2 \sin(3x) + 1$$